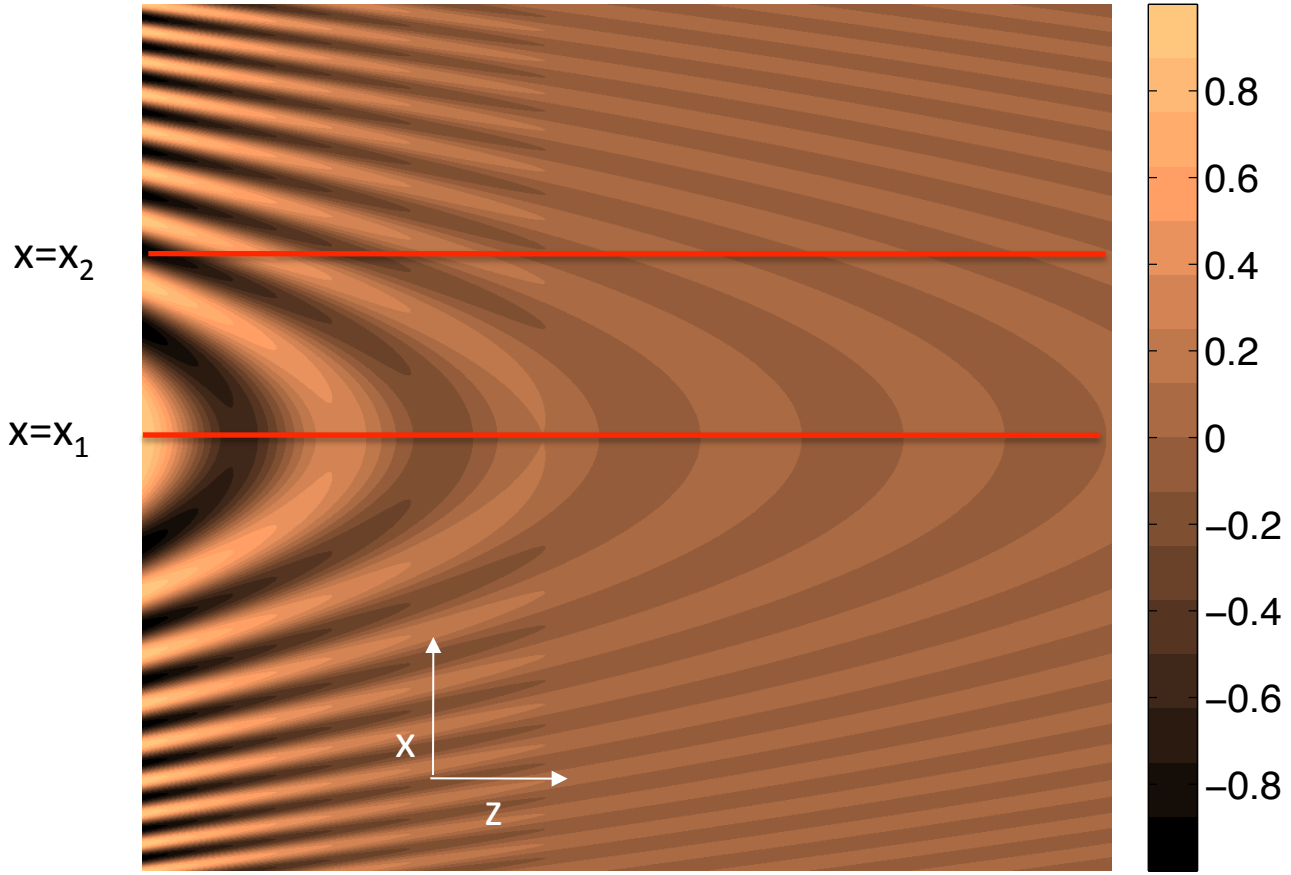
PHYS 408 Group Activities for Jan 20 2010

0-Suppose a wave's real electric field in the y direction is shown below on the left (white = 1V/m, black = -1V/m, color scale on right) at some fixed time.

a) Sketch $E_y(z)$ at the two slices at $x=x_1$ and at $x=x_2$

b) If it is possible sketch the wavefronts of this wave in the x-z plane

c) If it is possible sketch the k-vectors of this wave in the x-z plane.



