The Mathematics Attitudes and Perceptions Survey: an Instrument to Assess Expert-like Views and Dispositions Among Undergraduate Mathematics Students.

Abstract

One goal of an undergraduate education in mathematics is to help students develop a productive disposition towards mathematics. A way of conceiving of this is as helping mathematical novices transition to more expert-like perceptions of mathematics. This conceptualization creates a need for a way to characterise students' perceptions of mathematics in authentic educational settings. This article presents a survey, the Mathematics Attitudes and Perceptions Survey (MAPS), designed to address this need. We present the development of the MAPS instrument and its validation on a large (N = 3411) set of student data. Results from various MAPS implementations corroborate results from analogous instruments in other STEM disciplines. We present these results and highlight some in particular: MAPS scores correlate with course grades; students tend to move away from expert-like orientations over a semester or year of taking a mathematics course; and, interactive-engagement type lectures have less of a negative impact, but no positive impact, on students' overall orientations than traditional lecturing. We include the MAPS instrument in this article and suggest ways in which it may deepen our understanding of undergraduate mathematics education.

Keywords: Undergraduate mathematics education, attitudes towards mathematics, perceptions of mathematics.

Introduction

One primary goal of an undergraduate education in mathematics is to transition students to more expert-like conceptions of and ways of thinking about mathematics. This is more than learning the requisite subject content knowledge—students should gain domain-specific ways of thinking and an appreciation for the place of the subject relative to other academic pursuits. Schoenfeld [1, p. 341] states that "people develop their understandings of any enterprise from their participation in the "community of practice" within which that enterprise is practiced." Essentially, being in a mathematics course and working with mathematics and mathematicians serves as an apprenticeship in mathematics. The social and sociomathematical norms of the discipline are seldom taught explicitly but are nonetheless experienced by the students. It is through this context that students develop their mathematical understandings, perspectives, and ways of communicating. It is then desirable to be able to recognize key perspectives, attitudes, emotions, and epistemological and ontological beliefs—which we refer to collectively as *beliefs*—that affect students' development in mathematics and to identify educational events that impact these factors. These beliefs are significant in their own right, and also can have a direct impact on students' learning.[2]

Measures of Student Beliefs

There has been, and continues to be, great interest by researchers in analysing students' beliefs and perspectives and attitudes towards mathematics. For example, Petocz et al. [3] surveyed approximately 1,200 undergraduate students from five countries about their views of mathematics and how it fits in with their future study and career plans. The authors found students' responses were best characterized with a five-level hierarchical framework. The lowest level, *Numbers*, centred around viewing mathematics as concerning number and calculation, whereas the highest level, *Life*, conceives of mathematics as being a way of thinking integral to living in the world.

In addition to open-ended surveys, instruments have been developed to assess various aspects of students' relationship with mathematics, including attitudes toward, beliefs about, and anxiety when solving or anticipating a mathematical task.[4,5,6,7] An example of such an instrument, the Conceptions of Mathematics Survey (CMQ), was developed from qualitative studies of how students view mathematics as a discipline.[8,9,10] Respondents are given scores on two scales: fragmented and cohesive. Those students with a fragmented conception of mathematics view the subject as a collection of disconnected procedures and facts, while those with a cohesive conceptions tend to view math as an interconnected web of knowledge. Cohesive conceptions tend to be positively correlated with course grade, while fragmented scores are often negatively correlated, highlighting the importance of students' views of a subject in relation to their academic performance.[9,10,11]

The above surveys share two limiting characteristics: unidimensionality and an absence of input from mathematicians in the design and subsequent results. Unidimensionality necessarily limits the range of perspectives intended to be considered in the survey and may lead to confounded results. For example, mathematicians would presumably score highly on the cohesive scale of the CMQ and low on the fragmented scale, though, as identified in [12], an expert number theorist may agree with a statement such as "[f]or me, math is the study of numbers," which is intended to contribute to the fragmented scale, but experts in other domains may disagree.

Data from mathematicians would enable an instrument to be used in expert-novice studies and can provide insight into exactly what orientations to mathematics are held by mathematicians. The CMQ, for example, can provide some insight into how a student views mathematics, but does not necessarily indicate how close this view is to a practising mathematician's. Conversely, when asked to complete the CMQ as they think their archetypal students would, mathematicians tend to underestimate their students' cohesive conceptions and overestimate their fragmented conceptions.[12] This identifies a disconnect between mathematicians and their students and highlights the need for an instrument that will consider both their views concurrently.

Measures of teacher beliefs

Not much is known about university instructors' views of mathematics and how these views influence teaching practice. There is, however, a large body of literature on the influence of primary and secondary teachers' views on their practice. It is known that a teacher's belief about the nature of mathematics impacts the way mathematics is presented and taught.[13,14,15,16] Teachers' beliefs about mathematics and mathematical activity has a direct relationship with the sociomathematical norms of the classroom,[17] teachers' practices and student learning,[18] and teachers' goals for learning.[19]

A number of teacher belief surveys have been constructed; for example the *Beliefs About Mathematics and Teaching* [20] and the *Mathematics Teaching Efficacy Beliefs Instrument*.[21]

Noticeably absent from these studies are the students' perspectives. If beliefs about mathematics affect a teacher's practice, do these benefit those students with similar beliefs? Are those students with different beliefs disadvantaged?

Novice-expert studies

The studies cited above explore students' and teachers' perspectives of mathematics separately. If we start with the assumption that an undergraduate education is intended, at least in part, to develop in students more expert-like perspectives toward a discipline, then there is a need to not only identify student and teacher/instructor perspectives, but also how these are positioned relative to each other. It is known, for example, that mathematical novices and experts approach mathematical tasks in fundamentally different ways.[22,23,24] These approaches are informed by a number of factors in addition to and distinct from knowledge gained from experience with mathematical situations, including beliefs about the nature of mathematical activity. This suggests that an improvement in students' expert-like behaviour in mathematics may follow a shift in their views of mathematics.

In intervention studies, or studies of curriculum change in general, there is a need to assess the students' conceptions of the field to determine if these shift towards those of experts. One approach to this is to quantify students' conceptions and contrast these results to those of experts in the field. This has been performed in a number of settings through, for example, asking "what is mathematics?",[25] identifying a *desired direction of change*,[26] or the creation of a beliefs instrument.[27] Taken together, these studies acknowledge that an expert-like perspective is not a singular trait, identifying a need for a multi-dimensional novice/expert instrument.

Perhaps the most notable expert/novice instruments in undergraduate STEM education is the family of Colorado Learning Attitudes about Science Surveys (CLASS). Initially developed for physics,[28,29] CLASS surveys have extended to biology,[30] chemistry,[31] earth and ocean sciences,[32] and computer science.[33] Each of these surveys are multidimensional, being comprised of a number subscales that quantify aspects of expert-like perspectives. The questions for the original physics CLASS were distilled from interviews with physics students and instructors and the resulting category structures were developed with rigorous statistical methods.[28,34] The survey was completed by physicists and each question was checked for expert consensus. Student responses are marked relative to this consensus: +1 for a response in the same direction (ie. agree/disagree) as the expert consensus and 0 otherwise. These scores constitute the students' expert-like orientations in each of the subscales and an overall expertise index. Similar approaches were employed in the development of the subsequent CLASS surveys.

