Upper-division students' difficulties with Ampère's law

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This study presents and interprets some conceptual difficulties junior-level physics students experience with Ampère's law. We present both quantitative data, based on students' written responses to conceptual questions, and qualitative data, based on interviews of students solving Ampère's law problems. We find that some students struggle to connect the current enclosed by an Ampèrian loop to the properties of the magnetic field while some students do not use information about the magnetic field to help them solve Ampère's law problems. In this paper, we show how these observations may be interpreted as evidence that some students do not see the integral in Ampère's law as representing a sum and that some students do not use accessible information about the magnetic field as they attempt to solve Ampère's law problems. This work extends previous studies into students' difficulties with Ampère's law and provides possible guidance for instruction.

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I. INTRODUCTION

Ampère's law is one of the fundamental laws of electromagnetism (E&M). In integral form, it is written as

$$\oint \boldsymbol{B} \cdot d\ell = \mu_0 I_{enc}, \tag{1}$$

where $\boldsymbol{B}$ is the magnetic field, μ_0 is the permeability of free space, and $d\ell$ is an infinitesimal length vector of a loop enclosing a current I_{enc}. This law is commonly taught in both introductory physics and upper-division E&M. Yet Ampère's law is conceptually difficult for both introductory physics students [1] and upper-division physics majors [2] (many of whom have seen Ampère's law multiple times). Manogue *et al.* [2] describe many potential difficulties for upper-division students with Ampere's law, although much of their paper is anecdotal. In this paper, we describe our observations of upper-division students' struggles with Ampère's law.

This paper is an outgrowth of a broader effort to transform the teaching of upper-division E&M [3]. Part of our data comes from students' written responses to conceptual Ampère's law questions. One of these questions was administered to students at the end of the semester as part of the Colorado Upper-Division Electrostatics (CUE) assessment [4,5]. The rest were assigned before and after students worked on a tutorial [6] devoted to Ampère's law. These responses provide evidence that upper-division students do struggle with Ampère's law.

To better understand some of the specific mistakes students make, we also examine several videotaped interviews one of us (SVC) conducted as part of the project to transform upper-division E&M. In each interview, a student works on one or more problems while verbalizing his or her reasoning. For this paper, we focus our attention on the interviews of students solving Ampère's law problems.

Like previous studies involving interviews [1,7–9], we limited our investigation to a small number of undergraduate physics students (eight) and four experts (three graduate students and one faculty member). One advantage of this procedure is that we can probe individuals' problem-solving behaviors in detail. This small-scale qualitative approach is complementary to larger, more quantitative measures of students' abilities [4,5]. Given our small number of interviewed students, we cannot say how frequent any of these difficulties with Ampère's law are in the broader population of all upper-division physics students. Instead, we only attempt to describe some of the possible difficulties upper-division physics students can have when applying Ampère's law [10].

In our analysis, we focused on students' explanations and justifications, as in previous studies on Ampère's law [1]. By focusing on students' explanations and justifications we hope to sidestep a key difficulty in studying students' struggles with Ampère's law: Students are often able to simply remember a particular problem and the steps needed to solve it. While Ampère's law represents an important early step in gaining skill with vector calculus, it can only be used to find analytic solutions for an extremely limited set of problems. Many, if not all, of these problems are discussed in detail in E&M textbooks (e.g., Griffiths's *Introduction to Electrodynamics* [11]). Additionally, and as noted elsewhere [1,2,12], solving an Ampère's law problem can be reduced to an algorithmic procedure. Students can use their memories of a particular problem and follow this algorithm without understanding the relevant physics [1]. Simply observing whether or not a student correctly solves an Ampère's law problem may tell us very little about what that student understands. Instead, a better measure of that student's understanding may be achieved by looking at his or her explanations and justifications.

This paper is organized as follows. In Sec. II we describe the course from which we drew our interviewed students. Section III presents data from the pre- and post-tutorial assessments and the CUE. The students and the interviews are described in Sec. IV. We present the difficulties we observed

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in Sec. V. Section VI includes a discussion of our results and their implications for instruction and future studies.

II. COURSE DESCRIPTION

The students participating in this study were all enrolled in their first semester of a two semester, upper-division course sequence on E&M (hereafter known as "junior E&MI"). The majority are from the University of Colorado at Boulder (CU), although in Sec. III we present some data from non-CU students for comparison purposes. At CU, junior E&MI uses Griffiths's textbook [11] and covers electro- and magnetostatics (Griffiths's chapters 1–6) [3,13]. This transformed course includes many research-validated physics education research (PER) techniques typically implemented at the introductory physics level, such as peer instruction [14] and weekly tutorials [6]. In terms of Ampère's law, this course spent five lectures on current distributions, Ampère's law, and the relationship between Ampère's law and the Biot-Savart law. Students had one tutorial and one homework assignment on these topics. All students interviewed for this study were drawn from this transformed version of junior E&MI.

