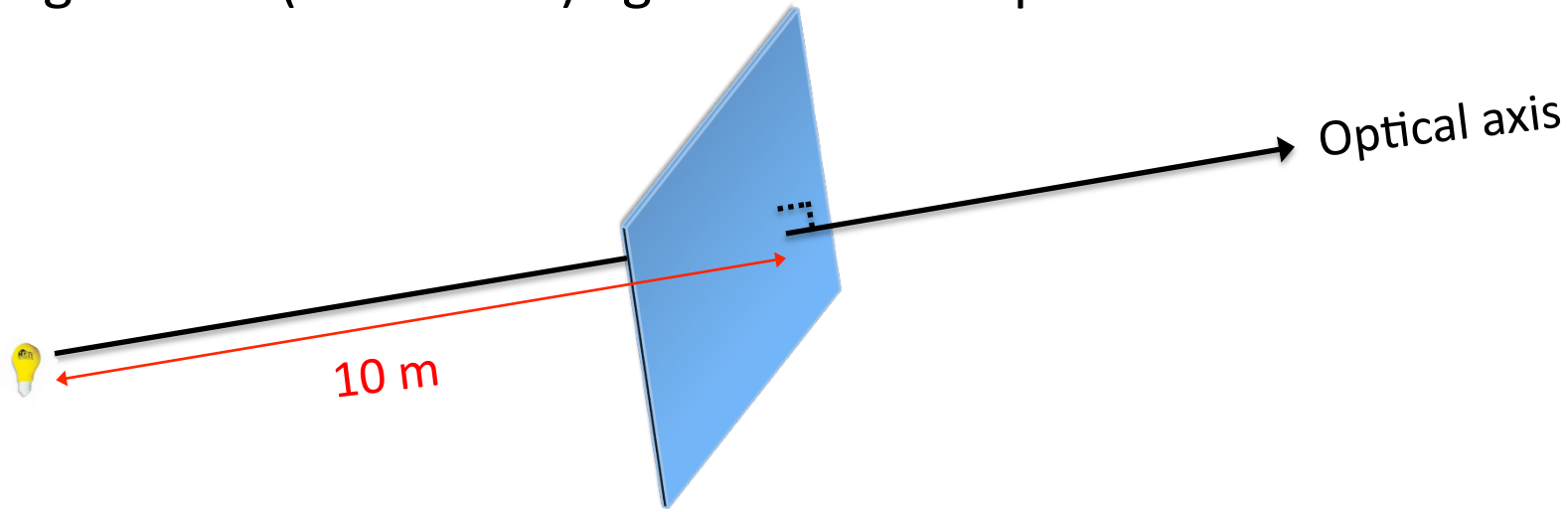
PHYS 408 Group Activities for Jan 18 2011

Please complete these exercises without opening the textbook. If you have questions talk with your group mates or raise your hand and Ellen or David will come to help

1- What is the exact amplitude and phase of a wave emitted by a single color (λ=550 nm) light bulb in the plane shown below?