Bonus: determine the functional form of E(x,z)

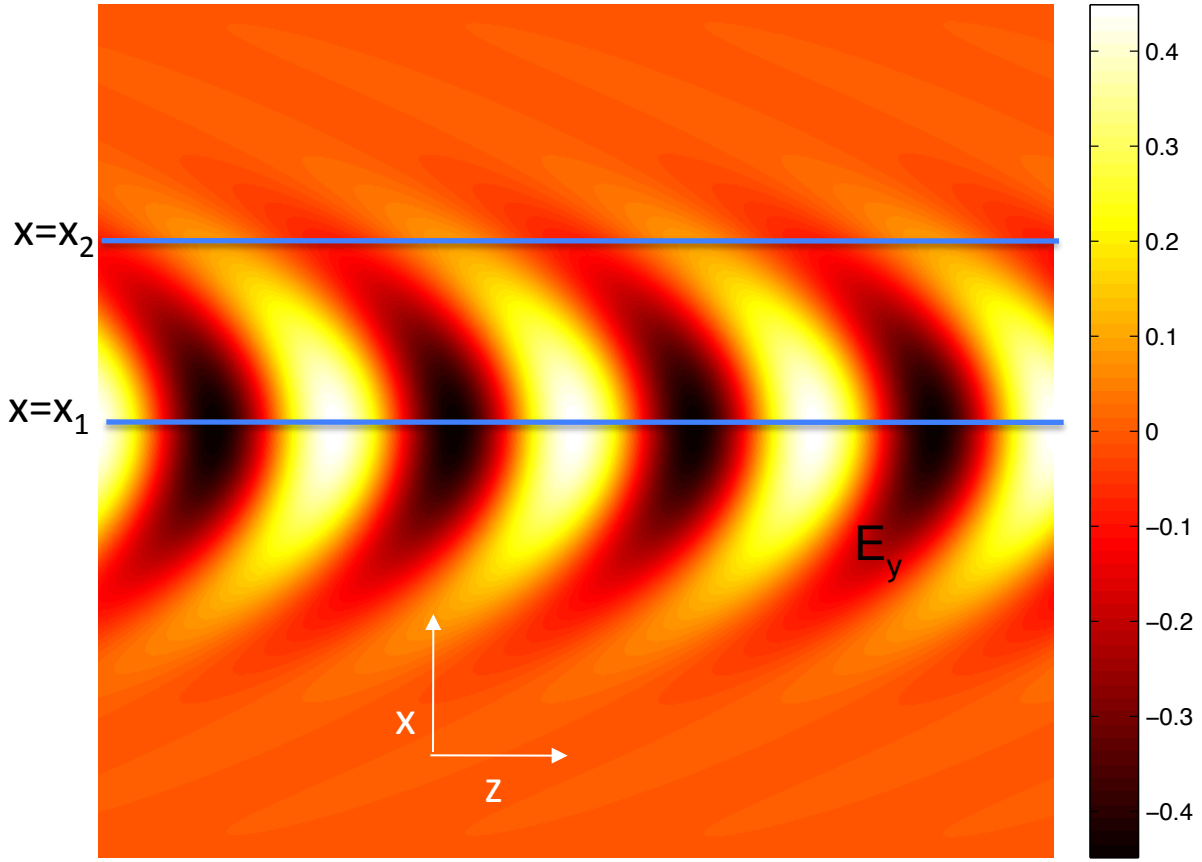
PHYS 408 Group Activities for Jan 20 2010

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Bonus: determine the functional form of E(x,z)

Many of you had questions regarding the $\vec{k}$ vector, λ & the phase ϕ

in general $\nabla\phi = \vec{k}$

example: spherical wave $\sim e^{ikr} = e^{i\phi}$

$\nabla\phi = \nabla(kr) = k\hat{r} = \vec{k}$ w/ ∇ operating in spherical coordinates

$|\vec{k}| = \frac{2\pi}{\lambda} = k$

example plane wave @ angle θ $\sim e^{i(k\cos\theta z + k\sin x)} = e^{i\phi}$

$\nabla\phi = \nabla(k\cos\theta z + k\sin\theta x) = k\cos\theta\,\hat{z} + k\sin\theta\,\hat{x}$

$= k_z\hat{z} + k_x\hat{x} = \vec{k}$

$|\vec{k}| = \sqrt{k_z^2 + k_x^2} = k\sqrt{\cos^2\theta + \sin^2\theta} = k = \frac{2\pi}{\lambda}$

example ~~fresn~~fresnel approx $\sim e^{ikz}e^{ik\frac{x^2+y^2}{2z}} \sim e^{i\phi}$

$\nabla\phi = \nabla\left(kz + k\frac{x^2+y^2}{2z}\right) = \left(k - k\frac{x^2+y^2}{2z^2}\right)\hat{z} + \frac{kx}{z}\hat{x} + \frac{ky}{z}\hat{y}$

$|\vec{k}| \neq k$!