Results established through the use of CLASS surveys include demonstrating correlations between expert-like beliefs on the physics CLASS and self-rated interest in physics, variation in responses according to degree year, intra-year shifts away from expert-like conceptions, and program-level selection of students with expert-like orientations.[28,29,30,35,36,37]

The Current Study

Previous work has shown that expert/novice instruments must be made domain-specific. If the questions are too general—asking about overall perceptions of science, for example—

students are unable to commit to an answer. For example, Adams, et al. [28] report that, when asked “Understanding science basically means being able to recall something you’ve read or been shown,” students tend to respond with, “it depends on whether you mean biology or physics.” Therefore, the existing CLASS survey are not readily applicable to undergraduate mathematics.

The current article presents an adaptation of the CLASS to undergraduate mathematics: the Mathematics Attitudes and Perceptions Survey (MAPS). We begin by presenting the categories that emerged through exploratory and confirmatory factor analyses of a large MAPS data set. We then review the literature supporting the educational relevance of the categories identified in our MAPS instrument. Next, we discuss the development of MAPS and present some initial observations on the relationship between MAPS scores and course grades. We conclude by discussing ways in which MAPS can help uncover and improve students' experiences in mathematics courses.

MAPS Categories

Expert-like behaviour is multifaceted and complex, making a succinct description of it a difficult task. As we present in our Methods section, seven factors of expert-like behaviour in and views of mathematics have emerged from our development and testing of the MAPS instrument: 1) confidence in, and attitudes towards mathematics (Confidence), 2) persistence in problem solving (Problem Solving), 3) a belief about whether mathematical ability is static or developed (Growth Mindset), 4) motivation and interest in studying mathematics (Interest), 5) views on the applicability of mathematics to everyday life (Real World), 6) learning mathematics for understanding (Sense Making), and 7) the nature of answers to mathematical problems (Answers). We freely admit that this list is not exhaustive, nor will or should it be. We do, however, think that these seven factors capture a representative picture of expert-like approaches to and views of mathematics. What follows is a brief review of the literature concerning each of our factors.

Confidence in mathematics

Representative statement (MAPS #17): "No matter how much I prepare, I am still not confident when taking math tests."

Confidence in mathematics is a person's perceived ability to successfully engage in mathematical tasks. Confidence is known to affect a student's willingness to engage with a task, the effort they expend in working the task, and the degree to which they persist when encountering setbacks.[38] Self-reported confidence level can also aid in identifying understanding and misconceptions. Hasan et al. [39] report on a study in which students recorded their confidence in their responses to multiple choice physics questions. The resulting Certainty of Response Index (CRI) scores were compared to question correctness. Those questions with a high CRI and high average correctness are likely to be widely understood by the class, whereas those with a high CRI and low average correctness indicate widespread misconceptions; see also [40] for a refinement of the approach in [39]. The approach of having students report their confidence in their responses has proven useful in understanding the complex relationship between performance on conceptual and procedural mathematical tasks: the students in the study of Englebrecht et al. [41] had no more misconceptions about concepts than they did about procedures.

In a study of first-year engineering students, a regression analysis of student performance revealed a significant correlation between confidence level and course grade.[42] A similar result was obtained in [43] for achievement and mathematics self-efficacy—a term often used interchangeably with confidence. Perhaps the most noteworthy result in [43] is that confidence was a better predictor of continuation in a mathematics-intensive program than either performance or achievement. At the same time, achievement can have a positive influence on confidence. This creates a complex interplay between confidence and achievement: higher achievement leads to greater confidence which leads to affording more opportunities to achieve. Confidence is therefore a dynamic, rather than static trait that is shaped and influenced by the educational setting.

Persistence in Problem Solving

Representative statement (MAPS #24): “If I get stuck on a math problem, there is no chance that I will figure it out on my own.”

How students approach solving a non-routine mathematical problem (i.e., one where they can “get stuck”) is just as important as their ability to solve that problem. It is now well established that experts and novices differ in how they solve problems. Experts have a wealth of knowledge—in terms of knowledge of facts and definitions, but also of problem types and solution strategies—and this aids in their problem solving. Experts also attend to different features of problems than novices. In a replication of the seminal work of Chi, Feltovich, & Glasser,[44] Schoenfeld and Herrmann [22] had mathematics problem solvers, both expert and novice, group problems from a given set according to their perceived similarity. Experts grouped the problems according to their *deep structure*, that is, according to the underlying principles needed to solve them. Novices tended to group the problems according to their *surface structure*, concerning superficial features of the problem setup. Moreover, experts engage metacognitive skills while solving problems, monitoring their own progress, looking for relevant choices among their broad set of known solution strategies, and willing to abandon strategies when they are judged to be no longer applicable. Lacking these various aspects of expertise, novices are more likely to attend to surface features and thus identify inappropriate strategies, then apply those inappropriate strategies with little or no reflection, and (prematurely) feel that their options are exhausted.[1] We may thus consider perseverance in problem solving as a relatively distinct from issues of anxiety or laziness and more in terms of the ability to select appropriately from a sufficiently large set of strategies and to continue selecting and attempting strategies based on one's progress.

Growth Mindset

Representative statement (MAPS #5): "Math ability is something about a person that cannot be changed very much."

This category rates students' belief about whether mathematical ability is innate or can be developed. Those with a *fixed mindset* believe that ability is not learned, rather being an intrinsic property of the person. This fixed ability changes little, if at all, even in educational settings. A *growth mindset*, on the other hand, recognizes that abilities are not innate and can be acquired and improved consciously and effortfully. The current consensus in various scientific disciplines is that intellectual ability is not fixed and can be developed, even in the case of those with extreme ability.[45]

Though the concept of a fixed/growth mindset has been present in the educational literature for decades,[46] it was the popular account of Dweck [47] that brought the effect of mindset on educational outcomes to the fore. Dweck [47] estimates that 40% of primary and secondary school students in the United States have a fixed mindset, 40% have a growth mindset, and 20% have a mixed perspective. Mindset, both the teacher's and students', is known to have a profound influence on educational achievement. Students with a fixed mindset tend to acquiesce after experiencing setbacks. They disengage from learning with an acknowledgement that their efforts will not produce results. Growth mindset students, however, are more willing to redouble their efforts in the face of a challenge; they believe that underachievement can be rectified with greater effort.

