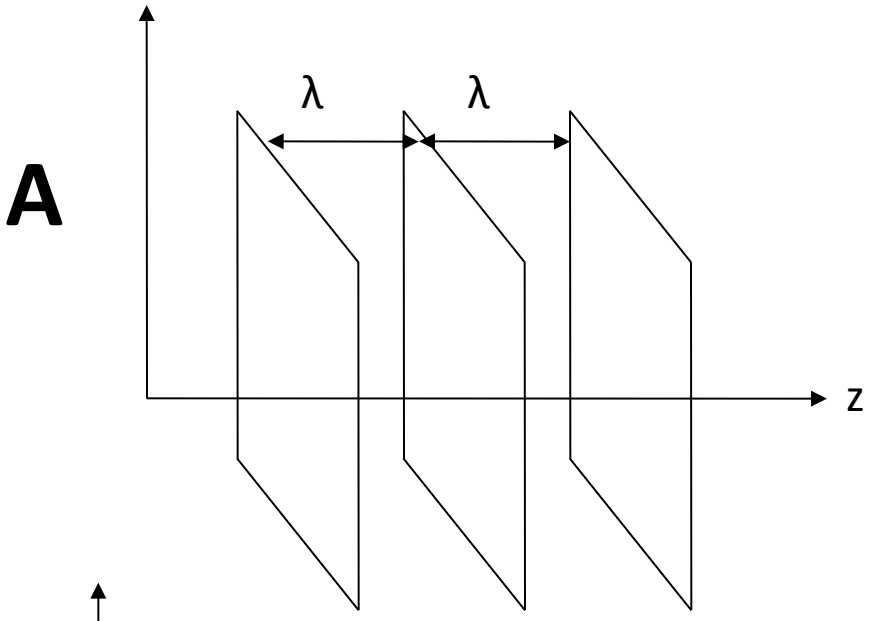
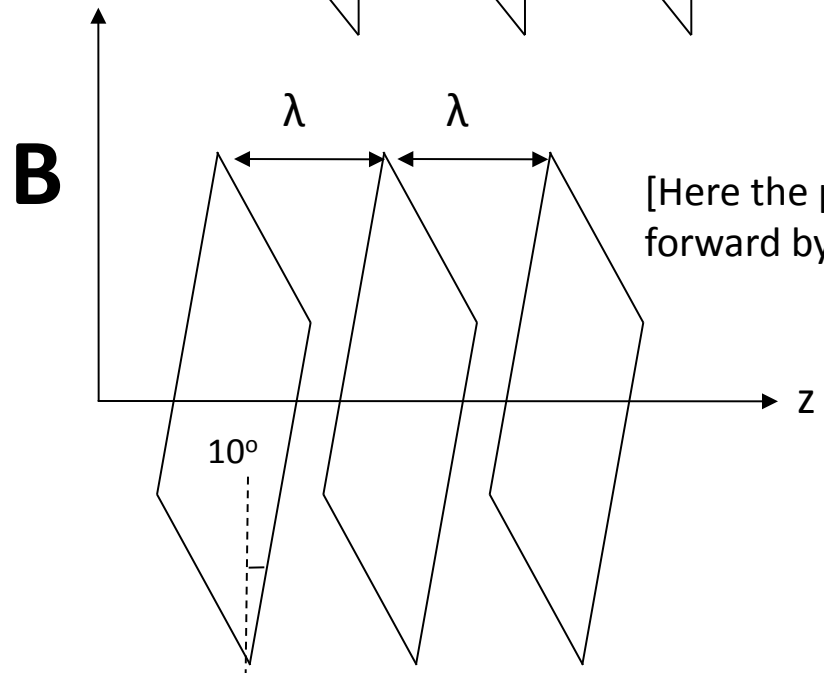
PHYS 408 Group Activities for Jan 13 2011

Please complete these exercises without opening the textbook. If you have questions talk with your group mates or raise your hand and Ellen or David will come to help

Many of you explained that a plane wave can be described by planes of constant phase. These planes were described to be parallel 2D planes separated by λ. Are both A and B valid depictions of possible planes of constant phase for a plane wave propagating in the z direction?

Explain your answer using the equation of a plane wave traveling in the z direction.





