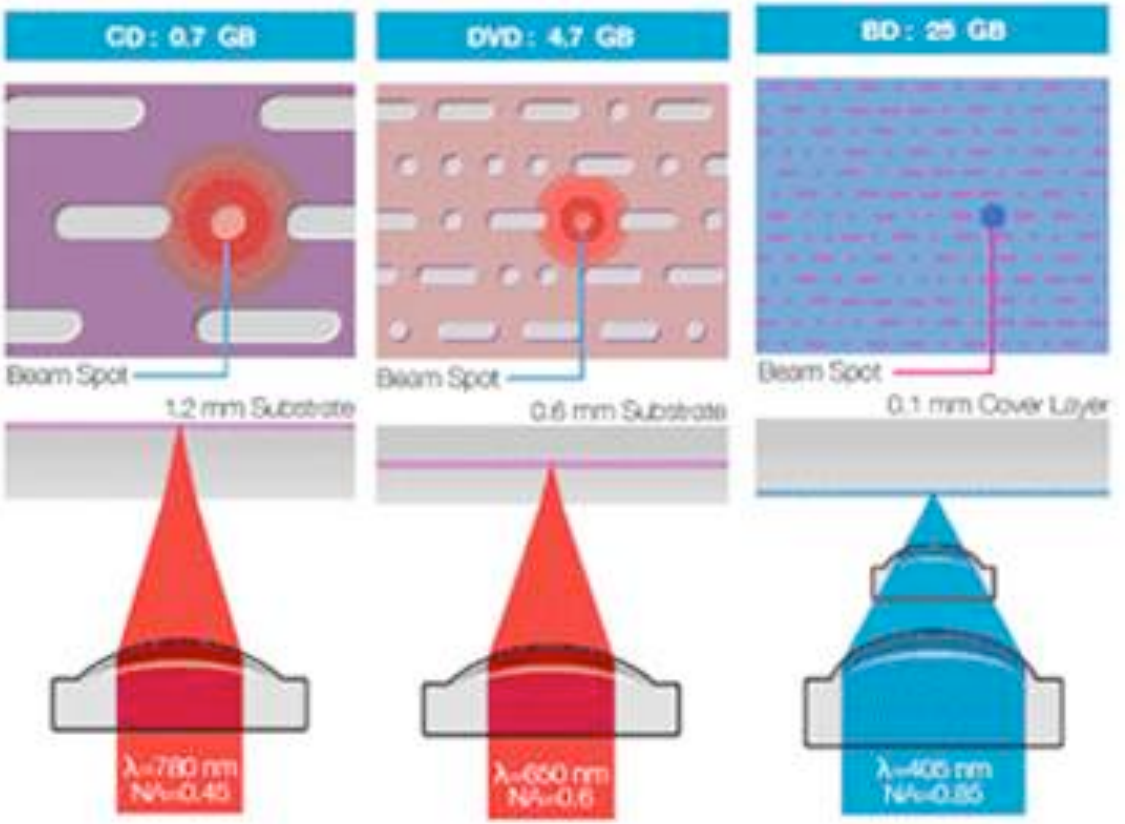
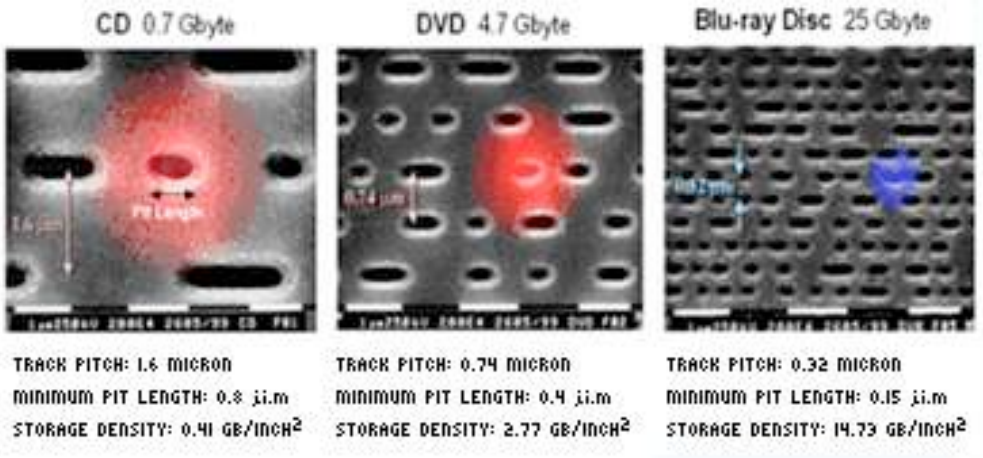
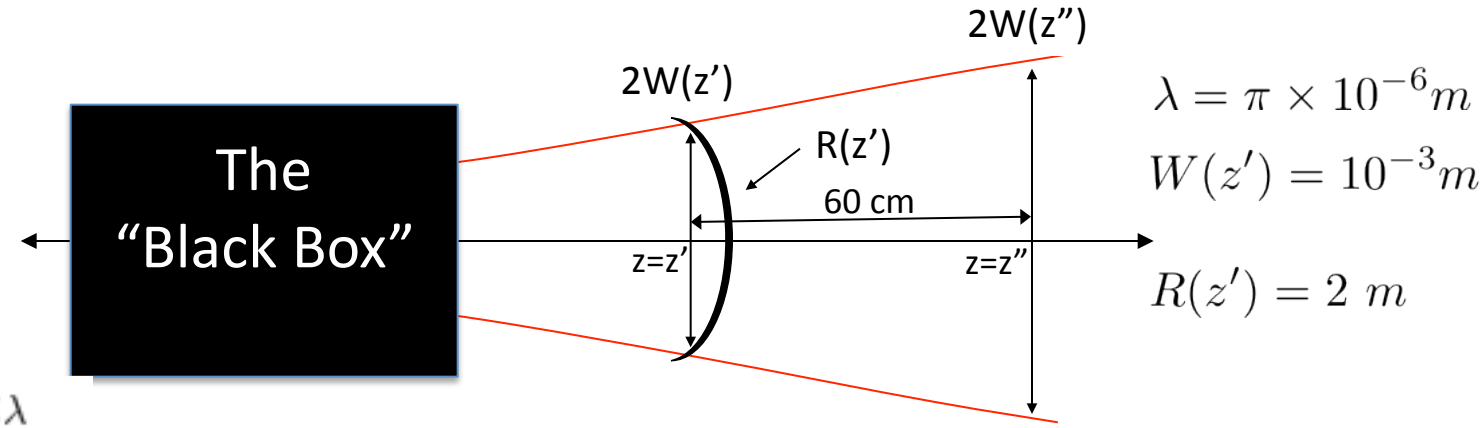
Gaussian Beams through Optical Components…why would we care?