Teacher mindset is known to affect student achievement through the implicit or explicit structuring of the educational setting. Perhaps the most visible manifestation of teacher mindset is in the practice of "ability grouping". Students are identified as having a high or low ability and segregated accordingly, often physically separated. The teacher then creates differentiated educational tasks for the groups. This practice is not intrinsically detrimental—perhaps the groups differ in their prior educational experiences and have different aggregate skill sets—but in practice students perceive ability grouping as separating the "smart" from the "dumb".[48] This grouping not only negatively affects the lower ability group, but can hinder the progress of the higher ability students.[48,49,50,51]

Research on mindset has been extensively conducted in mathematics education; see [52,53] for brief reviews. Most importantly, interventions intended to shift mindset from fixed to growth have shown to be effective in mathematics education.[52,53]

Interest in mathematics

Representative statement (MAPS #32): "I only learn math when it is required."

This scale quantifies students' interest in engaging with mathematics. The literature on interest as a psychological construct is vast and diverse and has come to encompass or overlap with other constructs such as attention and surprise.[54] Here we use a restricted notion of interest: a student's active willingness to engage in mathematical situations.

Interest may not have a direct effect on academic achievement in a particular course but may affect overall academic achievement in mathematics by influencing a student's selection of mathematics courses. For example, Köller and Baumert [55] found no correlation between student interest and Grade 7 and 10 course achievement, but did find that interest was a predictor of advanced course selection. This, in turn, affected overall achievement in high school mathematics: interest early on influenced course selection which affected preparedness for Grade 12 mathematics. The authors also found that achievement correlated with interest: those students with greater mathematics grades expressed a greater interest in mathematics.

Relationships between mathematics and the Real World

Representative statement (MAPS #15): "Reasoning skills used to understand math can be helpful to me in my everyday life."

The transference of domain-specific knowledge to novel situations, either in or outside that domain, is a main goal of all education. Transference has been an especially sticky issue in mathematics education and has motivated restructurings of a number of K-16 mathematics curricula internationally to include more authentic, contextualized problems. The idea being, if students learn mathematics through more contextualized problems the more they will recognize connections between mathematics and other domains. Ultimately this is hoped to improve transference. Though seeing these connections may not directly impact a student's academic outcome in mathematics, we recognize the importance of connection-making. This category is intended to quantify a student's ability to recognize connections between mathematics and other contexts.

Another use of contextualized problems in mathematics education has been to motivate students to deepen their study. In this regard, if an intervention improves a student's ability to make connections to authentic situations, it may also improve their motivation to study mathematics and have longer-term effects on their academic achievement in mathematics.

Sense Making

Representative statement (MAPS #11): "In math, it is important for me to make sense out of formulas and procedures before I use them."

This category is intended to quantify students' perspectives on the nature of their personal mathematical knowledge. Students tend to structure and apply their mathematical knowledge in two broad ways: as certain tools to solve learned problem types or as a coherent body of knowledge that can be interpreted and applied equally to known and novel problems.[8,9,10]

Extensive research on how students approach acquiring and structuring knowledge has been performed at the tertiary level. In one of the first studies in this vein, Marton and Säljö [56] found that students took two qualitatively different approaches to learning a given text. The *surficial* approach involved memorizing what the students identified as key, examinable material. Students who took a *deep* approach attempted to see the ideas present in the text and to relate these to their existing knowledge. Those who took a surficial approach had a poor recollection of the text compared with those who took a deep approach. It must be noted that approaches to study are not static characteristics of students. Rather, a student will adopt a given approach to study based on any number of factors—their interest, prior knowledge, the tasks they experience in the educational situation, among others; see [57,58] for extended reviews.

These surficial/deep categories are known to relate to overall academic achievement, but exactly how is a matter of debate.[59,60] The general trend is that surficial approaches are negatively, while deep approaches are positively, correlated with course grade.[61] In the context of undergraduate mathematics, Maciejewski and Merchant [11] found that how study approaches correlate with course grade depends on the course emphasis. Essentially, those courses that emphasize reproduction of procedures do not discourage surficial approaches and only slightly encourage deep approaches. Courses with higher-level tasks discourage surficial approaches. The lesson here is that how a student acquires and structures knowledge matters to their academic achievement and this structuring of knowledge is influenced by the educational setting.

The Nature of Answers

Representative statement (MAPS #9): “I expect the answers to math problems to be numbers.”

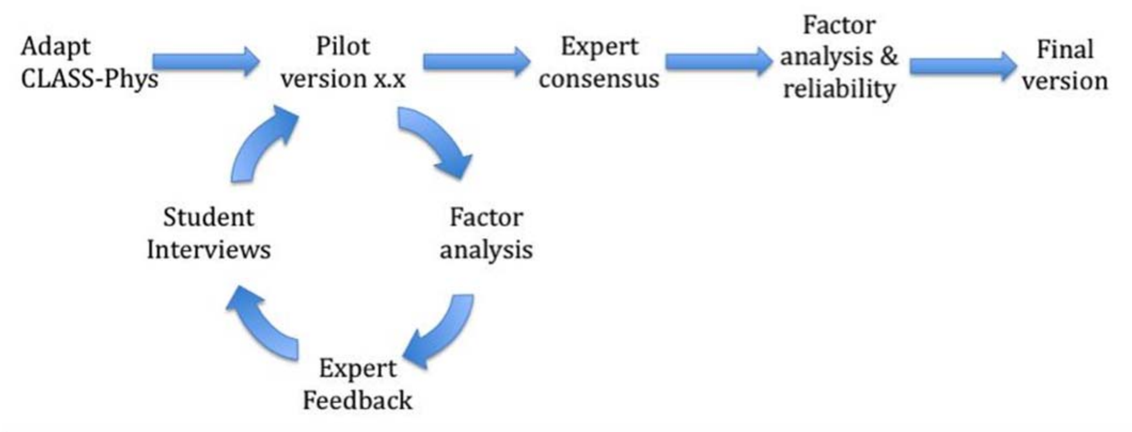
This category characterizes students’ views on the nature of solutions to mathematics problems. Students may view answers in mathematics as being either right or wrong and the solutions supporting these answers as having a certain degree of rigidity. These views can affect students’ conceptions of mathematics and ultimately their achievement in mathematics.

For example, in a series of interviews with undergraduate students, Crawford, Gordon, Nicholas, and Prosser [8] found two prevailing perspectives on the nature of mathematics. The *fragmented* view is one where mathematics is perceived as a collection of facts with little underlying structure. A *cohesive* view acknowledges the interconnectedness of mathematical ideas. This result informed the creation of the *Conceptions of Mathematics Questionnaire* that assigns respondents values on fragmented and cohesive scales.[9,10] These scores are known to correlate with students' approaches to studying mathematics and their academic outcome in mathematics: a fragmented view correlates to a surficial approach to study and lower grades, while a cohesive view correlates to a deep approach and higher grades.