Why not ?? => b/c fresnel is approximation!

0.5- From Maxwell's equations we derived the Helmholtz wave equation

$$\nabla^2 \vec{E} + k^2 \vec{E} = 0$$

a) what were the assumptions (if any) in this derivation?

we then considered started considering the the scalar wave equation

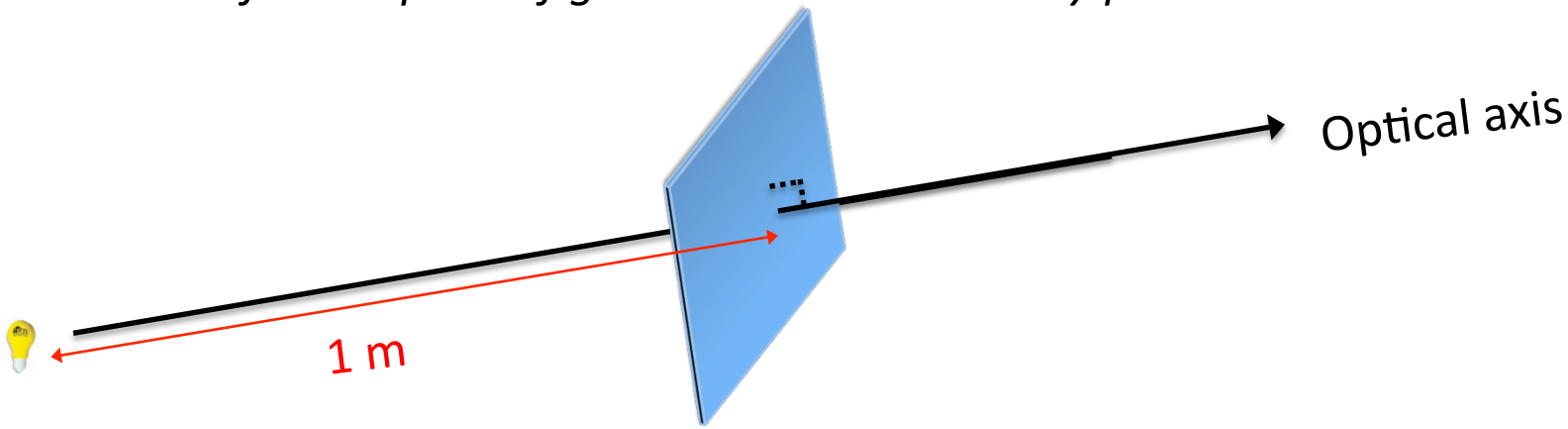
$$\nabla^2 U + k^2 U = 0$$

b) What is U in relation to $\vec{E}$?

c) Why are we using U rather than $\vec{E}$?

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1-Lets use some real numbers to see how "good" the Fresnel approx is. Calculate the phase error in the Fresnel approximation in a observation plane normal to the radial direction if you are 1 cm off the optical axis and you are a distance 1 m from the source. *Hint: the first step is to figure out what I mean by phase error…*



Bonus: on the figure below plot (as a function of z) the boundary showing when the Fresnel approx is valid

