1-Midterm reschedule Feb 8? or 10?

2-Lab partners by Feb 1

3- “Extra” materials during in-class activities

no computers open

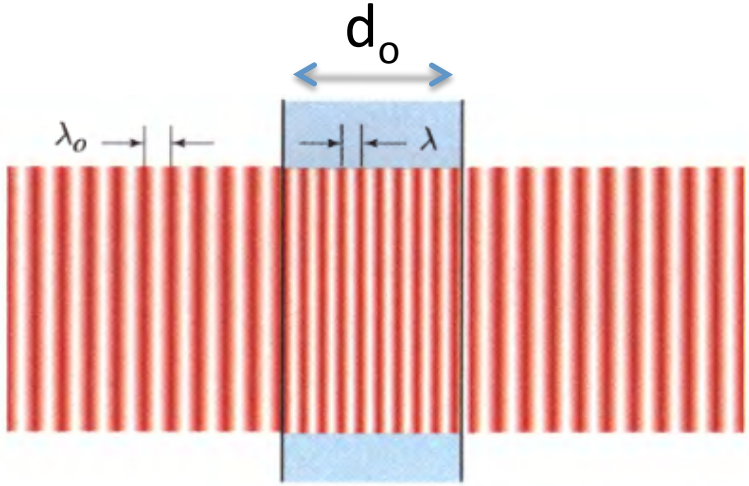
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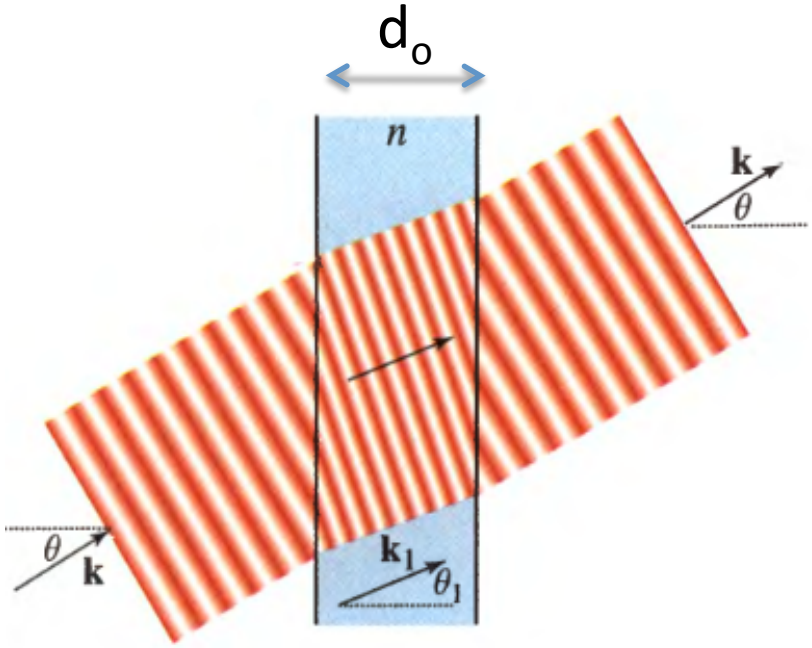
PHYS 408 Group Activities for Jan 25 2011

Please complete these exercises without opening the textbook. If you have questions talk with your group mates or raise your hand and Ellen or David will come to help

1-To see how different optical components (lenses, etc) affect wave propagation we are going to keep track of the changes to the wavefronts by looking at the transmittance. $$t(x,y) = \frac{E(x,y,z=d)}{E(x,y,z=0)}$$

Find t(x,y) for these two cases





Images from B. Saleh and M Teich, Fundamentals of Photonics, 2nd Edition. (2007) Wiley and Sons

2-Based on your answers…what physically does t(x,y) represent?