III. INDICATIONS OF A PROBLEM

Do E&MI students struggle with Ampere's Law? The limited literature on this topic indicates the answer is "yes." Manogue *et al.* [2] list several difficulties they observed while teaching E&MI. They note that students may struggle to correctly determine the magnitude and direction of the magnetic field, choose an Ampèrian loop, extract $\boldsymbol{B}$ from inside $\oint \boldsymbol{B}\cdot d\ell$, use curvilinear coordinates, and understand current densities [2]. Our study provides empirical support for some of these difficulties, as we discuss below.

Conceptual assessments developed at CU also indicate that junior E&MI students are not completely facile with Ampère's law. For example, we developed an open-ended conceptual diagnostic—the Colorado Upper-Division Electrostatics assessment, or CUE [5]—in order to document student learning difficulties in this course. The CUE was developed, refined, and validated using think-aloud interviews, faculty feedback, and prior research into common student difficulties [5]. The CUE's grading rubric shows high inter-rater reliability for both overall scores and for scores on individual items [5]. Question 17 on the CUE (Fig. 1) asks students where the magnetic field of an infinite nonmagnetizeable cylinder with a uniform volume current is at its maximum. The answer is that the magnetic field is largest at the edge of the cylinder. A complete explanation includes the following elements: (1) $\oint \boldsymbol{B}\cdot d\ell$ relates the magnetic field and the radius of the Ampèrian loop to I_{enc}, (2) $\oint d\ell$ (the circumference of the Ampèrian loop) increases linearly with the radius of the Ampèrian loop, (3) I_{enc} increases quadratically with the radius of the Ampèrian loop until the radius of the Ampèrian loop equals the radius of the cylinder, (4) I_{enc} reaches its maximum when the radius of the Ampèrian loop equals the radius of the cylinder.

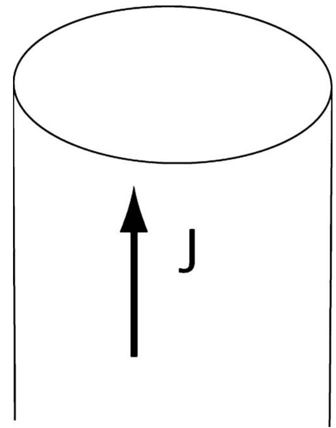

FIG. 1. Question 17 from the CUE [5]: Consider an infinite nonmagnetizeable cylinder with a uniform volume current density J. Where is the $\boldsymbol{B}$-field maximum? Explain how to determine this. The detailed grading rubric for this question assigned two points for correctness and five points for reasoning.

Figure 2 shows the average total score (postinstruction) on this question for students from six different classes. Five were transformed using the research-based course materials developed at CU (see Sec. II)—four at CU (CU1–4) and one at a small liberal arts college (nonCU). These transformed courses are compared to two traditionally taught courses at another large research institution, combined into a single group (Trad). Students in the traditionally taught courses have an average score of only $28 \pm 3\%$ on this problem. While the students in the transformed course outperform those in the traditionally taught courses (averages range from $36 \pm 4\%$ for CU2 to $69 \pm 10\%$ for nonCU), their performances still do not meet faculty expectations. In order to determine the nature of their difficulties, we show the same results broken into two components: The average "correctness" score in each class and the average "reasoning" score in each class (Fig. 3). It is clear that many students in the transformed courses were able to adequately provide a correct answer (i.e., "at the edge of the cylinder"); their low scores on this problem stem from their difficulty in justifying that answer. For example, one CU student answered the question by writing "at surface, use amperes [*sic*] law, great-

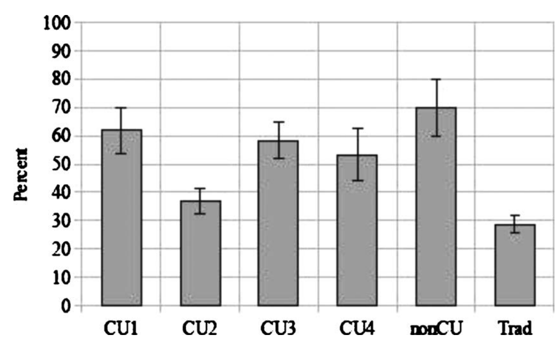

FIG. 2. Average percentage correct on CUE question 17. The number of students in each group who took the CUE postinstruction are as follows: CU1=20, CU2=42, CU3=27, CU4=35, nonCU =31, and Trad=27.

