# Teaching Standard Deviation by Building from Student Invention

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First-year physics laboratories are often driven by a mix of goals that includes the illustration or discovery of basic physics principles and a myriad of technical skills involving specific equipment, data analysis, and report writing. The sheer number of such goals seems guaranteed to produce cognitive overload, even when highly detailed "cookbook" instructions are given. Recent studies indicate that this approach leaves students with a poor conceptual understanding of one of the most important features of laboratory physics and of the real world of science, in general: the development of an understanding of the nature of measurement and its attendant uncertainty.[1] While students might be able to reproduce certain technical manipulations of data, as novice thinkers they lack the mental scaffolding that allows an expert to organize and apply this knowledge.[2,3] Our goal is to put novices on the path to expertise, so that they will be able to transfer their knowledge to novel situations.

One proven method[4] of getting students to explore underlying structure is to have them complete activities as a preparation for future learning. Like the inquiry-based techniques that are being used in a variety of laboratory contexts,[5] these so-called "invention activities" actively engage the students and are intended to stimulate creative thinking. However, they differ from many such methods by being brief, highly structured activities that are intended to precede explicit instruction and reinforcing practice. In these tasks, students are provided with a set of very deliberately selected cases, and their aim is to invent a compact description (typically mathematical) that generalizes across the given cases. They need not identify the correct answer, as the purpose of the exercise is to groom students for future learning. Explicitly, the invention activity facilitates students in detecting important structure in the given cases and in building an organizational scaffolding that prepares them to understand conventional descriptions. Once the activity has been completed, the students can then be told the expert knowledge and then follow up with practice (e.g., the students get a chance to collect and analyze their own data). Studies on the added benefits of the invention-then-telling approach reveal profound differences when students are presented with more expert-like tasks that include learning new related ideas and applying their knowledge to new situations.[6,7]

An effective invention activity has several specific features,[8] of which the three most notable are:

1. ***A clear goal.*** The task should present a clear and challenging goal of developing a compact and consistent description or representation of the fundamental attributes across the cases. The solution usually involves integrating several features into one single representation. Typically, the description integrates several features into one representation, e.g., a ratio.
2. ***Contrasting cases.*** The task should include multiple cases simultaneously. Contrasting cases assist in the development of early knowledge because they help learners to notice new features or structure and to develop new interpretations. Learning to perceive has been described in terms of observing what distinguishes one thing from another,[9] and contrasting cases are a powerful way to help people discern differentiating properties.[10-12] Cases should systematically vary on key parameters so students try to see how these variations relate at a deeper, structural level. A good test of cases is to consider whether the cases are structured so that a reasonable (but incorrect) description based on a subset of them would fail to work for the remainder.
3. ***Student collaboration.*** The task should be done by pairs or groups of students, which carries the advantages of a greater number of ideas and some peer instruction.

One must also pay close attention to: context (the task should involve things relatively familiar and meaningful to the students); level of difficulty (the task should be structured so that students typically achieve partial success, e.g., always capable of getting started but seldom finding the "correct" answer); absence of jargon (the task should be free from subject-specific vernacular, which commonly triggers students to attempt recall of formulae they have already learned rather than inducing a response more closely related to dealing with the development of a new process).

## An example invention activity

Using a diagnostic assessment being developed at UBC, we found that students were not acquiring key conceptual understanding of statistics, measurement, and uncertainty, in accord with related studies.[1] As a consequence, we set out to design a collection of invention activities to produce an improved understanding of key concepts that were creating difficulties. Here we present an example of an invention activity that has been quite successful. The lab was an independent part of the first semester of an introductory calculus-based physics course intended for students planning to take higher level courses in physics and astronomy. The activities described below were used in six sections with about 25 students in each during the fall 2008 semester.

In the first part of the invention activity, the students are provided with four normally distributed data sets and a set of instructions, as follows.

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With fresh water supplies diminishing, the use of flow meters to regulate water consumption will become increasingly important. You are a new technician at a metal working shop which frequently uses fresh water to cool and lubricate several of its machines. In anticipation of upcoming mandatory regulation, you have proposed to your new bosses that the shop determine its current level of water usage. They like your idea so much that they have put you in charge of making this measurement.

In researching this question you quickly learn that flow meters can be very costly to design and build so that they will always give the exact same measurement; however, less expensive ones also exist. These cheaper models do not always give the exact same reading for consecutive measurements, only close to the same. This is okay because your particular application has more generous tolerances —you do not need to break the bank and get the very best device.

You have found four different models (Schwartz Water Flow Meter, MegaBonn 3000, Wiemanator Carlatron, and Jimmy Dees Flomometer), for the same lesser price, that all perform well-enough for your intended application. The manufacturers have each provided data (below) on the flow rate of water (in units of milliliters per second), as measured by their device and through the same standardized test equipment. But a picture is worth a thousand words, and you want to convert these data into a useful graph for easy comparison.

Specifically, you must invent a procedure for graphically representing the water flow data for each of the four devices. There is more than one way to do this, but you have to use the same procedure for each device, so that a fair comparison may be made between graphs. Outline your procedure for converting the data provided below into a useful graphical representation, and show the resulting graph for each data set. The only rule is that the exact same procedure must be used for each device in creating the graphical representation.

The students are then presented with four complete data sets, whose main features are described in Table I.