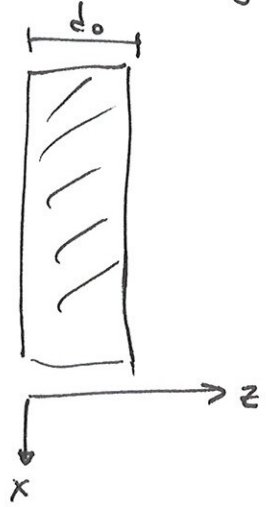
3- In either case there should be no x or y dependence in the transmittance. Describe how you can argue this by counting wavefronts from z=0 to z=d in the z-direction and showing it is independent of x,y

1-)

for the first case, the wave has a general form

$$E = E_0 e^{ik_0 z}$$

taking $z=0$ @ start of glass plate



$$t(x,y) = \frac{E_0 e^{ik_0 n d}}{E_0}$$

$$= e^{ik_0 n d}$$

---

general form of wave in glass plate is:

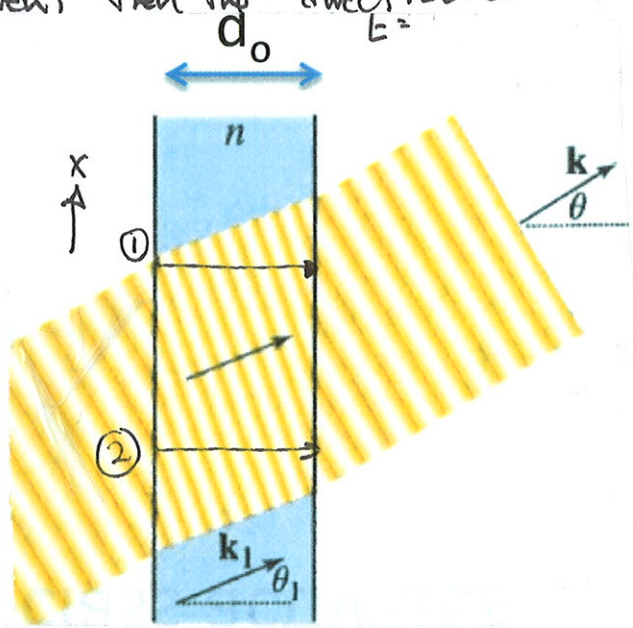
$$E = E_0 e^{ik_0 n(\cos\theta_1 z + \sin\theta_1 x)} \quad \text{where } n\sin\theta = n\sin\theta_1$$

$$t(x,y) = \frac{E_0 e^{ik_0 n(\cos\theta_1 d + \sin\theta_1 x)}}{E_0 e^{ik_0 n \sin\theta_1 x}}$$