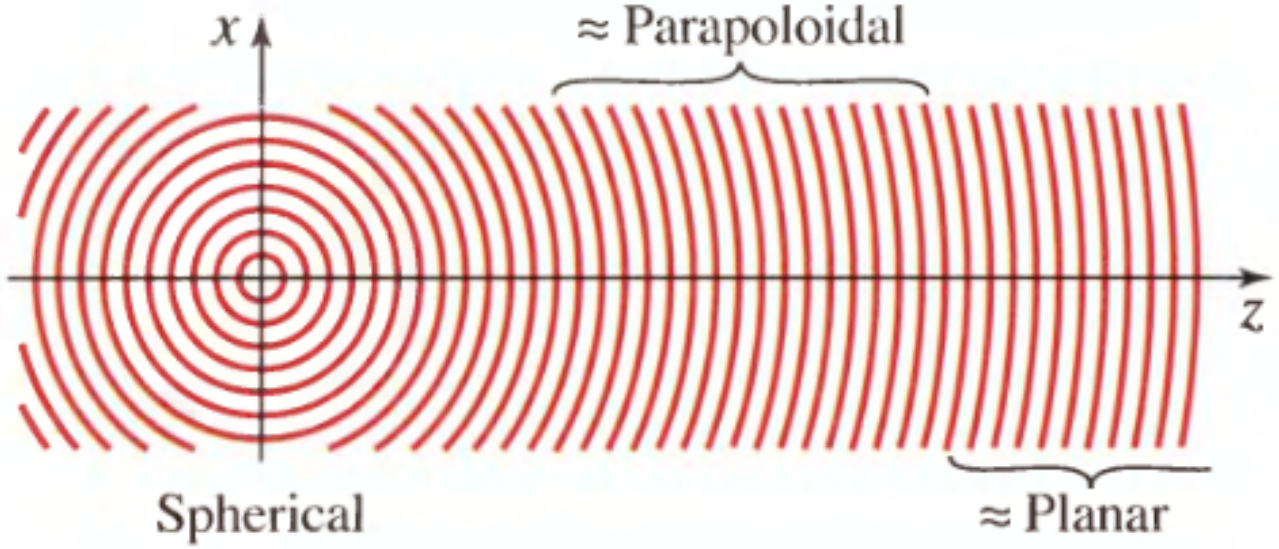


Image from B. Saleh and M Teich, Fundamentals of Photonics, 2nd Edition. (2007) Wiley and Sons

Calculation of phase error for Fresnel Approx



Spherical Wave $u(r) \sim \frac{A}{r} e^{ikr}$

$\Rightarrow \phi_{exact} = k\sqrt{(10^{-2})^2 + 1} = 2\pi \cdot 10^6 \left(1.000049998 75\right)$

Fresnel Approx $u(r) \sim \frac{A}{z} e^{ikz} e^{ik\left(\frac{x^2+y^2}{2z}\right)}$

$\phi_{approx} = kz\left(1 + \frac{x^2+y^2}{2z^2}\right)$

$= 2\pi \cdot 10^6 \left(1 + \frac{(10^{-2})^2}{2}\right)$

$= 2\pi \cdot 10^6 (1.00005)$

... so phase error:

$\delta\phi = |\phi_{exact} - \phi_{approx}| = 2\pi \cdot 10^6 (1.25) \cdot 10^{-9} \sim 2\pi 10^{-3} (kz)$

$\Rightarrow$ small so approx is good

When constant defines a paraboloidal surface

$$E \approx \frac{A}{z} e^{ik\left(\frac{x^2+y^2}{2z}\right)} e^{ikz}$$

Plane wave carrier

= A(r)

-complex amplitude that modulates carrier wave

-<u>slowly varying envelope</u> w.r.t. carrier oscillations

Write down some proof that will convince myself or Ellen that A(r) is slowly varying with respect to carrier wave. Include sketches of |A(r)| and arg[(A(r)]

~~For~~ From last class we showed The envelope $A(r) = \frac{A}{z} e^{ik\frac{x^2+y^2}{2z}}$

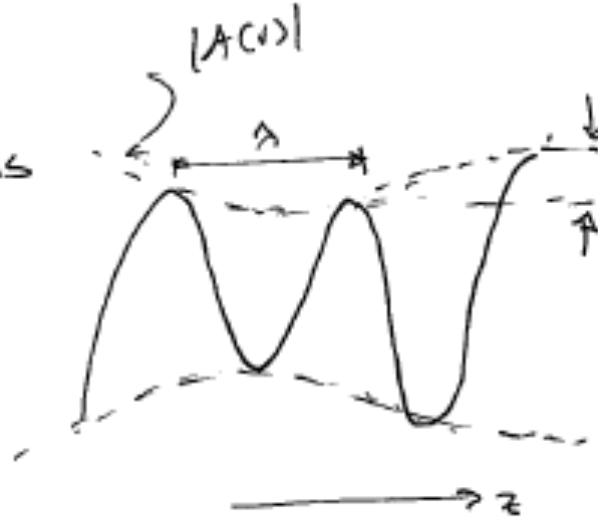
varys slowly wrt to The plane wave carrier. As $A(r)$ is complex need to look @ $|A(r)|$ & $\arg[A(r)]$ → ie on length scale $\Delta z \sim \lambda$