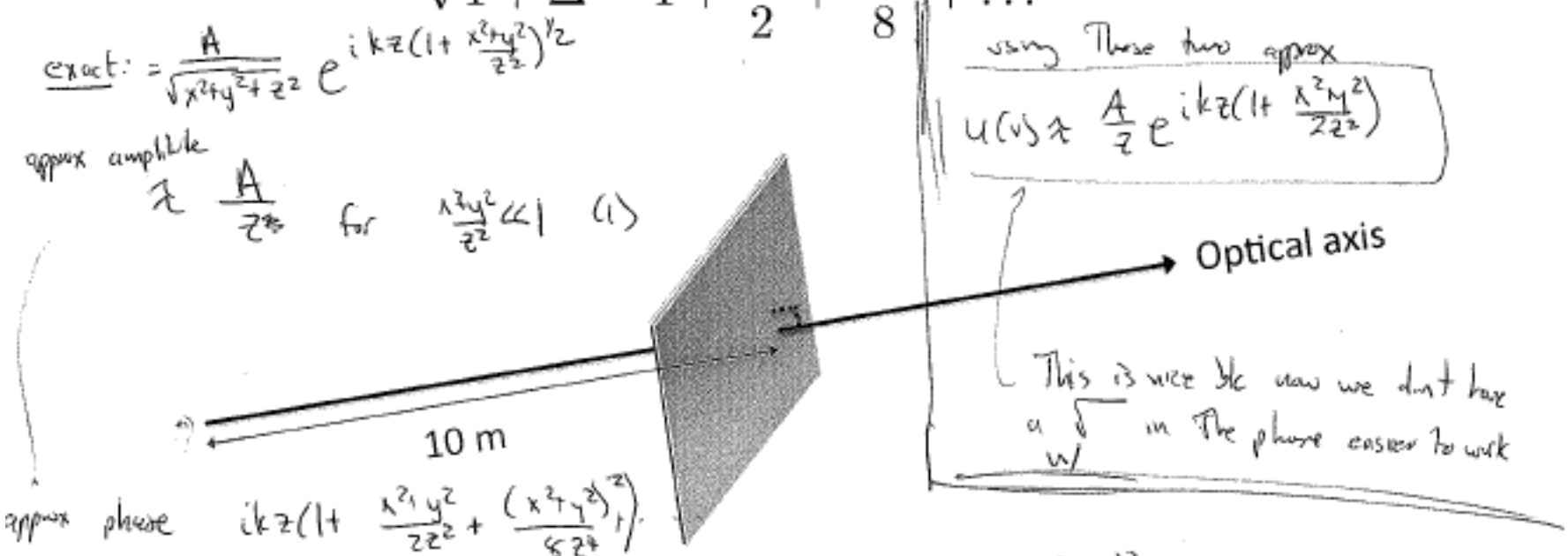
Sketch the wavefronts in that plane shown above. Do these wavefronts have the same amplitude?

2-Repeat using the Frensel approximation…what advantages does this approximation offer? Under what conditions is this approx valid? Hint: $\sqrt{1+\Delta} = 1 + \frac{\Delta}{2} + \frac{\Delta^2}{8} + \ldots$

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PHYS 408 Group Activities for Sept 23 2010

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3-A point source (λ=1 micron) is located at x,y,z=0. Sketch and label a single wavefront from this source a distance of $|\vec{r}| = 1$ projected into a plane that includes the vector $\vec{r}$. Note this is a DIFFERENT plane then the previous exercise.

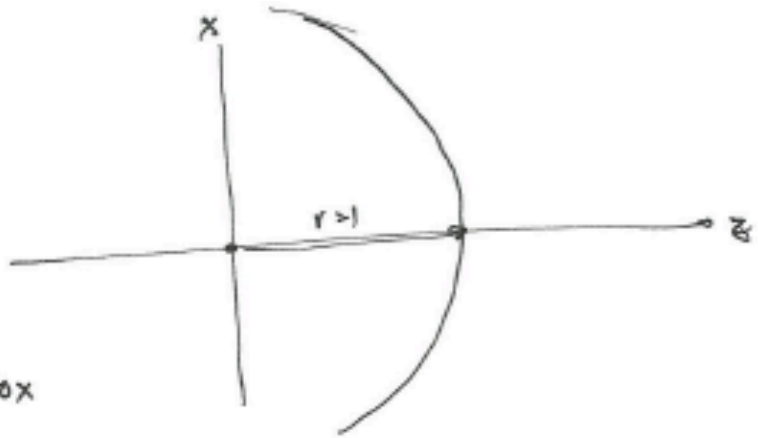
4-Repeat with your Fresnel approximated form on same sketch

5- Now repeat 1&2 at a point 10x as far from the origin.