Images Source: www.blu-raydisc.com

PHYS 408 Group Activities from before break



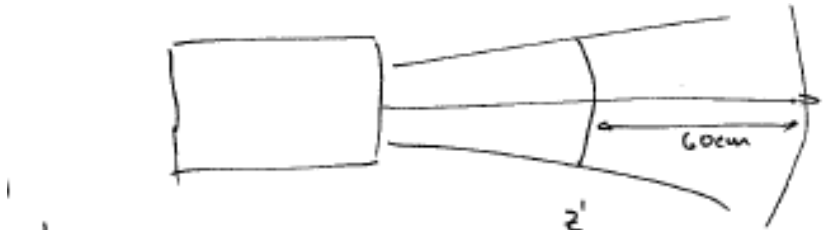
$$q(z) := z - iz_0.$$

$$\frac{1}{q(z)} = \frac{1}{R(z)} + \frac{i\lambda}{\pi w^2(z)}.$$

A laser located somewhere in a black box shines a beam out. Say you have measured the beam width W(z') and radius R(z') at z=z' (note that you don't know what z' is).

Using the q(z), find the location of the minimum waist (z')

Using the q(z), find the beam width 60 cm away [W(z")]



$w(z') = 10^{-3}$ m

$R(z') = 2$m

$\lambda = \pi \times 10^{-6}$

$$\frac{1}{q'} = \frac{1}{R(z')} + i\frac{\lambda}{\pi w^2(z')}$$

$$\frac{1}{q'} = \frac{1}{2} + i$$

$$q' = \frac{2}{5} - i\frac{4}{5}$$

Thus minimum waist location is $\frac{2}{5}$ m to the left of z'