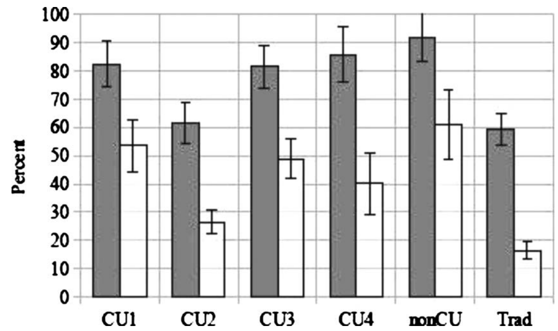

FIG. 3. Average score for both correctness (gray bars) and reasoning (white bars) on CUE question 17.

est I_{enc}." The transformed course emphasized reasoning ability and conceptual understanding, which may result in students' higher scoring on this aspect of the question, but students still do not provide adequate reasoning for their answers. As we discuss in Sec. V below, we observed many instances during individual interviews in which students struggled to justify their answers.

Another indication that junior E&MI students have not mastered Ampère's law comes from a question students answered in an on-line quiz after working on a tutorial (out-of-class worksheet) activity on Ampère's law (Fig. 4). Students were asked to determine which of the given Ampèrian loops are useful for learning something about the magnetic field. The correct answers are loop (a) and loop (b). Students lost points for choosing incorrect loops. After instruction and the tutorial on Ampère's law, students average $23.0 \pm 10\%$ on this question. The most common wrong answer is that both loops (b) and (d) are useful since these are the only loops that enclose any current. This data provide further evidence that, even in the transformed courses which include a substantial conceptual focus, students experience significant conceptual difficulties with Ampère's law.

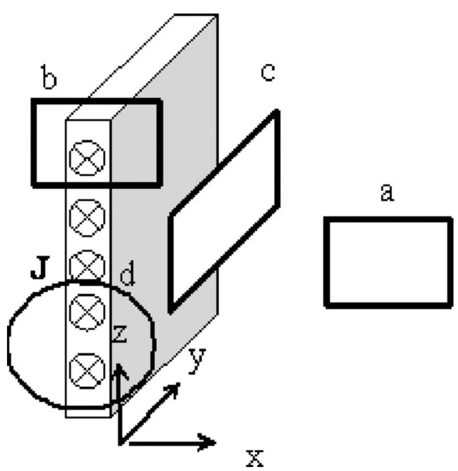

FIG. 4. The post-Ampère's law tutorial question: We have a large (infinite) sheet with a uniform current density J flowing down it. The current runs in the $+y$ direction, as shown. (The sheet is infinite in the y and z directions.) Consider several small Ampèrian loop choices, labeled a-d. List ALL of these which might prove useful in learning something quantitative about $\boldsymbol{B}(x,y,z)$ *somewhere*. For each loop, explain briefly why you did NOT choose it if you didn't, or what it's useful for if you did.

The quantitative data presented here suggests that students struggle with Ampère's law even at the end of their junior E&MI courses and even in courses that implement some research-supported practices. In the following sections, we examine the interviews of a few junior E&MI students in detail to better understand the nature of some of their difficulties.

IV. PARTICIPANTS AND INTERVIEWS

Why do junior E&MI students struggle with Ampère's law? To help answer this question, we look at videotaped interviews of individual students working on Ampère's law problems. One of us (SVC) began interviewing students during the spring 2008 semester as part of the transformation of junior E&MI [3]. Originally, six students were selected to participate in semistructured think-aloud interviews conducted at regular intervals throughout the semester. These six students all volunteered in response to an e-mail sent out to the entire class. They were given a small monetary incentive for their time. These interviews were intended to complement more quantitative assessments of students' difficulties and the effects of the transformed course [4,5].

Each student was interviewed individually and only SVC and the student were present at each interview. During an interview, the student solved one or more junior-level E&M problems while talking through his or her procedures and thought processes. The interviewer asked occasional questions to clarify the student's statements or to provide prompts for him or her to articulate his or her thinking. The problems were drawn from material that had previously been covered in class since the last interview. All interviews were videotaped and SVC wrote field notes after each interview.

In order to establish the external validity (i.e., the generalizability of our results) [15], SVC chose the interviewed students such that they represented a broad range of skills and abilities (based on their performances on the first exam). In order to ensure we had an accurate understanding of what each interviewee was claiming (what Otéro and Harlow [15] call internal validity), SVC continually asked follow-up questions during the interviews. One of the six volunteering students was unable to elucidate his thought processes in sufficient detail to be useful for further study. For this type of research, finding students who can articulate their thinking is important, as noted by previous PER studies [7] and recommendations for qualitative research [15]. The interviews of the remaining five students (hereafter known by their pseudonyms Brian, Camille, Elaine, Mitchell, and Wayne) are included in the observations we report in Sec. V below. Specifically, we focus on the interviews in which these students solve the solenoid problem (see Fig. 5).