6-Comparing these two cases…what can you say about the Fresnel approx as you increase your observation distance from the source?

7-Calculate the phase error in your approximation in a observation plane normal to the radial direction if you are 1 cm off the optical axis

3) For simplicity take the plane including $\vec{r}$ to be the x-z plane a single wave front @ $|\vec{r}|=1$ is a circle for the exact spherical wave solution



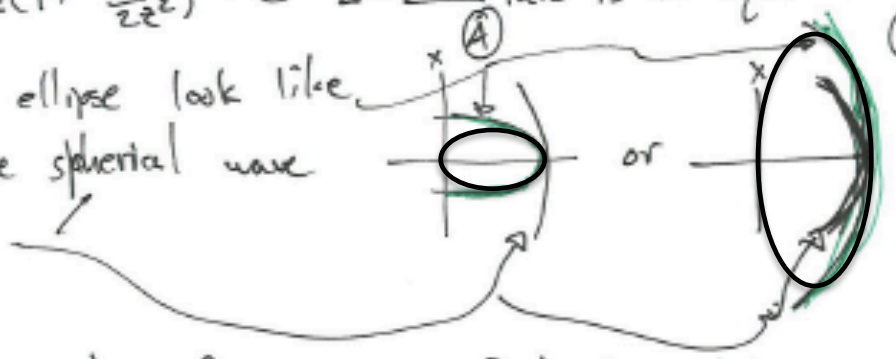
4 $u(r) \sim e^{ikz\left(1+\frac{x^2+y^2}{2z^2}\right)}$ for fresnel approx

in x-z plane ($y=0$), wave fronts are:

$$kz\left(1+\frac{y^2}{2z^2}\right) = C \quad \text{(C: constant)}$$

This is an equation for a ellipse

so what does this ellipse look like relative to the spherical wave front?



There are a number of ways to find the relative curvature

- Take 2nd derivative & look @ curvature
- See where circle & ellipse intercept x-axis (w/ z very small)

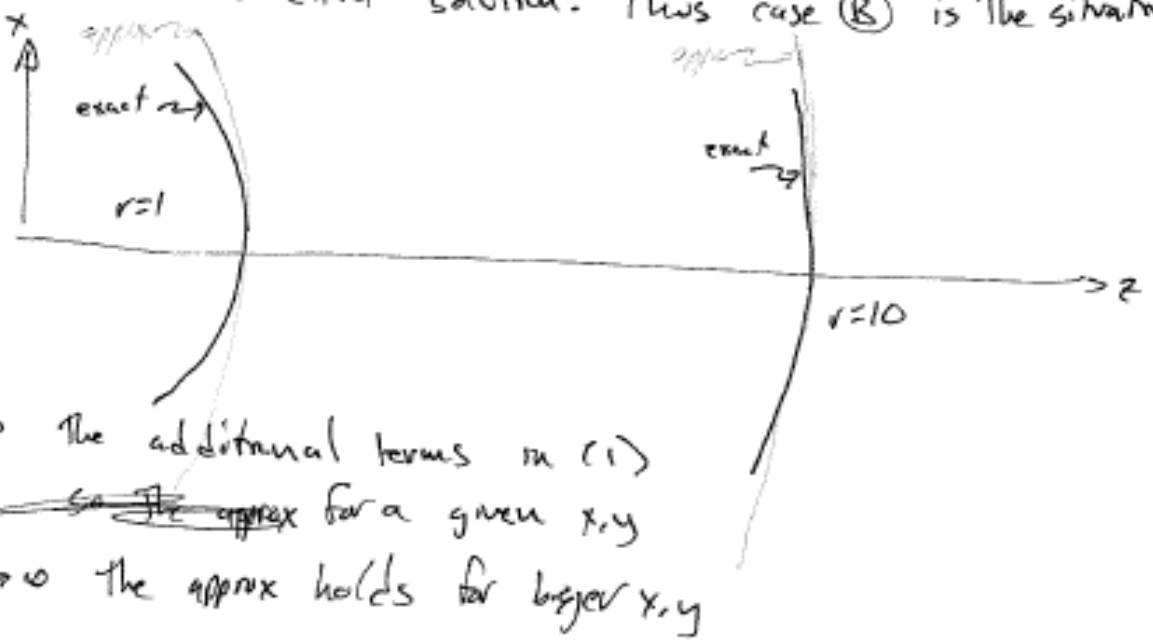
5 Here is a hand waving description:

The exact solution is: $\sim e^{ikz\left(1+\frac{x^2+y^2}{2z^2}+\frac{(x^2+y^2)^2}{8z^4}+\dots\right)}$ (1)

The approx solution is $\sim e^{ikz\left(1+\frac{x^2+y^2}{2z^2}\right)}$ (2)

That add to the phase

Thus the exact solution has more terms dependent on x & y. So if we want to stay on a crest (keep phase constant) as x, y increase z needs to decrease more for the exact solution. Thus case (B) is the situation:



6 So as $z \to \infty$ the additional terms in (1) contribute less ~~So the approx~~ for a given x, y meaning as $z \to \infty$ the approx holds for bigger x, y

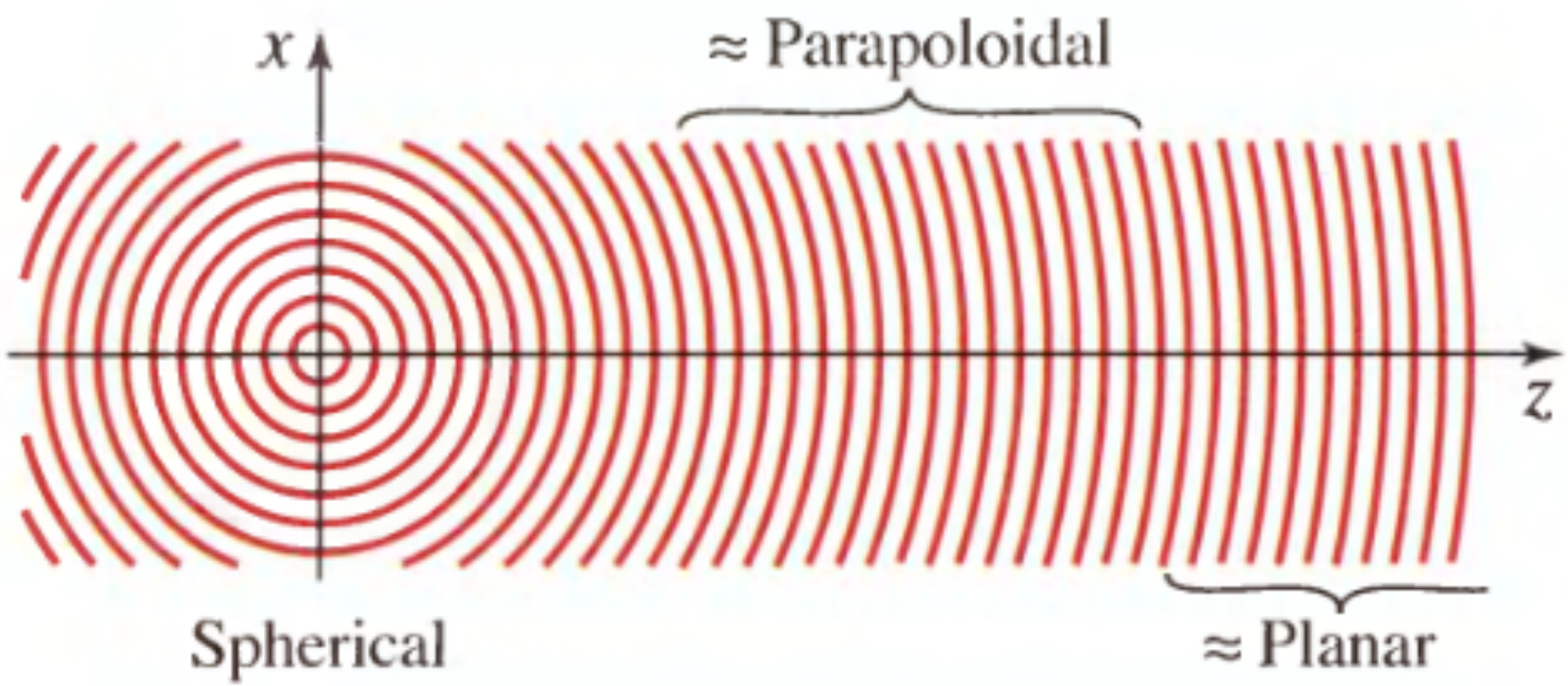


Image from B. Saleh and M Teich, Fundamentals of Photonics, 2nd Edition. (2007) Wiley and Sons

We can only measure the intensity of E&M waves…so why keep track of the phase?



"Hillmorton radio masts" by G-Man - Photo by G-Man May 2005. Licensed under Public Domain via Wikimedia Commons - https://commons.wikimedia.org/wiki/File:Hillmorton_radio_masts.jpg#/media/File:Hillmorton_radio_masts.jpg

The Fresnel Approximation of $E = \frac{A}{r} e^{ikr}$

is $E \approx \frac{A}{z} e^{ik(z+\frac{x^2+y^2}{2z})}$ and we called this a

"paraboloidal" wave with paraboloidal phase fronts. But some students have pointed out to me that $C = k(z + \frac{x^2+y^2}{2\,z})$ is not a paraboloidal surface. What is it?

So if they are the wavefronts, then what is parabolodial about this wave?

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When constant defines a paraboloidal surface