Methods

MAPS was adapted from the CLASS-Phys instrument using a similar process to that used to adapt CLASS-Phys to CLASS-Chem and CLASS-Bio.[30,31] Statements from the CLASS-Phys were modified initially simply by replacing the word "physics" with the word "math." Statements were then modified, added and dropped based on the results of student and faculty interviews, as well as whether statements could achieve expert consensus and satisfactory factor analysis results. The initial version of MAPS was developed in Fall 2010, and piloted with students in both Fall 2010 and Spring 2011. It then underwent several iterations of revision, with subsequent versions being administered in Fall 2011/Spring 2012 and Fall 2012/Spring 2013. The current version of MAPS was then administered in Fall 2013 and Fall 2014. A final factor analysis and model confirmation were performed on this latest version of MAPS to establish the categories listed here. Last, reliability and concurrent validity—patterns in course levels and correlations with course grades—were measured for the final version of MAPS. This development process is summarized in Figure 1.

<Figure 1 here>



Pilot Runs

Each version of the survey was administered to students using an online survey system. Respondents were given unlimited time to complete the survey, but typically completed it in less than ten minutes. Depending on the course, completion of the survey was either entirely voluntary and anonymous, or it was non-anonymous and carried a very small participation credit (less than 1%) towards their course grade. As for the CLASS-Phys instrument, our administration of MAPS contained a "filter statement," that instructed students to respond "Agree" in order to verify that students were reading the statements. The student's response data was discarded if he or she did not answer "Agree" to this statement.

Student Interviews

A total of 19 students were interviewed at two different time points during the development of MAPS. Five students were interviewed on version 1 in Summer 2011, and 14 students were interviewed on version 3 during Fall 2012. During these interviews, students read each statement aloud to the interviewer, and gave their response choice from the 5-point scale along with an explanation for their response. In most cases, students freely supplied an explanation for their response, but when they did not the interviewer would prompt them to explain their choice. The purpose of these interviews was to ensure that the wording and meaning of the statements were clear to students and that their responses agreed with their verbal explanations. In addition, these validation interviews were to ensure that responses that agreed with the expert orientation were indeed due to expert-like attitudes and perceptions and that novice orientation responses were indeed due to novice-like attitudes and perceptions.

The student interviews on version 1 in Summer 2011 resulted in minor rewordings of three statements that students found to be unclear or ambiguous. The more extensive validation interviews on version 3 in Fall 2012 resulted in dropping five statements from the survey, because students did not interpret the statement in a consistent way or because their response choice did not agree with their verbal explanation. As an example, we dropped the statement "If I get stuck on a math problem on my first try, I usually try to figure out a different way that works" because we found that students would agree with this statement for a range of possible reasons, that did not necessarily correspond to an expert-like attitude. For instance, some students would interpret "a different way" to include soliciting help from friends or looking in books or on the web; other students would interpret it to mean working alone. By contrast, others explained agreement by reasoning: "well, I have no other option than to try again."

Expert Feedback

We solicited feedback from mathematics experts at two time points. First, after the initial version of MAPS was drafted and piloted with students in Fall 2010/Spring 2011, we invited a group of 17 experts (6 mathematics faculty, 1 postdoc, and 10 graduate students) to a focus group on MAPS. At the start of the session, each participant completed the survey on paper and we collected it. We then proceeded through a guided discussion about the goals of the survey, the proposed groupings of statements, and the individual statements themselves. In addition, participants were invited to provide further feedback in written form at the end of the session. From this first set of expert feedback we found that experts had differing opinions on many of the questions regarding approaches to learning mathematics (eg. "When I solve a math problem, I find an example that looks like the problem given and follow the same steps.") As a result, 4 questions about approaches to learning mathematics were removed from the survey. In addition, the group of experts suggested several attitudes that were not included in the survey. These mathematicians felt most strongly that such an instrument should include questions probing students' interest in the subject and confidence in solving mathematics problems and/or anxiety when doing exams. For this reason, we drafted 6 entirely new statements that were then piloted in the next version of the survey. (eg. #17 on the final version: "No matter how much I prepare, I am still not confident when taking math tests.")

We also individually interviewed 10 mathematics faculty in Summer 2012 on version 2 of the survey. The interview process mirrored that of our student validation interviews, in that faculty were asked to read the statements aloud, give their response choice, and explain the reasoning for their response. As with the student interviews, the purpose of these interviews was to ensure that the wording and meaning of the statements was clear to experts, and that their chosen response agreed with their verbal explanation. The purpose of this stage was to ensure that MAPS assesses attitudes and perceptions that are of value to experts, and also to allow faculty an opportunity to provide feedback on the individual statements. As a result of these interviews, we reworded four statements and dropped nine additional statements due to either unclear wording, lack of value to the experts who would purportedly use MAPS, or poor observed loadings in the exploratory factor analysis.

Expert Consensus

After completing validation with students in Fall 2012, in Fall 2013 we invited all members of the mathematics department at the University of British Columbia to complete the survey. A total of 58 responses were collected, 36 from faculty members which comprise our pool of expert responses. The major academic interests of these faculty members are: pure mathematics (N = 20), applied mathematics (N = 9), teaching focus (N = 3), other (N = 4), for a total of N = 36.

We used these 36 expert responses to determine whether there was expert consensus on each MAPS statement. Statements with greater than 25% neutral responses, and those with less than 80% agreement when the neutral responses were removed, were considered to not have a consistent expert view. For all but 6 statements, there was a consistent expert response, that was then characterized as the "expert response" and used to define the expert view.

Four statements did not achieve expert consensus and were dropped from subsequent versions of MAPS. These statements all had less than 80% agreement among faculty, after removing the neutral responses. These statements and the agree/disagree proportions are:

1. "I study math to learn things that will be useful in my life outside of school" (51.7%/48.3%)
2. "To understand math, I sometimes relate my personal experiences to the topic being studied" (55.6%/44.4%)
3. "I find it difficult to memorize all the necessary information when learning math" (42.9%/57.1%)
4. "Doing math puzzles is very interesting for me." (76.7%/23.3%)

It is interesting to note that the first two of these statements suggest that, in contrast to CLASS-Phys, experts do not uniformly relate the mathematics they learn to their personal life, since these questions had a nearly even agree/disagree split.

Two additional statements, MAPS #22 and #31 in the final version, did not achieve consensus, but have been retained in the survey because they provide valuable information about the mindset (growth vs. fixed) of the student population. These statements had too large a proportion of neutral responses, 36.1% and 30.6% respectively, in the expert pool to achieve consensus. In addition, even after removing the neutral responses the experts were split on whether they agree or disagree with statement 31, with 40% of experts agreeing and 60% disagreeing with this statement. Since the mindset of students is an important factor in their approach to learning and success in a course, but is not necessarily an agreed-upon subject among mathematicians,[62] these two statements were retained in the survey but are not scored as part of the expertise rating.

Among those statements with sufficient expert consensus as described above, there was a mean consensus (after Neutral responses removed) rate of 94% and a mean Neutral response of 9%.

In what follows, a student response for a statement was scored as 1 if the student agreed with the expert direction (e.g., if the expert consensus was to disagree with the statement, students answering “Strongly Disagree” or “Disagree” would score 1 for that statement), or 0 otherwise, meaning either “Neutral” or the opposite direction of the experts.