Our analysis of the interviews followed an iterative path. The videos of the interviews were reviewed and transcribed. Based on our observations of the tapes and the transcripts, we developed several coding schemes. Our codes were not based on any previous theory, but instead emerged from our data analysis. We applied the coding schemes to the videos

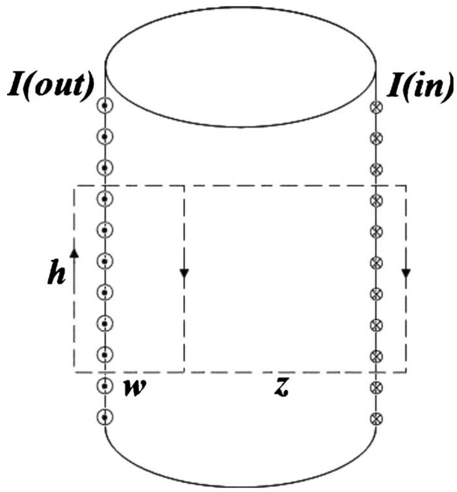

FIG. 5. The solenoid problem: Find the magnetic field of a very long solenoid with n closely spaced turns per unit length on a cylinder of radius R and carrying a steady current I. Adapted from example 5.9 in Griffiths [11]. The Ampèrian loops shown above were not given to students.

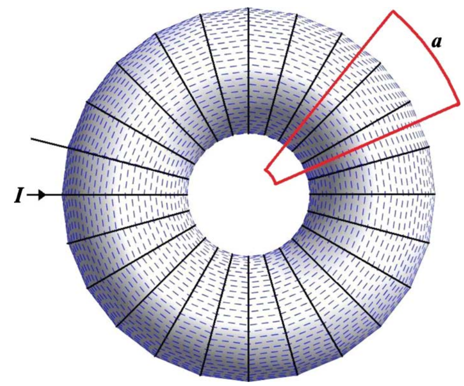

FIG. 6. (Color) The toroid problem: Find the magnetic field of a toroid with a long wire wrapped around it with n closely spaced turns per unit length, a steady current I, and radii as indicated in the figure. Adapted from example 5.10 in Griffiths [11]. The Ampèrian loop a shown above was not given to students.

and transcripts. We rejected or revised any code that was too general to provide us worthwhile details on students comments or too specific to make cross-student comparisons [10,15]. Our goal was to establish the reliability of our conclusions by triangulating our observations and inferences across interviewees. As such, we met to discuss the coding process and to compare our codings. We also presented video clips, transcribed segments, and our interpretations to other physicists and physics education researchers for discussion and feedback. In order to check our codes, we also solicited a new round of volunteers for interviews (whih ultimately led us to further revise our coding scheme to that presented in Sec. V below).

Our second set of interviewed students included three students who were currently enrolled in the spring 2009 version of junior E&MI. At this time we also interviewed four experts (three physics graduate students and one physics faculty member). These experts provided valuable comparisons and contrasts to the interviewed undergraduates. The three new undergraduate students will be referred to by the pseudonyms Alistair, Allison, and Michael. All three were recruited and compensated in the same way as the students in the original round of interviews. The interviews were videotaped and conducted in the same manner as before, except CSW also sat in on the interviews. As before, we asked each student to solve the solenoid problem. To further check the validity of our conclusions, we also asked the students to solve the toroid problem (see Fig. 6). The toroid problem and its solution, unlike the solenoid problem, were not explicitly covered in class (although they are discussed by Griffths [11]). We added the toroid problem under the advice of colleagues who suggested we examine how students approach a problem with which they may not be as familiar. The experts also solved both the solenoid and toroid problems. Data from these new interviews were combined with data from the original five interviews (for a total of eight interviewed junior E&MI students and four experts) to form the *data corpus* which we describe and interpret in Sec. V below.

V. OBSERVED DIFFICULTIES

What difficulties do students experience with Ampère's law in junior E&MI? Below, we list many of the problems we observed during the interviews. These problems can be split into two categories: difficulties connecting I_{enc} to the properties of the magnetic field, and not using information about the magnetic field. As we discuss in Sec. VI, some, but not all, of these observed difficulties match the findings of previous studies [1,2].