$\therefore$ @ $z=0$ $\quad q = -i\frac{4}{5}$

@ $z = z''$ $\quad q'' = \left(\frac{2}{5} + \frac{60}{100}\right) - i\frac{4}{5}$

$$q'' = 1 - i\frac{4}{5}$$

$$\Rightarrow \frac{1}{q''} = \frac{5}{41}(5 + i4)$$

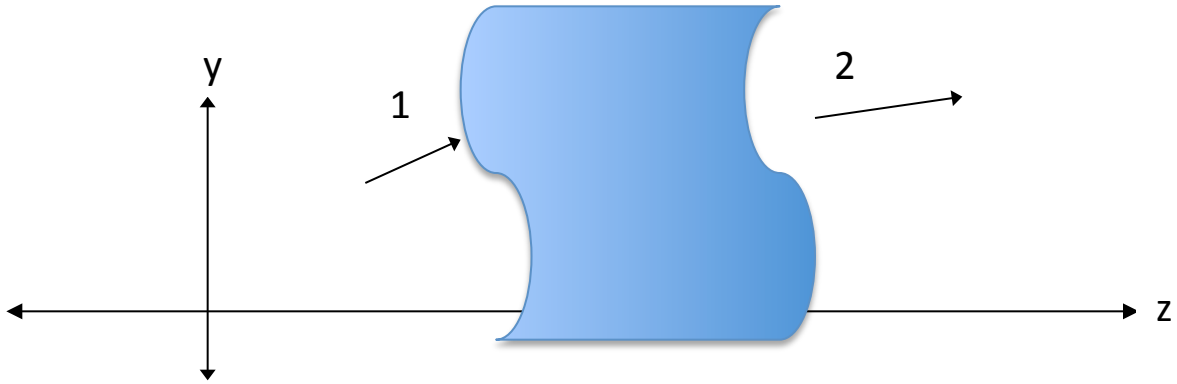
so $\frac{20}{41} = \frac{\lambda}{\pi w^2(z'')}$

$$\Rightarrow w^2 = \frac{41}{20} \cdot 10^{-6}$$

So we can track R(z) and W(z) of the Gaussian beam as it propagates via q(z)..what about if the beam goes through lens, etc?

PHYS 408 Group Activities for Feb 22 2011

Please complete these exercises without opening the textbook. If you have questions talk with your group mates or raise your hand and David or Ellen will come to help



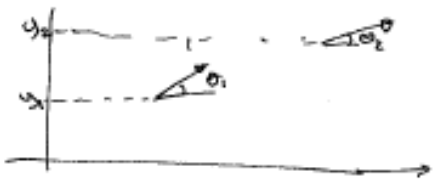
1-What is the 1x2 column vector that we can characterize a generation ray (such as ray 1 and ray 2)? Why are only 2 elements required?

2-Using this parameterization for rays 1 and 2, what is the most general linear transform between 1 and 2? Again state your reasoning in your own words. What have you assumed about the rays? Why is this assumption required?

3-If there is only free space (a distance *d*) from ray 1 to ray 2, what is the appropriate transform? What is the transform if from ray 1 to ray 2 is a dielectric boundary with n_1 and n_2 respectively

Derive the transform for a spherical dielectric boundary

1



note a ray has infinite length so don't quantify its length

A two column vector is $\begin{bmatrix} y \\ \theta \end{bmatrix}$

y is height off z axis (or optical)

θ is angle makes with respect to z axis

Only two elements are required b/c we are looking in 2-D

2

The most general linear transform in a 2x2 matrix:

$$\begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix}$$

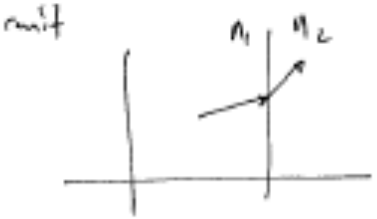
for this transform we have made the paraxial approx. w/o it there wouldn't be a linear transform between y_2 & θ_1

3



from $1 \to 2$ $\theta_1 = \theta_2$, so $C=0$, $D=1$

$y_2 = y_1 + \theta d$, so $A=1, B=d$



from $1 \to 2$ $y_1 = y_2 \Rightarrow A=1$ $B=0$

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$\Rightarrow n_2 = \frac{n_1}{n_2}\theta_1$