[Here the planes are tilted forward by 10 degrees]

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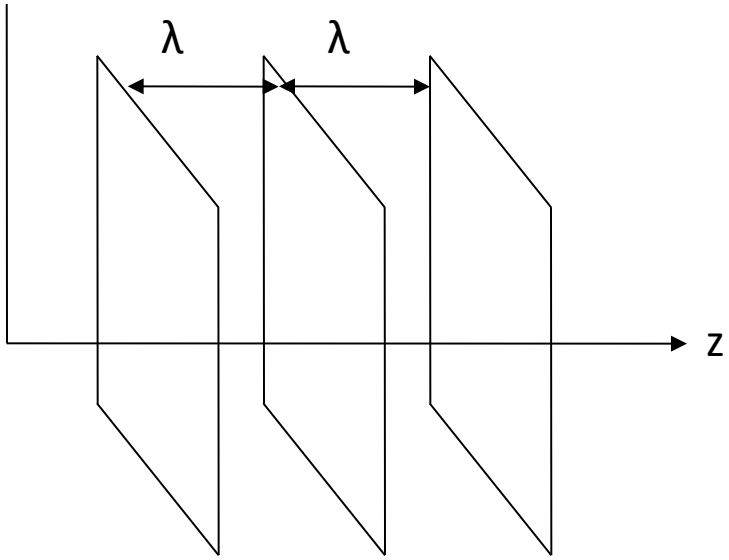
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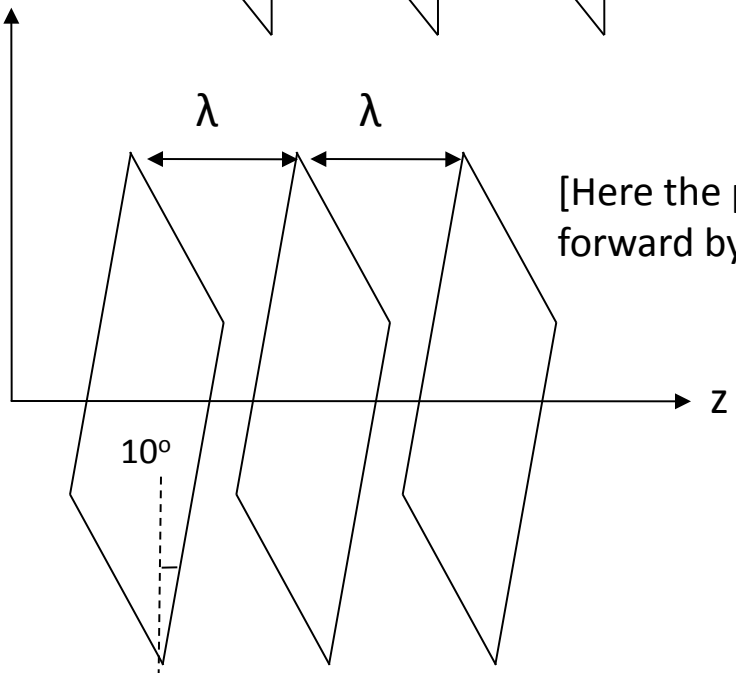
only A is valid for a plane wave in $\hat{z}$

$E = E_0 e^{ikz}$, so planes of constant phase should not depend on x, y. This is not true for B

A



B



[Here the planes are tilted forward by 10 degrees]

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1. With this electric field $\vec{E} = \hat{z} E_o e^{i(\vec{k}\cdot\vec{r}-\pi)}$ $\vec{k} = k_x\hat{x} + k_y\hat{y} = 3\hat{x} + 4\hat{y}$ and a magnitude of 10 V/m draw a diagram (to scale) of the wavefronts of field in the x-y plane with labels showing k, λ, k_x, k_y and the strength of the electric field E_o Please also label the frequency of the wave on your picture

2. Also indicate where surfaces of constant intensity in your sketch

3. In your reading the curl of the complex amplitude of the electric field (in certain circumstances) was simplified to:

$$\nabla \times \vec{E} = i\vec{k} \times \vec{E}$$

what are the specific conditions where this expression holds true? Prove your answer.

4. Using the above expression, along with

$$I = <|\vec{S}|> = \frac{1}{2}|Re(\vec{E} \times \vec{H}^*)|$$

and one of Maxwell's eqns, find an expression the time-averaged intensity in terms of only the complex amplitude of the electric field (E) and some constants. Why is this result useful?

