PHYS 408 2010/11 term2

Homework problem set #1, problems 1-3
Assigned: 07/01/2011
Written Solution due 14/01/2011 5 PM in Hennings 328
If you collaborated with anyone, please write their name at the top of your HW

Problem 1 (20 marks)

For monochromatic fields (i.e. single frequency ω), we like to simplify our life by taking out the time dependence….and deal with complex fields. **By convention**, we always pay a bit more attention to the **positive** frequency component (i.e. the term oscillating with a dependence of exp{-iωt}).

For instance, take the real field : $\vec{\mathcal{E}}(r,t) = E_o \hat{y} cos(k_o z - \omega t + \phi)$

We can always decompose a real field as: $\vec{\mathcal{E}}(r,t) = \frac{\vec{E}^+}{2} e^{-i\omega t} + \frac{\vec{E}^-}{2} e^{i\omega t}$

Complex Amplitude

Pos. Freq Component

So applying this decomposition to our example real field above is:

$$\vec{\mathcal{E}}(r,t) = \hat{y}\frac{E_o}{2}(e^{i(kz+\phi)}e^{-i\omega t} + e^{-i(kz+\phi)}e^{i\omega t})$$

Therefore, the complex amplitude associated with the real field above is:

$$\vec{E}^+ = \vec{E}(r) = \hat{y}E_o e^{i(kz+\phi)}$$

Find the complex amplitude of these fields:

a) $\vec{\mathcal{E}}(r,t) = 2\hat{x}sin(-ky - \omega t)$

b) $\vec{\mathcal{E}}(r,t) = \hat{x}sin(ky - \omega t) - \hat{x}cos(ky + \omega t)$

Therefore, **by convention**, if we always took the positive frequency (i.e. exp{-iωt}) of the real field it's always trivial to reconstruct the real field from a complex amplitude. We simply multiply the complex amplitude by exp{-iωt} and take the real part.

For instance, take the complex amplitude: $\vec{E} = 4\hat{y}e^{-ik_o z + i\phi}$

What do we do to reconstruct the real field?, we always, by convention, took the positive frequency components :

So the real field corresponding to the complex amplitude above is:

$$\vec{\mathcal{E}}(r,t) = Re\left[4\hat{y}e^{-ik_o z + i\phi}e^{-i\omega t}\right] = 4\hat{y}cos(-k_o z - \omega t + \phi)$$

Find the real field represented by these complex amplitudes:

c) $\vec{E} = i\hat{x}e^{ik_o y - i\pi}$

d) $\vec{E} = 10\hat{z}e^{-ik_o x - \pi/6}$

Problem 2 (40 marks)

Consider the following two electric fields (written as either a complex amplitude or a real electric field) that might represent electromagnetic plane waves

i) $\vec{\mathcal{E}}(r,t) = E_o(4\hat{y} - 4\hat{x})cos(k_o y - \omega t)$

ii) $\vec{E} = E_o \hat{y} e^{-i(5k_o z - \pi/4)}$

a) First explain the difference between k and k_o where k appears in the Helmholtz wave equation.
b) Now determine which of these fields satisfy the Helmholtz wave eqn? If they do, what is the relationship between k and ω ? Remember the ω is the independent variable (and not k). Why?
c) Which are electromagnetic waves? You must check all Maxwell's eqns to determine this fact For those that are, give an expression for the real electric field or complex amplitude. For those that are not, which of Maxwell's Eqns do they violate?

Problem 3 (40 marks)

The intensity, I, of a electro-magnetic wave can be calculated via the time-averaged Poynting Vector

$$\mathrm{I} = < |\vec{\mathcal{S}}| > = \frac{1}{T}\int_{t_o}^{t_o+T} |\vec{\mathcal{S}}| \,\mathrm{dt} = \frac{1}{T}\int_{t_o}^{t_o+T} |\vec{\mathcal{E}} \times \vec{\mathcal{H}}| \,\mathrm{dt}$$

where T is long compared to an optical cycle $\frac{2\pi}{\omega}$

or the intensity can be calculated via the complex Poynting vector

$$I = < |Re[\vec{S}]| > = |Re\left[\frac{1}{2}\vec{E} \times \vec{H}^*\right]|$$

a) Pick one of the electric fields you found in Problem 2 that is an electromagnetic wave and produce an expression for the real magnetic field and the complex amplitude of the magnetic field.

b) Calculate the intensity using via both methods and show they are equal.