$|A(r)| = \frac{A}{z}$

so $\Delta|A(r)| = \left|\frac{A}{z} - \frac{A}{z+\Delta z}\right| = \frac{A(z+\Delta z) - Az}{z(z+\Delta z)} = \frac{A\Delta z}{z(z+\Delta z)} \overset{\Delta z \ll z}{\simeq} \frac{A\Delta z}{z^2}$

as $\frac{\Delta z}{z^2} \ll 1$ for $\Delta z \sim \lambda$, $z \gg \lambda$

$\Rightarrow$ a plot of This



$\Delta|A(r)|$ varys slowly over λ

The corresponding change in phase is:

$\Delta \arg[A(r)] \simeq k(x^2+y^2)\frac{\Delta z}{z^2}$

Question: What is The sketch for The phase picture?

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$$E \approx \frac{A}{z} e^{ik\left(\frac{x^2+y^2}{2z}\right)} e^{ikz}$$
$$\approx A(r) e^{ikz}$$

When A(r) is slowly varying we call this a paraxial wave. We've seen the amplitude |A(r)| doesn't change much over length scale

2-Perhaps more important is the k-vectors of a paraxial wave…what is intrinsic about its k-vectors? Prove your answer… *HINT: what is the relationship between wavefronts and k-vectors and how do you calculate the k-vector from the phase?*

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The $\vec{k}$ vectors are mostly along the $\hat{z}$ axis. To prove this remember $\vec{k}$ are $\perp$ to wavefronts (surfaces of constant phase). Thus by taking $\vec{\nabla}\phi$ where $\phi = k\left(\frac{x^2+y^2}{2z}\right) + kz$ ⟹ complete this to prove $\vec{k}$ is pointing mostly in $\hat{z}$ direction



6 continued

k vectors are normal to wavefronts so close to original the paraxial wave vectors have a small angle θ_1

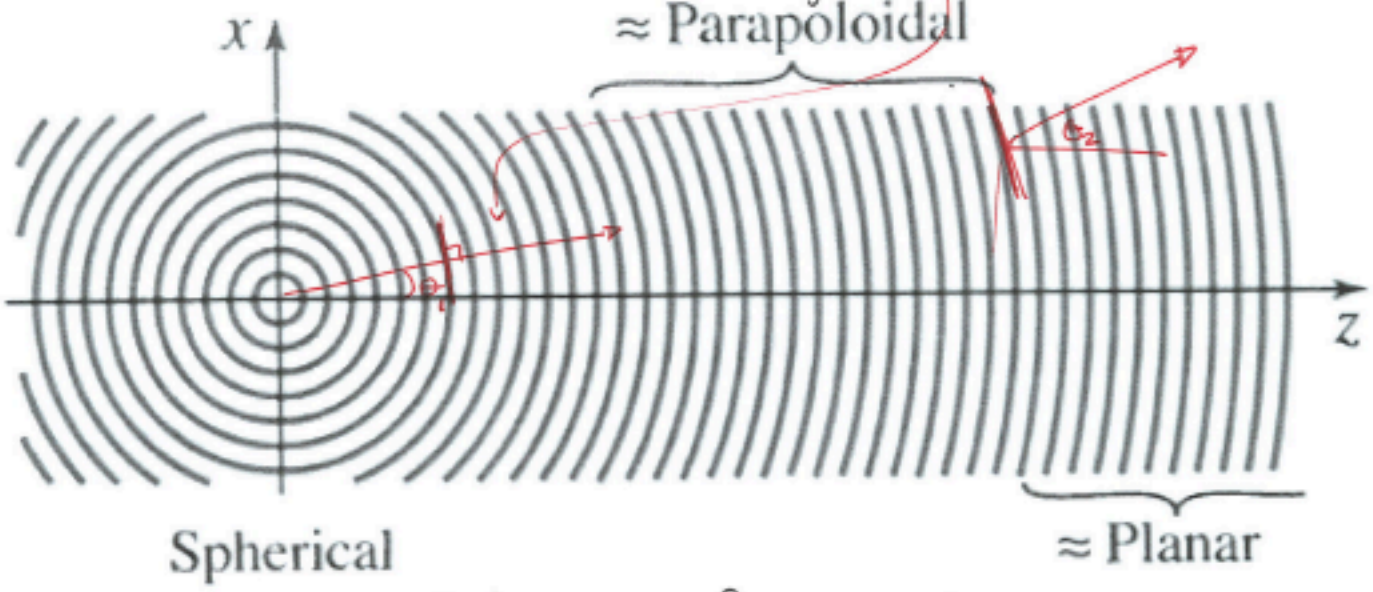


Image from B. Saleh and M Teich, Fundamentals of Photonics, 2nd Edition. (2007) Wiley and Sons