A. Incorrect applications of Ampère's law

We observed that some junior E&MI students made incorrect inferences about the properties of the solenoid's magnetic field based on their knowledge of I_{enc}. Specifically,

(i) Some students reason that $I_{enc}=0$ implies $\boldsymbol{B}=0$; and
(ii) Some students claim the magnetic field of a solenoid cannot have a radial component because the problem's solution does not depend on the width of the Ampèrian loop.

As we discuss below, these problems may be interpreted as evidence that students do not think of the integral $\oint \boldsymbol{B}\cdot d\boldsymbol{\ell}$ as representing a sum.

To correctly solve the solenoid problem, one must recognize that there is no magnetic field outside of the solenoid. Some students struggled to justify this statement. For example, both Michael and Mitchell drew an Ampèrian loop like the one shown in Fig. 5 of height h and width $w+z$. They claimed that this loop demonstrated that there is no magnetic field outside the solenoid since $I_{enc}=0$ for this loop. Elaine and Brian both drew Ampèrian loops located entirely outside the solenoid and also claimed that these loops imply $\boldsymbol{B}=0$ since $I_{enc}=0$, as the following dialog from Elaine's interview demonstrates:

Elaine: Uh, you can draw a loop out here [outside the solenoid] and show there's no I enclosed.
Interviewer: mm-hmm

Elaine: And because there's no I enclosed, the $\boldsymbol{B}\cdot d\ell$ is going to be zero.
Interviewer: mm-hmm
Elaine: Um, so $\boldsymbol{B}$'s going to be zero because you've got the length of the loop.

The problem with this argument is that $I_{enc}=0$ does not automatically mean the magnetic field is zero. For example, an Ampèrian loop drawn entirely outside the solenoid could still have $I_{enc}=0$ if there was a constant magnetic field running parallel to the solenoid. $I_{enc}=0$ is a necessary, but not sufficient, condition for the magnetic field to be zero.

What significance can we attribute to this mistake? We can interpret this error as evidence that some students do not view the integral in Ampère's law as representing a sum. One piece of evidence in favor of this interpretation is that students, such as Wayne and Camille, who explicitly wrote $\oint \boldsymbol{B}\cdot d\ell$ as the sum of $\int \boldsymbol{B}\cdot d\ell$ for each side of their Ampèrian loops typically did not make this mistake. Michael is the one exception to this statement. Although Michael did write $\oint \boldsymbol{B}\cdot d\ell$ as the sum of $\int \boldsymbol{B}\cdot d\ell$ for each side of his loop, he forgot a minus sign in his sum. The minus sign is a result of which direction one decides to integrate around the loop. Once Michael realized his mistake, he also realized why his argument for $\boldsymbol{B}=0$ outside of the solenoid was incomplete. In fact, a second piece of evidence in favor of this interpretation is the fact that students stopped arguing $I_{enc}=0$ implies $\boldsymbol{B}=0$ when the interviewer explicitly reminded them that the integral represents a sum. For example, consider the exchange between Brian and the interviewer immediately after Brian claimed $I_{enc}=0$ implies $\boldsymbol{B}=0$:

Interviewer: Okay, so if we have the integral, and again it's a closed integral-
Brian: mm-hmm
Interviewer:—so if we know that the closed integral of $\boldsymbol{B}\cdot d\ell$ equals zero around there, how does that show us that $\boldsymbol{B}$ equals zero?
Brian: Well, because you can still break up the components but, oh, I see what you're saying. Just as I did here [Brian points to a previous solution] I can break it [the closed loop integral] up.

The fact that students ceased arguing that $I_{enc}=0$ implies $\boldsymbol{B}=0$ once they broke $\oint \boldsymbol{B}\cdot d\ell$ into a sum is consistent with our interpretation that they were not originally thinking of the closed loop integral as a sum.

Students' arguments regarding the components of the magnetic field provide another piece of evidence that they do not always view $\oint \boldsymbol{B}\cdot d\ell$ as a sum. The magnetic field of a solenoid has no radial component. One can argue that the magnetic field cannot have a radial component using the Biot-Savart law and the right hand rule or by invoking $\nabla\cdot\boldsymbol{B}=0$. Manogue *et al.* [2] and Griffiths [11] alternatively use a symmetry argument [16]. When asked why the magnetic field of a solenoid has no radial component, five students used one or more of the reasons listed above. The other three junior E&MI students instead offered a different, incorrect rationale.

Camille, Elaine, and Mitchell all argued against a radial component by noting that the problem's solution does not depend on the width of the Ampèrian loop they selected. For example, Camille said:

"I mean, the, with this loop that I drew right here, um, it, it will, no matter how far this extends within the cylinder or within the solenoid, um, it's still going to have the same enclosed current, which means that this integral here is always going to have to be the same."