Student data

The majority of the student data comes from first and second year courses at a large, research-based Canadian university that attracts high-performing students both locally and from abroad. Data from a separate institution, a medium-sized American university drawing primarily from the local geographical area, was collected from a variety of students, including pre-service elementary and secondary teachers, college algebra, and calculus students, was also included in the analysis and later used as part of the reliability verification. Responses were collected using online and paper surveys, with some instructors offering extra credit for completion of the instrument. The types of courses that comprise the dataset are presented in Table 1.

Type of Course	Number of Courses	Number of Students	Notes
Differential Calculus (“Calculus 1”; First-year)	4	1647	200-600 each from versions tailored to Life Sciences, Physical Science & Engineering,

			Commerce & Social Sciences, and a two-semester (double the usual time) version
Integral Calculus ("Calculus 2"; First-year)	3	990	333-600 each from versions tailored to Life Sciences, Physical Science & Engineering, Commerce & Social Sciences
Multivariable Calculus ("Calculus 3"; Second-year)	2	261	
Introductory Proof (Second-year)	1	83	
UCA Math	18	430	Different institution and population.
Total		3411	

Factor Analysis

Our process of uncovering the factor structure underlying the MAPS survey began by dividing student responses into two groups, uniformly at random. The first group of student data (N = 1705) was used for an exploratory factor analysis and the second (N = 1706) for a confirmatory factor analysis. For an accessible introduction to factor analyses, see [63].

The first step in the exploratory factor analysis phase was to remove from the data set students who had at least 80% of the same responses as the expert consensus; for the purposes of uncovering a factor structure, the responses from such students are so high across potential categories that they would mute differences between factors. In all, 118 responses were removed, leaving 1587. This set was determined to be suitable for factor analysis as it provided a ratio of 51 responses per statement and a high Kaiser-Meyer-Olkin (KMO) factoring adequacy value of 0.88. The scree plot and parallel analysis for this data suggested an eight-factor structure; based on this the routine was run for 7, 8, and 9 factors looking for stable patterns in factor groupings, using the oblique rotation “oblimin” in attempting to accentuate categories of statements while accepting that factors would not likely be orthogonal in this type of data.[64] Loadings were computed using the fa.poly() function from the psych package of the statistical software R; this particular method uses the polychoric correlation matrix for the variables, which is more appropriate for dichotomous variables, recalling that all responses had been scored as 0 or 1 by this point. When restricting our category choices to contain at least three statements loading at least 0.34 on a factor, we arrived at a model with 7 such categories where the groupings made statistical sense and were identical or at least similar across the 7, 8, and 9 factor versions of the routine. Statement 27 (“I think it is unfair to expect me to solve a math problem that is not similar to any example given in class or the textbook, even if the topic has been covered in the course”) did not qualify for any categories under these conditions, but has been retained in the final instrument as part of the overall “Expertise” score. Finally, we attached category names to the factors (like "Confidence" and "Interest") based on the themes we could identify and how they matched with existing constructs in the literature.

Confirmatory Factor Analysis and Reliability

We combined the subscales identified in the exploratory stage with the set of expert-consensus statements as an additional large category to create a structured model for confirmation. A confirmatory factor analysis was performed with the data not used in the exploratory phase (N = 1706) using the cfa() function of the "lavaan" R package, version 0.5-18;[65] we note that this data set still included "expert" student respondents. Key indicators of model fit are the χ^2 value for model fit for which we report a value of χ^2 = 10221, the Root Mean Square Error of Approximation (RMSEA) with a value of 0.034 and 90% confidence interval of [0.032, 0.036], the Standardized Root Mean Square Residual (SRMR) with a value of 0.032, and comparative indices: Comparative Fit Index (CFI) of 0.924 and Tucker-Lewis Index (TLI) of 0.906. These all suggest a good model fit for the combined pool of responses.

The confirmation routine was also attempted with specific course populations, those with at least 100 respondents in the same course, within the full data set, representing the diversity of the data that went into the model. Similar fit numbers to those above emerged in all cases.

With the full pool of student data (N = 3411; includes the "expert" student responses), we found a Cronbach's alpha value of 0.87 (95% confidence interval [0.86, 0.88]) for the whole instrument, without the filter statement, indicating good reliability. Alpha values for the categories ranged from 0.55 to 0.70, which are lower than that for the entire instrument. This is likely due to the small number of items in most categories.

Results

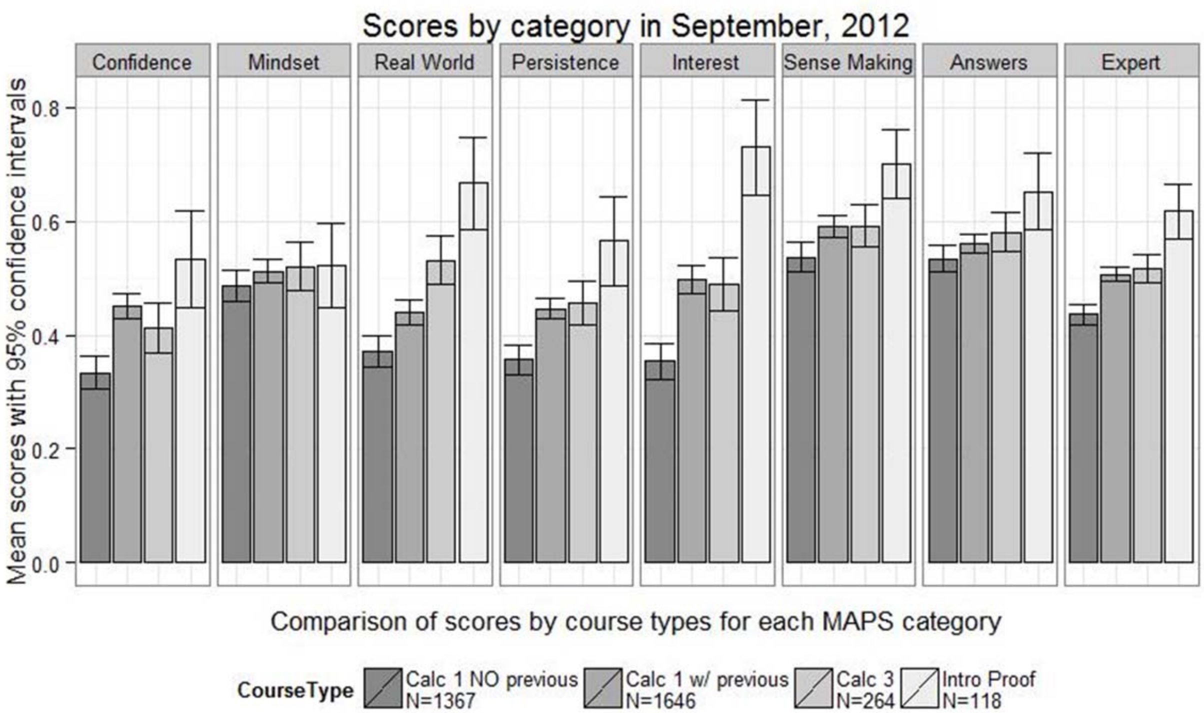
In this section we report highlights from the MAPS data we have collected to date. The intention here is to identify common or expected trends in MAPS data. These trends may help interpret and frame results from subsequent MAPS implementations.