further away from origin) we can accept a point further from origin. This means $\theta_2 > \theta_1$ i.e. we can accept a bigger angle & still have the paraxial approx

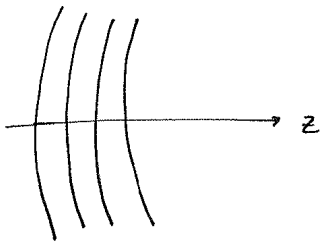
→ you can see this mathematically by finding the k vector via $\nabla\phi = \hat{k}$

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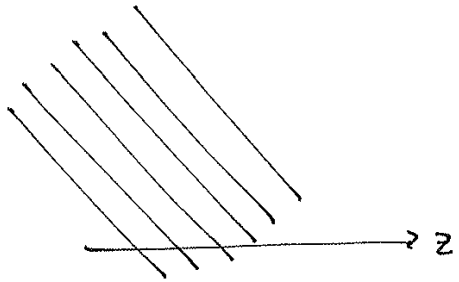
This is an old Midterm Exam question on paraxial waves:

3- For a few different waves, a sketch of their respective wavefronts are given below. For each wave indicate whether it fulfills a condition of being a paraxial wave RELATIVE TO THE Z-AXIS. Justify your answer to receive any marks

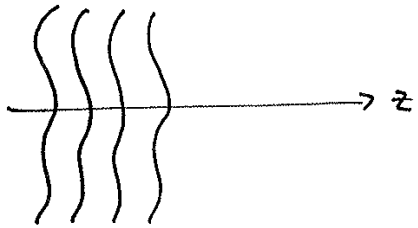
A



B



C

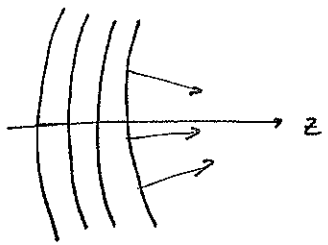


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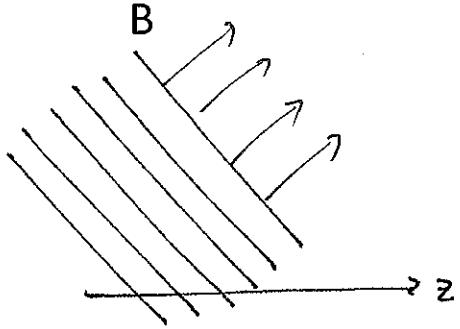
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A



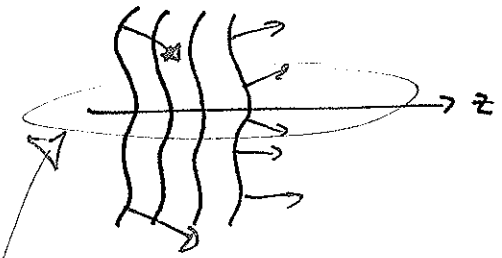
paraxial as $\vec{k}$ vectors are close to z-axis

B



not paraxial wrt to z-axis

C



if you consider entire region The not paraxial but if you consider only small region around z-axis curvature is small => small angles Thus can be considered paraxial w/in This region

$$E \approx A(r)e^{ikz}$$

We derived one such $A(r)$ by approximating a spherical wave. However there are other possibilities for $A(r)$...but there must be some restrictions on $A(r)$ for $E \approx A(r)e^{ikz}$ to approximate an electromagnetic wave

The paraxial wave equation

$$\nabla_T^2 A + i2k\frac{\partial}{\partial z}A = 0$$

For HW #3 you need to derive this eqn. from the Helmholtz wave equation...and physically relate the approximations to the wave (just writing the approximations will net you zero marks). Hint: use activities from this class!

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2-So we are worrying a lot about wavefronts, their shape, and there accompanying k-vectors. Why?

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