In other words, one can use an Ampèrian loop of any width w (Fig. 5) and calculate the same value for the solenoid's magnetic field. Based on this fact, Camille, Elaine, and Mitchell concluded that the magnetic field cannot have a radial component. This argument ignores the fact that the closed loop integral is insensitive to any hypothetical radial magnetic field because $\boldsymbol{B}\cdot d\ell$ along the horizontal sides of the loop cancels in the final integral.

Why did Camille, Elaine, and Mitchell all make this mistake? As noted above, one possible interpretation is that, in this context, these students did not think of the closed loop integral as representing a sum. This interpretation is bolstered by the fact that these students abandoned this argument when asked to think about the integral as a sum.

The mistakes cited in this section suggest that some students do not think of the closed loop integral in Ampère's law as representing a sum. In this regard, we may postulate that some students may not be activating a particular problem-solving resource that Sherin calls "parts-of-a-whole" [9]. This term refers to a student's ability to see a whole (e.g. $\oint \boldsymbol{B}\cdot d\ell$) as being composed of many parts (e.g. $\int \boldsymbol{B}\cdot d\ell$ for each side of the loop). The activation (or nonactivation) of this resource has been identified by Meredith and Marrongelle as critical to some students' abilities to successfully solve problems in electromagnetism. However, these are not the only errors students can make when applying Ampère's law. Indeed, Camille, Elaine, and Mitchell's difficulty in justifying why there is no radial magnetic field may have another interpretation. As we discuss next in Sec. V B, this may be an indication that some students do not use accessible information about the magnetic field when applying Ampère's law.

B. Not using information about the magnetic field

Ampère's law problems typically ask students to calculate the magnetic field for a given situation [11]. Yet in order to apply Ampère's law, a student must already know some information about the magnetic field, such as the direction in which it points or where its magnitude is zero [2,11]. This information is needed to select an Ampèrian loop such that the dot product in $\oint \boldsymbol{B}\cdot d\ell$ is easy to evaluate. This information is also needed to evaluate $\oint \boldsymbol{B}\cdot d\ell$. Our observations indicate that students do not always use information about the magnetic field that is accessible to them. One possible manifestation of this issue is students' erroneous argument against a radial component to the magnetic field (to which we provide another explanation in Sec. V A above). Additionally, we also observed during the interviews instances in which students

(i) Did not choose an Ampèrian loop based on the direction in which the magnetic field points; and

(ii) Did not use the fact that the magnetic field is zero in their calculations.

Together, these observations suggest that students do not always use information about the magnetic field when working on Ampère's law problems. We defend this interpretation below.

We first return to Camille, Elaine, and Mitchell's fallacious argument against a radial magnetic field. In Sec. V A, we suggested that this problem could be due to not viewing the closed loop integral in Ampère's law as representing a sum. However, another possible interpretation is that these students were not really thinking about the solenoid's magnetic field when they set up their Ampèrian loops. Both Camille and Elaine drew their Ampèrian loops without giving any indication via words or gestures that they chose their loop based on the direction in which the solenoid's magnetic field points. The fact that all three of these students used their Ampèrian loops to try to make inappropriate conclusions about the magnetic field, plus the fact that at least two of them gave no indication they were even thinking about the magnetic field when they chose their Ampèrian loops, suggests that some students are not using information about the magnetic field to select and use their Ampèrian loops.

Like Camille, Elaine, and Mitchell, Brian appeared to neglect information about the magnetic field when he set up his Ampèrian loop. In fact, he first focused on calculating I_{enc} without first defining an Ampèrian loop. After he wrote down $I_{enc}=nI$ (which he called the "total current"), the interviewer questioned him about what he was calculating:

Interviewer: Total current where?
Brian: Per, oh yeah you're right. Um, that's turns per length…I have to multiply by length to get my total current, so $n\ldots n$ times my unit length times I is my total current.
Interviewer: Again, but total current where? I'm not quite sure where.
Brian: Oh, the current in the problem going around.
Interviewer: Okay, but like, total current for the whole solenoid? Total current? I'm not quite sure where you're calculating this.
Brian: Um, well, I've got one coil current going around, right? And that's just I.
Interviewer: Okay.
Brian: But I have n numbers per length.
Interviewer: Okay.
Brian: So if I multiply n by length then that would just give me turns.
Interviewer: uh-huh
Brian: And the number of turns times I is going to be my total current going around.

Notice that Brian never specified which length he was multiplying by. He first mentions an Ampèrian loop when he turns to evaluating the closed loop integral $\oint \boldsymbol{B}\cdot d\boldsymbol{\ell}$.