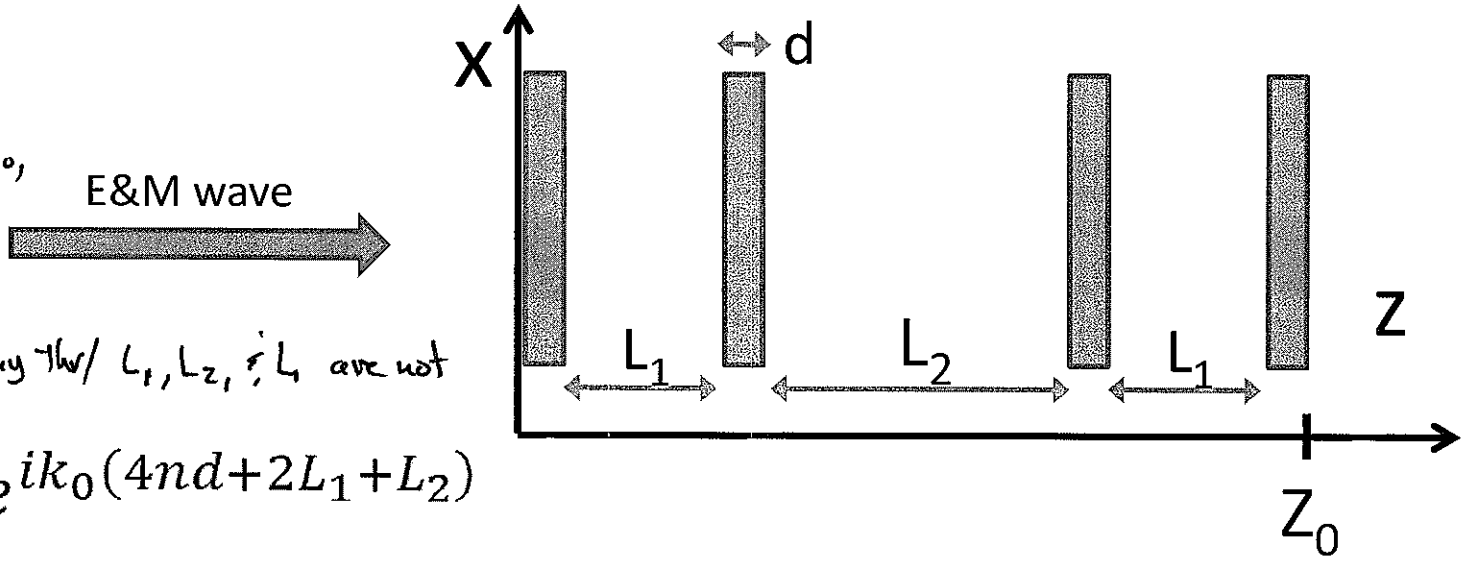
$$= e^{ik_0 n d_0 \cos\theta_1}$$

2-) $t(x,y)$ accounts for the phase shift picked up by the wave in the $\hat{z}$ direction (which is different than the direction of propagation)

3- Wavefronts represent phase accumulation, so by counting the number you can get a measure of $t(x,y)$. @ positions ① & ② one counts 7 wavefronts. Indeed anywhere in x you will count the same #, thus $t(x,y)$ should be independent of x,y



5- An E&M wave with complex amplitude $\vec{E}(\vec{r}) = E_0\hat{x}e^{ik_0z}$ is incident on 4 pieces of glass as shown below. All 4 pieces of glass have a thickness ***d*** and index of refraction ***n*** (vacuum between the pieces of glass). What is the complex amplitude of the field immediately after the 4th piece of glass on the right (at $z=z_0$)?



① we want the field @ $z=z_0$, so there shouldn't be any z dependence ⇒ A & C are out

② B is out b/c the phase delay thru/ $L_1, L_2,$ & L_1 are not accounted for

(A) $E_0\hat{x}e^{ik_0z}e^{ik_0(4nd+2L_1+L_2)}$

(B) $E_0\hat{x}e^{ik_0(4nd)}$

(C) $E_0\hat{x}e^{ik_0z}e^{ik_0(4nd)}$

③ E is out b/c the phase factor $e^{ik_0z_0}$ would account for an extra propagation distance of z_0 in vaccum