Overall MAPS averages

The first result is the overall MAPS category averages and distributions from our largest data cohorts, first and second year undergraduate mathematics courses, partitioned into four student groups: i) Calculus 1 with no previous calculus experience (Calc1-N); ii) Calculus 1 with previous calculus experience (Calc1-Y); iii) Calculus 3, second-year multivariable calculus (Calc3); and iv) second-year introduction to proof (IntroProof). These results are reported in Figure 2. The ranges for the individual-level overall expertise index are significant: some students had perfect agreement (an index of 1) and perfect disagreement (index of 0) in almost all student groups.

Many of the observed trends are not unexpected. In all categories, Calc1-N students have the lowest means, while IntroProof students have the highest. This is significant for all the categories except for Mindset, where no significant differences between the student groups are observed. IntroProof students had the greatest expert-like orientations to mathematics likely because the course was taken almost exclusively by mathematics and statistics majors. This is in contrast to first year calculus, where math majors constitute only a small fraction of all enrollments. A previous longitudinal CLASS study in physics corroborates this observation: physics degree recipients tend to have expert-like orientations to physics early on in their tertiary physics education.[36]

<Figure 2 here>



Correlations between MAPS scores and course grades

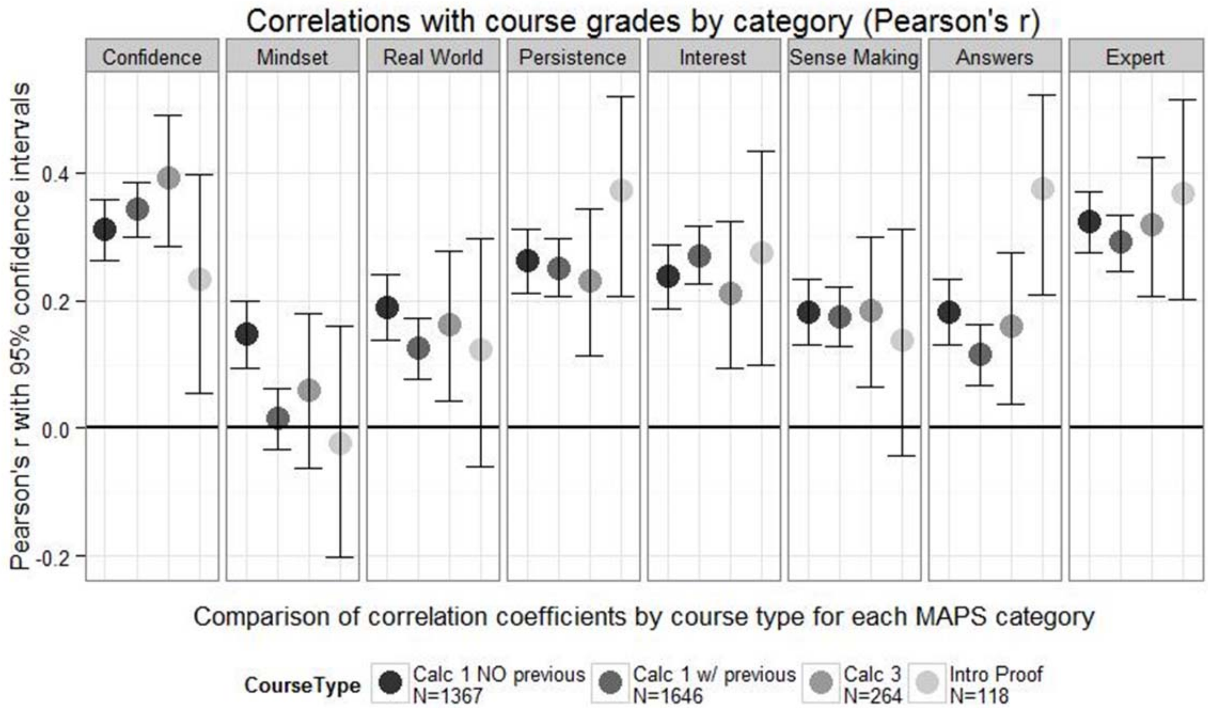
Next, we match the aggregate data, partitioned by course type as above, with course grades to identify correlations between MAPS subscales and grades. These correlations are presented in Figure 3. Course grades were determined in a similar manner within each course grouping, depending largely on traditional written exams in all cases, with some variety in exam setting depending on the specific instructors involved. While a thorough analysis has not been completed, the types of questions on the exams for the participating courses are similar to those reported in [11].

Of all categories, Mindset had the lowest correlations with grades across all groups and three of the four correlations, Calc1-Y, Calc3, and IntroProof, are not statistically significant. All other correlations are significant at the $p < 0.01$ level except for three in the IntroProof group: Confidence is significant to $p < 0.05$ while Real World and Sense Making are not significant. These null results are possibly due to the small IntroProof sample size ($N = 83$).

The first observation of the MAPS/course grade data is that the overall expertise index is correlated with course grade in each of the course groupings. These range from $r = 0.29$ for the Calc1-Y group to $r = 0.37$ for IntroProof.

The second is that the confidence subscale is the most highly correlated with course grades among all the subscales, ranging from $r = 0.23$ for IntroProof to $r = 0.44$ for Calc3. Persistence and Interest are also important predictors of course grades across all groups. Sense Making and Answers exhibit low correlations with course grades, but this may be, as identified in [11], due to lower-level, service mathematics courses neither discouraging surficial approaches nor encouraging deep approaches to learning. Upper-level courses, like the IntroProof course in this study, tend to emphasize deeper approaches to learning. This may account for the relatively high observed correlation between the Answers category and course grades in the IntroProof group.