Brian: Okay, and…um…I know that there is…we have no current outside the solenoid. $\boldsymbol{B}$'s going to be equal zero. $\boldsymbol{B}$ inside… I'm trying of think of what my $d\ell$ is…is it…can it just be …2…$2\pi R$…$\boldsymbol{B}2\pi R$ equals $\mu_0 I$ through, which is nLI.

Interviewer: So that's I through what?
Brian: Through…my Ampèrian loop.

The interviewer then asks Brian to sketch his loop. He draws a loop similar to the one in Fig. 6 and quickly realizes that it is inconsistent with his statement that $\oint \boldsymbol{B}\cdot d\boldsymbol{\ell}=\boldsymbol{B}2\pi R$:

"And, which are already, I hope that's right, I think it's nLI, and then my Ampèrian loop would be…this. You're right, it's not $2\pi R$, it's not this anymore. Dang. [Brian erases $2\pi R$.] I'm not sure what to do…it's going to be something like that though. I know that $\boldsymbol{B}$'s going to be $\mu_0 nLI$ over, like, $2s$ or something like that or s."

Note that when Brian mentions "s" he appears to be talking about the radial coordinate in a cylindrical coordinate system. However, we cannot say for sure exactly what Brian meant by "s" since he did not explicity choose a coordinate system. In fact, as the above quotations demonstrate, Brian attempted to solve the problem without first establishing an Ampèrian loop. Even though he eventually drew a useful loop for solving the problem, the fact that he struggled with how to use the loop to evaluate $\oint \boldsymbol{B}\cdot d\boldsymbol{\ell}$ suggests that he did not select this loop using information about the magnetic field.

We find further evidence in support of this interpretation in Michael and Alistair's interviews. Neither referred to the magnetic field when they chose their loops for either the solenoid or the toroid problems. Only after they selected their loops did they comment on the magnetic field. Michael, for instance, drew the Ampèrian loop a shown in Fig. 6 and claimed it was a good loop to use for the toroid problem because the loop is "both inside and outside" the toroid, like the loop he used for the solenoid problem. However, he never referenced the magnetic field when talking about why he chose any of his loops. Alistair likewise did not select an Ampèrian loop based on the direction of the magnetic field. Instead, he focused on drawing a loop that "encloses current." This reasoning echoes that of the students whose CUE and tutorial pre- and post-test results we discussed in Sec. III. When pressed by the interviewer to explain how the magnetic field affects his choice of loop for the toroid problem, Alistair replied "'cause you want to find the $\boldsymbol{B}$ at a specific r?" When asked to elaborate, Alistair said "if we changed r [the radius of his Ampèrian loop] we would be averaging the $\boldsymbol{B}$-field and we'd lose information." This was the closest Alistair got to stating that one should choose a loop in order to make the dot product in Ampère's law easy to evaluate. Alistair and Michael's interviews, combined with the interviews of Camille, Elaine, and Mitchell, provide evidence that some junior E&MI students are not using accessible information about the magnetic field to set up their Ampèrian loops.

This approach is in stark contrast to that of the experts. For example, one of the first steps the faculty member took was to visualize the magnetic field (this example is taken from when he solved the toroid problem):

"I was going to say, you know, uh, one of the first things about this is, uh, knowing something about the $\boldsymbol{B}$-field direction. So, um, this is one of those situa-

> tions, so, uh, I'll set that one up there [he moves the paper with the question prompt away from his white board]. And again, uh, for myself, what I'll tend to do on these geometries is make sure that I know something about, uh, about the directions of the currents and the directions of the $\boldsymbol{B}$-fields."

Note that this professor was not simply recalling an algorithm for producing a solution—he explicitly stated that he did not know the solution from memory and that the toroid problem was a "true problem" for him. Two of the three graduate students also began solving the Ampère's law problems by discussing the direction in which the magnetic field points and where it is zero before they set up their Ampèrian loops.

Finally, some students may not think about what they know about the magnetic field even as they try to calculate the magnetic field. For example, when Elaine was evaluating and adding together $\boldsymbol{B}\cdot d\ell$ for each side of the Ampèrian loop she used in the solenoid problem, she claimed that, for both sides of height h in Fig. 5, $\int \boldsymbol{B}\cdot d\ell = BL$, where L represents the length of the loop. Consequently, her answer was off by a factor of two since $\int \boldsymbol{B}\cdot d\ell = 0$ for one side since the magnetic field is zero outside the solenoid. This mistake is interesting, because Elaine stated the magnetic field was zero outside the solenoid, although she did not use this piece of information until prompted by the interviewer.