(D) $E_0\hat{x}e^{ik_0(4nd+2L_1+L_2)}$

(E) $E_0\hat{x}e^{ik_0z_0}e^{ik_0(4nd+2L_1+L_2)}$

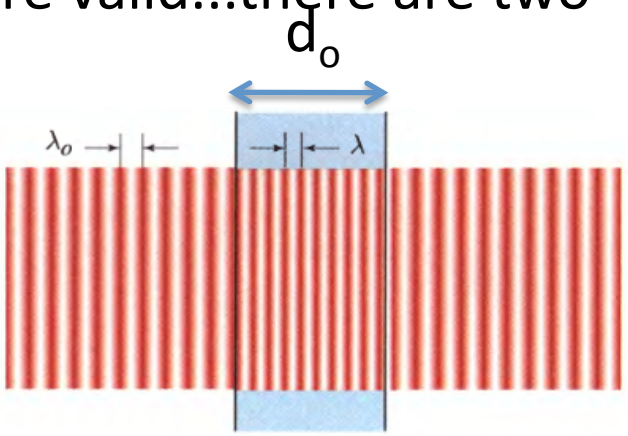
(F) something else

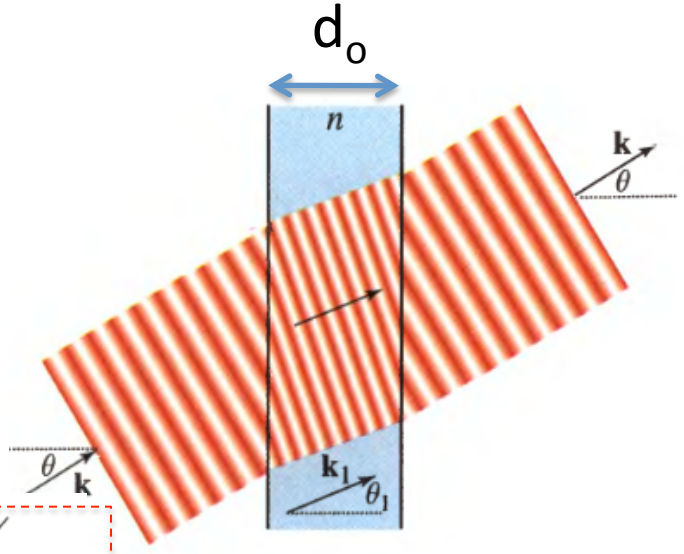
④ D correctly accounts for all phase delays

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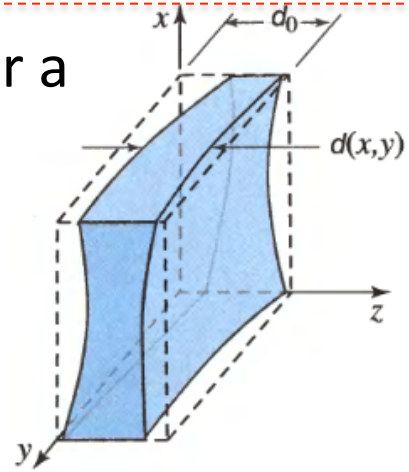
Please complete these exercises without opening the textbook. If you have questions talk with your group mates or raise your hand and Ellen or David will come to help

5-Under what two specific approximations can these two transmittances be equal? Give the limits where the approximations are valid...there are two





Derive the general result for a plate of varying thickness



Images from B. Saleh and M Teich, Fundamentals of Photonics, 2nd Edition. (2007) Wiley and Sons

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Please complete these exercises without opening the textbook. If you have questions talk with your group mates or raise your hand and Ellen or David will come to help

We've found that the complex transmittance of a lossless rectangular piece of glass is

$$t(x,y) = e^{i(nk_o d_o \cos\theta_1)} \approx e^{ink_o d_o}$$

If $\theta_1 \approx \frac{\theta}{n} \ll 1$ &

$\frac{k\theta_1^2 n d_o}{2} \approx k\frac{\theta^2}{n} d_o \ll \pi$

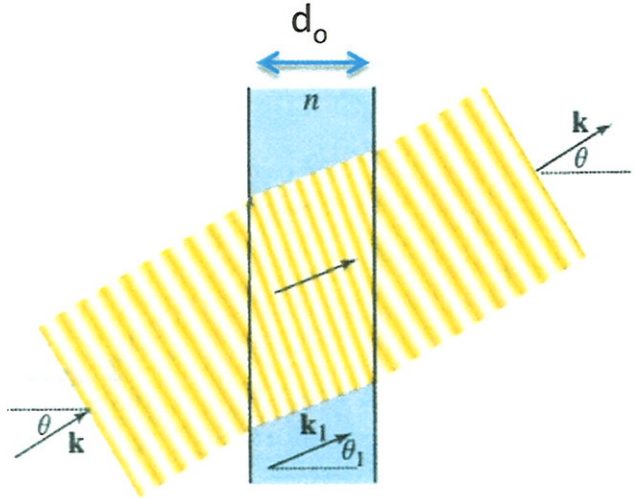


Image from B. Saleh and M Teich, Fundamentals of Photonics, 2nd Edition. (2007) Wiley and Sons