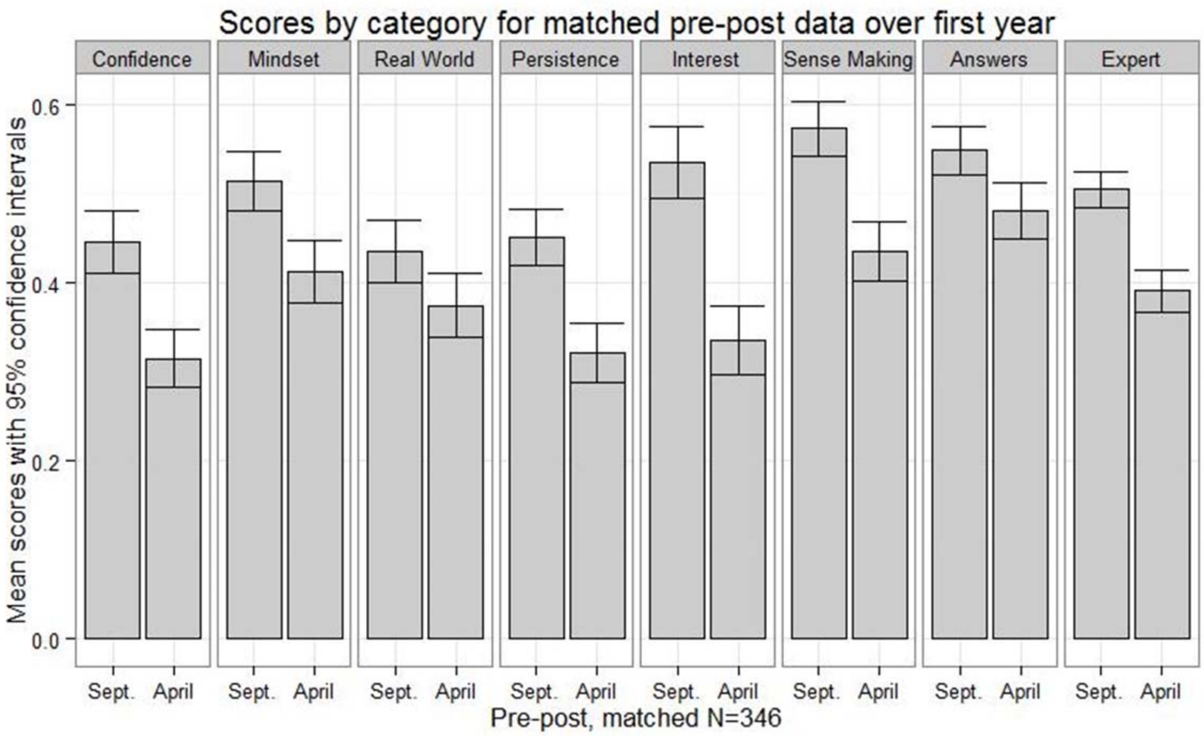
<Figure 3 here>



Academic year trends

The third result concerns differences in start and end of year MAPS scores. Students enrolled in a first year, one semester differential calculus course wrote the MAPS survey in September and those completing the follow-up integral calculus course wrote the MAPS in April. Those students who completed both of these surveys (N = 346) comprise the cohort for the following analysis. September and April means are presented in Figure 4. All MAPS categories, including the expertise index, saw declines over the academic year. Put differently, students enrolled in a first year calculus course sequence move away from expert-like orientations to mathematics over the duration of the academic year. This result is consistent with results from all CLASS-type surveys in other disciplines.[28,29,30,31,32,33] We are not able to elaborate on why the data exhibit these shifts, though we suspect that the nature of first year mathematics courses, with their emphasis on the reproduction of procedures, solving low-level, inauthentic problems, and a lack of emphasis on deeper approaches to learning is the cause.[66,11,12]

<Figure 4 here>

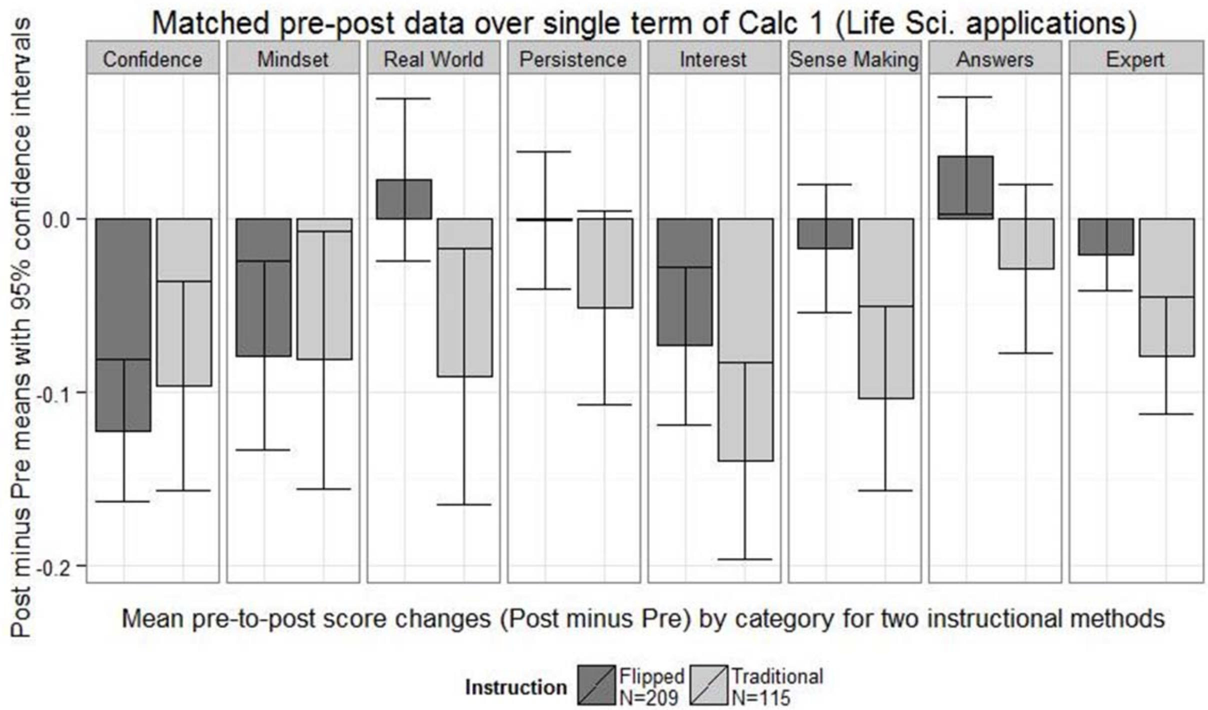


The effects of interactive engagement on MAPS scores

The fourth result comes from a control-group study of the effects of classroom “flipping” in a first-year, first-semester calculus course for students enrolled in life sciences programmes. The course was divided into two treatment conditions: flipped, where class time was devoted to interactive engagement activities, and traditional, with transmission-style lectures. For further details on the flipping implementation, see [67]. The pre/post data are presented in Figure 5. Scores in each of the MAPS categories declined over the semester in both treatment conditions. This is in line with the results reported above. However, the scores in the flipped condition declined less than those in the traditional condition (N_{Flipped} = 209, $M_{Flipped}$ = 51.64, $N_{Traditional}$ = 115, $M_{Traditional}$ = 46.34, $t(223) = 2.44$, $p = 0.01$, $d = 0.29$). This somewhat peculiar result corroborates CLASS-type results from interactive engagement studies in physics—it seems that interactive engagement teaching methods do *less harm* to students' expert-like orientations.[68]

A comparison between pre/post differences for the treatment and control groups is presented in Figure 5. In each category, except Mindset and Confidence, the difference in treatment means is greater than the difference in comparison group means, $p < 0.05$ for Interest and Persistence categories, and $p < 0.01$ for Real World, Sense Making, Answers, and Overall categories.