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$$\nabla \times \vec{E} = i\vec{k} \times \vec{E}$$

what are the specific conditions where this expression holds true? Prove your answer.

2) Looking @ $\vec{\nabla} \times \vec{E} = i\vec{k} \times \vec{E}$, The $\vec{\nabla} \times$ operator has spatial derivitives. The only way this equation could hold is if $\vec{E} \propto \vec{E}_0 e^{i(\vec{k}\cdot\vec{r})}$ i.e. a plane wave

($\vec{E}_0$: constant)

so This only holds true if $\vec{E}$ is a plane wave.

Full proof:

$\vec{k} = k_x\hat{x} + k_y\hat{y} + k_z\hat{z}$

$$\vec{\nabla} \times \vec{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial_x & \partial_y & \partial_z \\ E_x & E_y & E_z \end{vmatrix} \quad \text{w/} \quad \vec{E} = (E_{x_0}\hat{x} + E_{y_0}\hat{y} + E_{z_0}\hat{z})\, e^{i(k_x x + k_y y + k_z z)}$$

$$= \hat{x}(\partial_y E_z - \partial_z E_y) + \hat{y}(\partial_x E_z - \partial_z E_x) + \hat{z}(\partial_x E_y - \partial_y E_x)$$

(1) $$= i\hat{x}(k_y E_{z_0} - k_z E_{y_0}) + i\hat{y}(k_x E_{z_0} - k_z E_{x_0}) + i\hat{z}(k_x E_{y_0} - k_y E_{x_0})$$

(2) $i\vec{k} \times \vec{E} = i(k_x\hat{x} + k_y\hat{y} + k_z\hat{z}) \times (E_{x_0}\hat{x} + E_{y_0}\hat{y} + E_{z_0}\hat{z})$

completing The x in (2) will match up The terms in (1)

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assume plane wave — Maxwell Equation for time harmonic case

3) $\nabla \times \vec{E} = i\vec{k} \times \vec{E} = i\omega\mu_0 \vec{H}$

$\Rightarrow \vec{H} = \frac{\vec{k}}{\omega\mu_0} \times \vec{E}$

$\lambda = \frac{2\pi}{k}$, $\lambda = c$ so $I = \frac{1}{2}|Re(\vec{E} \times H^*)| = \frac{1}{2}|Re(\vec{E} \times \vec{k} \times \vec{E}^*)|\frac{1}{\omega\mu_0}$

$\vec{k}$ & $\vec{E}$ are $\perp$, so $\vec{k} \times \vec{E}^* = |k|E^* \hat{n}$ where $\hat{n} \perp \vec{k} \perp \vec{E}$

... Thus $I = \frac{1}{2}\frac{1}{\omega\mu_0}|Re(\vec{E} \times |k|E^* \hat{n})|$

$= \frac{1}{2}\frac{|k|}{\omega\mu_0}|E|^2$

- or -

$= \frac{1}{2}\frac{1}{c\mu_0}|E|^2 = \frac{1}{2}\frac{1}{\sqrt{\frac{\mu_0}{\epsilon}}}|E|^2$

This is nice b/c we can calculate intensity (measured quantity) directly w/ complex amplitude $\vec{E}$ (i.e. no need find $\vec{\Sigma}$ or $\vec{\mathcal{H}}$ or $\vec{H}$)

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5. Show the power delivered by this plane wave is infinite. *Hint: it is easiest to use results from Q4*

$$\vec{\mathcal{E}} = \hat{x}E_o cos(-kz - \omega t + \phi)$$

6. Another solution to the wave equation is a spherical wave…write down some reasons/examples why this would be a (better) model for light propagation as compared to a plane wave

7. Calculate power delivered by spherical wave by an completing an integral. *Hint:* $dA = r^2 sin(\phi) d\phi d\theta$

How many photons are delivered every second through a cross sectional of 1 cm^2 if λ=600 nm and E_o = 1 V/m (a typical laser pointer output)

Imagine a downtown street light that is 1 kW in total power. You are observing this light at Spanish Banks beach. How many photons are delivered every second through a cross sectional of 1 mm^2 (the size of your eye pupil) if λ=600 nm