We observed many cases in which students correctly stated in which direction a magnetic field pointed and where it is zero. This information is useful because it helps one select and use an Ampèrian loop, which in turn helps one evaluate both the left and right hand sides of Ampère's law. Yet we saw some junior E&MI students who did not use information they knew about the magnetic field at appropriate junctures in their problem solving procedures. This is in contrast to the experts we interviewed. We interpret this data as evidence that some junior E&MI students do not think about the magnetic field and its properties as they attempt to construct solutions to Ampère's law problems.

VI. DISCUSSION

We presented both quantitative and qualitative data regarding students' difficulties with Ampère's law. We also provided plausible interpretations for these difficulties. This study complements previous papers on students' difficulties with Ampère's law. On the one hand, we provide empirical support for some of Manogue *et al.*'s [2] claims (which they state are not research-based); on the other hand, we extend into the junior E&MI classroom the work of Guisasola *et al.* [1] on introductory physics students' difficulties with Ampère's law.

How does our work support Manogue *et al.*'s claims? First, we observed students who had trouble justifying why the magnetic field of a solenoid does not have a radial component. This observation agrees with Manogue *et al.*'s assertion that students might struggle determine the direction in which the magnetic field points and the variables on which its magnitude depends [2]. Second, Manogue *et al.* also claim that students might not visualize the integral in Ampère's law as a sum. This is our interpretation of some of the difficulties we observed, although we must note that the sum we consider for the solenoid problem only has four parts whereas Manogue *et al.* are thinking about the more general case of infinite sums [2]. Finally, we agree with Manogue *et al.* that students may encounter problems in choosing and using their Ampèrian loops. However, Manogue *et al.* claim that the primary mistake students make is choosing a loop based on a curve that already exists in the problem [2]. In contrast, we observed students who selected their loops based solely on whether or not it "enclosed current." Some students did not articulate why or how the magnetic field should influence one's choice of a loop. Nevertheless, our observations support many of Manogue *et al.*'s claims.

Note that there was one potential problem highlighted by Manogue *et al.* that we did not see. They claim that students have trouble evaluating the current term that appears in Ampère's law, especially since (depending on the situation) they may have to deal with a line, surface, or volume current. We did not see this difficulty, but this may be due to the fact that we gave them information about the magnitude and direction of the currents in the solenoid and toroid problems (see Figs. 5 and 6, respectively). Additionally, our problems did not have much variety in the nature of the current distributions. Our study is therefore unable to tell us if our students do have issues regarding line, surface, and volume currents.

Our work also complements Guisasola *et al.*'s [1] study of introductory physics students' conceptual difficulties with Ampère's law. They too examined students' justifications and explanations and found that many students give erroneous justifications and explanations or even no justifications or explanations, even when they chose the correct answer [1]. For example, Guisasola *et al.* found that some introductory physics students argue that when $I_{enc}=0$ the magnetic field must also be zero [1]. Our study shows that junior E&MI students make the same mistake. The fact that junior physics students make some of the same errors as introductory physics students is perhaps not surprising: A recent longitudinal study found that junior-level E&M courses may have little affect on students' conceptual knowledge of E&M [13]. Guisasola *et al.* note that at the introductory level, students may not consider all the variables in a problem, memorize algorithms without understanding their underlying logic, and overly simplify cause and effect relationships [1]. If left unaddressed, these problems might manifest themselves at the junior level, where students have the additional burden of rigorously applying the formalism of vector calculus to physical situations for the first time.

A student's struggles with Ampère's law may also tell us more than simply whether or not she understands Ampère's law. As Manogue *et al.* remark [2], "you have to be able to think like a physicist to do these problems." Ampère's law problems can promote many reasoning abilities that are valued by professional physicists, such as visualizing the physical situation of the problem, connecting mathematics and physics, and thinking metacognitively about what one does and does not understand. These reasoning abilities are taken from a list of learning goals from junior E&MI generated by both physics education research (PER) and non-PER faculty as part of the effort to transform how junior E&MI is taught [3–6]. Students' struggles with Ampère's law may be the "canary in the mine" that signals problems with students' masteries of these broader learning goals.

We see two fruitful avenues for future work. First, researchers may want to investigate in more detail students' struggles with the broader learning goals we mentioned above. During the process of coding and analyzing the interview data, we thought we saw distinct differences in how the students used mathematics [8,9], framed the nature of problem solving [17], and thought metacognitively [18]. However, we found our sample of students and problems too limited to adequately address any of these larger issues. Second, physicists may wish to devote more time to understanding students' difficulties with Ampère's law and developing resources (such as tutorials [6]) to help students overcome the difficulties presented here and in previous studies [1,2]. Such studies may also be informed by research into Gauss's law [1,19], with which Ampère's law has many similarities. Ambitious studies may focus on all of these issues, since all are likely connected to success with Ampère's law in particular and with physics in general.

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