<Figure 5 here>



Discussion

This article presents an instrument designed to measure beliefs and attitudes towards mathematics held by undergraduate students relative to mathematicians. The MAPS categories emerged from statistically rigorous analyses, were shown to be well-grounded in the research literature, and representative of the large set of epistemological beliefs, perceptions, and attitudes known to affect students' academic outcomes in mathematics. Additionally, we have intentionally kept the survey brief enough to be used as a pre and post test instrument in authentic course settings.

Results from our use of the MAPS survey are in line with results generated from its close cousins, the CLASS surveys. Generally, students move away from expert-like conceptions of mathematics over a semester or year-long mathematics course. Students in a second year, specialized course, report more expert-like orientations to mathematics than those in first year courses. Classes centred on interactive engagement, that occasion more authentic mathematical experiences, tend to push students away from expert-like conceptions less than traditional courses. Also, correlations were found between the MAPS subscales, including overall expertise index, and course grades, highlighting the importance of expert-like orientations for academic achievement.

An interesting future application of MAPS would be in monitoring how students' beliefs about mathematics change over the duration of an undergraduate degree. It is expected that, on average, students will shift toward more expert-like conceptions of their discipline.[69] However, this aggregate shift may not be caused by individual-level shifts. For example, Bates, et al. [38] and Madsen, McKagan, and Sayre [70] found that those students entering a physics program with more expert-like conceptions of physics were more likely to complete the program and that their conceptions of physics remained largely unchanged. This suggests that a physics program *selects* for physics-oriented students rather than *developing* an orientation to physics. We suspect the situation in mathematics is similar. Indeed, a common belief among mathematicians is that students who have productive dispositions towards mathematics which are more like professional mathematicians are more likely to be successful in a mathematics program. Many talented students leave STEM, often for reasons unrelated to their ability.[71] It is of great importance that departments foster and encourage growth, including more expert-like beliefs, instead of only catering to students already possessing that collection of beliefs.

This view of "good" mathematics students as having an innate ability in mathematics is echoed in our expert responses to our Mindset questions. Neither of the statements, "being good at math requires natural (i.e. innate, inborn) intelligence in math," (MAPS #22) and, "for each person, there are math concepts that they would never be able to understand, even if they tried" (MAPS #31) reached expert consensus. This seems to suggest that a fixed mindset is prominent among, at least some, mathematicians. This is puzzling since our other two mindset questions, "Math ability is something about a person that cannot be changed very much," (MAPS #05; expert consesus: Disagree), and "Nearly everyone is capable of understanding math if they work at it," (MAPS #06; Agree), did have consensus. This suggests an important topic for future research: is the belief that mathematical ability is innate and largely static common among mathematicians? Does this influence the way they teach?

As a final note, researchers interested in using MAPS should not necessarily feel restricted by our usage of it. There are many other possible results that the MAPS survey could help establish. For example, the various CLASS implementations have explored correlations between expert-like orientations and grades and how these orientations change over time. Indeed, MAPS could usefully be employed in any undergraduate mathematics education setting where student beliefs and perceptions are suspected to play a role.

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Appendix: The MAPS Instrument

The MAPS instrument consists of the following 31 questions and 1 filter statement. The survey can be offered online or in written form. Students respond to each question using a 5-point Likert format: “Strongly Disagree”, “Disagree”, “Neutral”, “Agree”, and “Strongly Agree”. The student receives 1 point for a question if their answer is in the same direction—that is, in the disagree or agree direction—as the expert consensus, given at the end of each question below. If the student responds in the opposite direction of the consensus, or a neutral response is given, they receive 0 for that question. The total expertise index is calculated by averaging the scores for all questions except 19, 22, and 31. Subscale scores are calculated analogously, with the question numbers comprising each category given in Table 2.

Table 2: MAPS categories and corresponding question numbers.

Category	Question
Growth Mindset	5, 6, 22, 31
Real World	13, 15, 21, 25
Confidence	1, 14, 17, 20
Interest	12, 26, 32
Persistence	8, 10, 24, 29
Sense Making	3, 4, 11, 18, 23
Answers	2, 7, 9, 16, 28, 30
No category but scored for expertise	27
Filter statement	19
Expertise (expert consensus)	all except 19, 22 and 31

The following questions are the MAPS instrument. The direction of the expert consensus follows each question in parentheses.

1. After I study a topic in math and feel that I understand it, I have difficulty solving problems on the same topic. (Disagree)
2. There is usually only one correct approach to solving a math problem. (Disagree)
3. I'm satisfied if I can do the exercises for a math topic, even if I don't understand how everything works. (Disagree)
4. I do not expect formulas to help my understanding of mathematical ideas, they are just for doing calculations. (Disagree)
5. Math ability is something about a person that cannot be changed very much. (Disagree)
6. Nearly everyone is capable of understanding math if they work at it. (Agree)
7. Understanding math means being able to recall something you've read or been shown. (Disagree)
8. If I am stuck on a math problem for more than ten minutes, I give up or get help from someone else. (Disagree)
9. I expect the answers to math problems to be numbers. (Disagree)
10. If I don't remember a particular formula needed to solve a problem on a math exam, there's nothing much I can do to come up with it. (Disagree)
11. In math, it is important for me to make sense out of formulas and procedures before I use them. (Agree)
12. I enjoy solving math problems. (Agree)
13. Learning math changes my ideas about how the world works. (Agree)
14. I often have difficulty organizing my thoughts during a math test. (Disagree)

15. Reasoning skills used to understand math can be helpful to me in my everyday life. (Agree)

16. To learn math, the best approach for me is to memorize solutions to sample problems. (Disagree)

17. No matter how much I prepare, I am still not confident when taking math tests. (Disagree)

18. It is a waste of time to understand where math formulas come from. (Disagree)

19. We use this statement to discard the survey of people who are not reading the questions. Please select Agree (not Strongly Agree) for this question. (Filter statement)

20. I can usually figure out a way to solve math problems. (Agree)

21. School mathematics has little to do with what I experience in the real world. (Disagree)

22. Being good at math requires natural (i.e. innate, inborn) intelligence in math. (Disagree)

23. When I am solving a math problem, if I can see a formula that applies then I don't worry about the underlying concepts. (Disagree)

24. If I get stuck on a math problem, there is no chance that I will figure it out on my own. (Disagree)

25. When learning something new in math, I relate it to what I already know rather than just memorizing it the way it is presented. (Agree)

26. I avoid solving math problems when possible. (Disagree)

27. I think it is unfair to expect me to solve a math problem that is not similar to any example given in class or the textbook, even if the topic has been covered in the course. (Disagree)

28. All I need to solve a math problem is to have the necessary formulas. (Disagree)

29. I get upset easily when I am stuck on a math problem. (Disagree)

30. Showing intermediate steps for a math problem is not important as long as I can find the correct answer. (Disagree)

31. For each person, there are math concepts that they would never be able to understand, even if they tried. (Disagree)

32. I only learn math when it is required. (Disagree)