$$E \approx \frac{A}{z} e^{ik\left(\frac{x^2+y^2}{2z}\right)} e^{ikz}$$

Plane wave carrier

= A(r)

-complex amplitude that modulates carrier wave

-slowly varying envelope w.r.t. carrier oscillations

7-Write down some proof that will convince myself or Ellen that A(r) is slowly varying with respect to carrier wave. Include sketches of |A(r)| and arg[(A(r)]

Conceptually this question is ~~asking~~ asking how to show the phase & amplitude of $A(r)$ changes slowly with respect carrier wave.

1st what is the length scale the carrier changes?

in $z \to z+\Delta z$; the phase changes by 2π when $\Delta z = \lambda$

So check if $|A(r)|$ & $\arg[A(r)]$ change much on this scale

$\Rightarrow |A(r)| = \frac{A}{z}$ so e $z \to z+\lambda$ $|A(r)| = \frac{A}{z+\lambda}$ plugging in # w/ $z \approx 1m$, $\lambda \approx 10^{-6}$

we see $A \to \frac{A}{1+10^{-6}}$

which doesn't change much

you can do the same approach for phase terms.

A more rigorous approach is

$$\Delta|A(r)| = \left|\frac{A}{z} - \frac{A}{z+\Delta z}\right| = \frac{A\Delta z}{z(z+\Delta z)} \overset{\Delta z \ll z}{\approx} \frac{A\Delta z}{z^2}$$ ← as $\frac{\Delta z}{z^2} \ll 1$ for $\Delta z \sim \lambda$, $z \gg \lambda$ amplitude changes little

$$\Delta \arg[A(r)] = \left| k\frac{x^2+y^2}{2z} - k\frac{x^2+y^2}{2(z+\Delta z)} \right| \approx k\frac{2\Delta z (x^2+y^2)}{2z^2}$$

$$= k(x^2+y^2)\frac{\Delta z}{z^2}$$

again for $\Delta z \sim \lambda$, $z \gg \lambda$

this expression is small as long as x^2+y^2 isn't too big (you don't go too far off z axis)