About 40% of the students created a histogram, with varying bin sizes, to represent this data. Half of the students, not unreasonably, produced a scatter plot of these data, graphing flow rate as a function of measurement number (i.e., 1, 2, ..., 10 or 1, 2, …, 20). The remaining 10% of the students came up with other creative graphical solutions. It is worth noting that most, if not all, of these students would have seen histograms in high school, but 60% of the class did not show any sign of transfer of this knowledge from high school. After about half an hour, sufficient time for all the groups to have started this plotting exercise, the instructor interrupted their work with a short lecture on how an expert would graphically represent this data, as shown in Fig. 1. In other words, the students were told how to create a histogram, along with the associated pros (e.g., easy comparison to the normal curve) and cons (e.g., pitfalls of poorly chosen bin size), but only after they had put substantial thought into what would be needed in a useful graphical representation.

In the second part of the invention activity, the students use the same four normally distributed data sets and receive a new set of instructions, as follows.

Now that you have created a graphical representation of the water flow data, a decision needs to be made concerning which device to purchase. In the interest of being able to best recommend one of these devices over another to your bosses, you have decided to assign a "blue-ribbon factor" to each of these four flow meters. This "blue-ribbon factor" will be a measure of how well the device measures the flow rate of water.

Specifically, you must invent a procedure for computing the "blue-ribbon factor" for each of the devices. There is no single way to do this, but you have to use the same procedure for each device, so that it is a fair comparison between the devices. Write your procedure and the "blue-ribbon factor" you compute for each device using the data provided above and/or the graphical representation provided below. From that, rank the performance of the devices in the order of best to worst. The only rules are that you:

1. Use the data provided from Part 1 and/or their graphical representation in histogram form. Each device always performs reproducibly, so a device only gets a single "blue-ribbon factor." The exact same procedure must be used for each device to determine its "blue-ribbon factor."
2. A small "blue-ribbon factor" implies that the device performs more reliably.

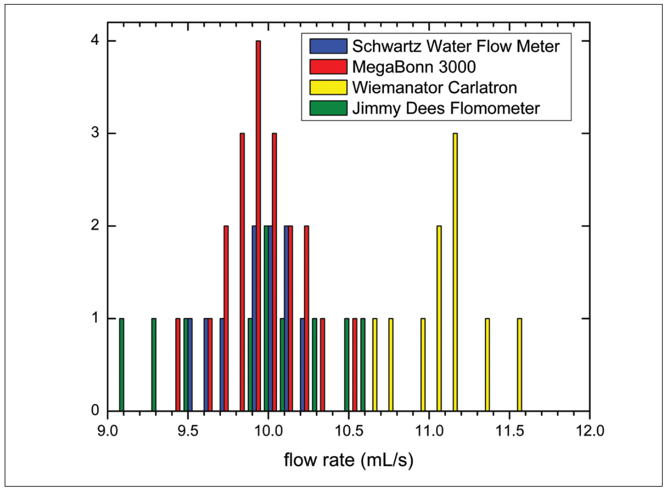


**Fig. 1. An example of the "expert solution" to the problem of creating a graphical representation of the normally distributed data sets.**

**Table I. A summary of the normally distributed data provided to the students. Note that the bottom three data sets vary systematically from the top data set by one feature at a time (first by the # of entries, then by the mean, and finally by the standard deviation).**

| ***Model Name*** | ***# of Entries*** | ***Mean*** | ***Standard Deviation*** |
|---|---|---|---|
| Schwartz Water Flow Meter | 10 | 9.94 mL/s | 0.25 mL/s |
| MegaBonn 3000 | 20 | 9.99 mL/s | 0.25 mL/s |
| Wiemanator Carlatron | 10 | 11.08 mL/s | 0.25 mL/s |
| Jimmy Dees Flomometer | 10 | 9.87 mL/s | 0.50 mL/s |

Fifty-five percent of the students successfully identified all three key features of the distribution (i.e., mean value, spread, and number of data points), with an additional 30% identifying two out of three key features. The student solutions to the second exercise were varied; for example, solutions typically involved combining the number of data points in the set with the mean and the spread of that data in some manner. Many students struggled with the best way to quantitatively describe the spread of the data, and spirited discussions were initiated as the students discussed with their peers the best way forward. Only a third of the students thought to quantify the spread by summing the squares of the differences from the mean; none successfully wrote down the formula for the standard deviation. (Some did know the right buttons to use on their calculator, i.e., to calculate the standard deviation, although none of them were able to describe what their calculator was actually calculating.)

To be clear, we did not expect them to invent for themselves the standard deviation over the course of a one-hour exercise. As before, after about 30 minutes had passed, the instructor interrupted their work with a short lecture on how the expert would tend to quantify this data. In other words, the students were told how to calculate the standard deviation. The instructor also took some time to comment on systematic error, motivated by the yellow data in Fig. 1.

## Measuring transfer

In our course, subsequent transfer assessment tools demonstrate that students are learning, retaining, and reapplying this knowledge. For example, when later presented with an opportunity to create a histogram, but without direct instruction to do so, 85% of the students did indeed represent their data in the form of a histogram (during a lab in which the students study the rate at which beta rays are emitted from a Sr-90 source). When asked to quantify the data in the same beta decay lab with no additional instruction, essentially all of the students calculated the standard deviation. Thus it is clear that these students have not just learned to create histograms and calculate standard deviations, they have developed the much deeper understanding associated with recognizing the underlying purpose of these techniques and when it is suitable to use them.

The activities described above have been designed to prime students' minds for subsequent lectures on creating histograms and calculating standard deviation. In asking students to invent original solutions to novel problems, the activities serve to prepare students to learn, which in turn should help problem solving in the long run. And problem solving is the single most important skill that educators can endeavor to hone in our students. (For example, school superintendents said the one thing that could help students learn would be for them to learn how to make choices. They wanted students to be able to "learn for themselves."[7] As well, employers commonly list problem solving as one of the skills they want from college grads.[13-15])


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