so $C=0$ $D = \frac{n_1}{n_2}$

$$\begin{bmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{bmatrix}$$

PHYS 408 Group Activities for Feb 22 2011

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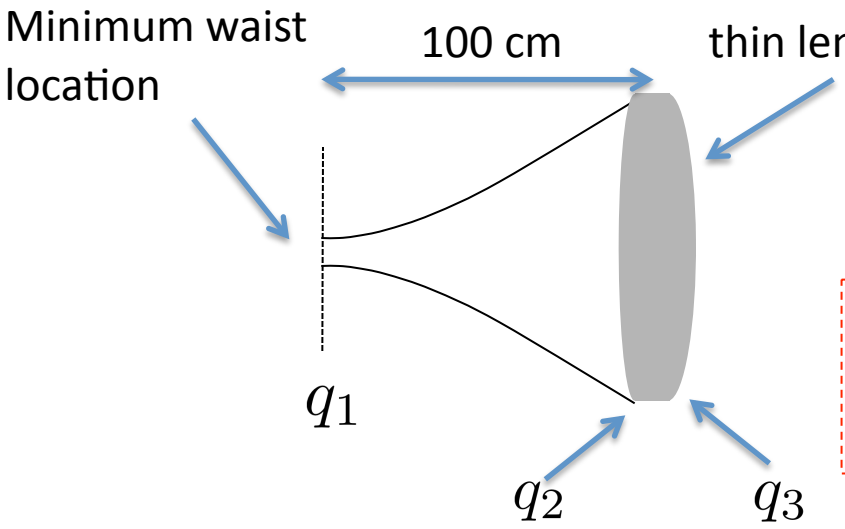
4-From your reading you know that $q_2 = \dfrac{Aq_1 + B}{Cq_1 + D}$

where $ABCD$ are the same elements as those from the ray matrices. In your own words, explain qualitatively why and how we can use these ray matrices in the context of Gaussian Beams? Are Gaussian Beams rays?

Do exercises 5&6 both with and without ABCD matrices

5-Given the fact that $q(z) = z - iz_o$ and $\dfrac{1}{q(z)} = \dfrac{1}{R(z)} + i\dfrac{\lambda}{\pi W^2(z)}$ $W_o = \sqrt{\dfrac{\lambda z_o}{\pi}}$

find the indicated $q(z)$'s with $\lambda = \pi \times 10^{-6}$ & $W_o = 10^{-3}$



6-Based on q_3 where is the minimum waist of the new beam located?

Bonus: prove the ABCD matrix of a thin lens with focal lengh f is $\begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}$

$q_1 = -i \quad (z_0 = 1)$

$q_2 = 1 - i$ $\qquad \frac{1}{q_2} = \frac{1+i}{2}$

$\frac{1}{q_3} = \frac{1}{q_2} - \frac{1}{f}$

$= \frac{1}{R_2} + i\frac{\lambda}{\pi w_2} - \frac{1}{f}$; $\qquad \frac{1}{R_2} - \frac{1}{f} = \frac{1}{R_3}$

$= \frac{1}{R_3} + i\frac{\lambda}{\pi w_3}$ $\qquad$ { $w_2 = w_3$

$= \frac{1}{2} + \frac{i}{2} - \frac{1}{\frac{1}{2}}$

$= \frac{i}{2} + \frac{1}{2} - 2$

$\frac{1}{q_3} = -\frac{3}{2} + \frac{i}{2}$; $\qquad \frac{i-3}{2}$

$q_3 = \frac{\frac{2}{(i-3)}(-i-3)}{(i-3)(-i-3)}$

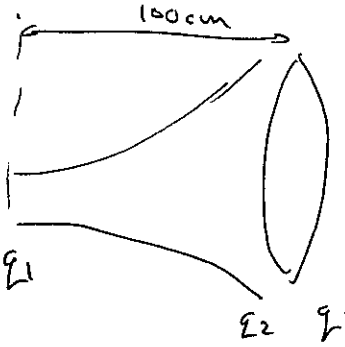
$q_3 = \frac{1}{10}(-6 + -2i)$

$= \frac{-6}{10} - \frac{1}{5}i \quad \Rightarrow \quad z_0 = \frac{1}{5}$ distance to minimum

.6m

clicker answer is A

we can also get q_3 from q_1 using cascaded ABCD matrices



$q_1 = -i$

from $q_1 \rightarrow q_2$ $\begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$d = 1m$

from $q_2 \rightarrow q_3$ $\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}$

$\Rightarrow$

total matrix $= \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix}$

$\Rightarrow q_3 = \frac{A_T q_1 + B_T}{C_T q_1 + D_T} = \frac{-i+1}{-2i-1} = \frac{-3-i}{5}$ which is the same answer as before