for $\theta_1 \ll 1$ $\quad \cos\theta_1 \approx 1 - \frac{\theta_1^2}{2}$

~~$t(x,y) \approx e^{ink_o d_o(1-\frac{\theta_1^2}{2})}$~~

$\Rightarrow t(x,y) \approx e^{ink_o d_o(1-\frac{\theta_1^2}{2})}$

if $\frac{nk_o d_o \theta_1^2}{2} \ll \pi$

$\Rightarrow t(x,y) = e^{ink_o d_o}$

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We've found that the complex transmittance of a lossless rectangular piece of glass is

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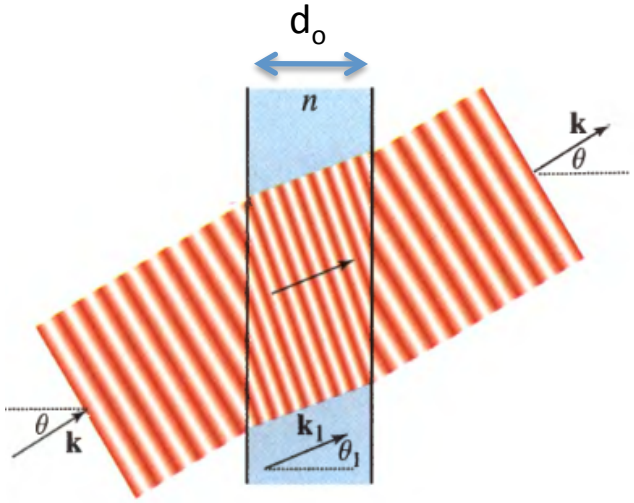


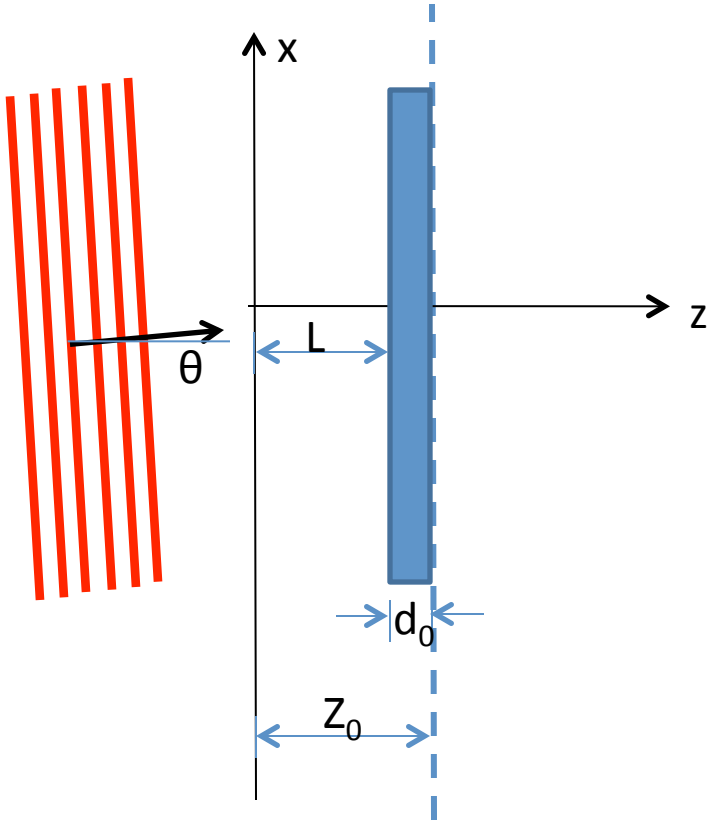
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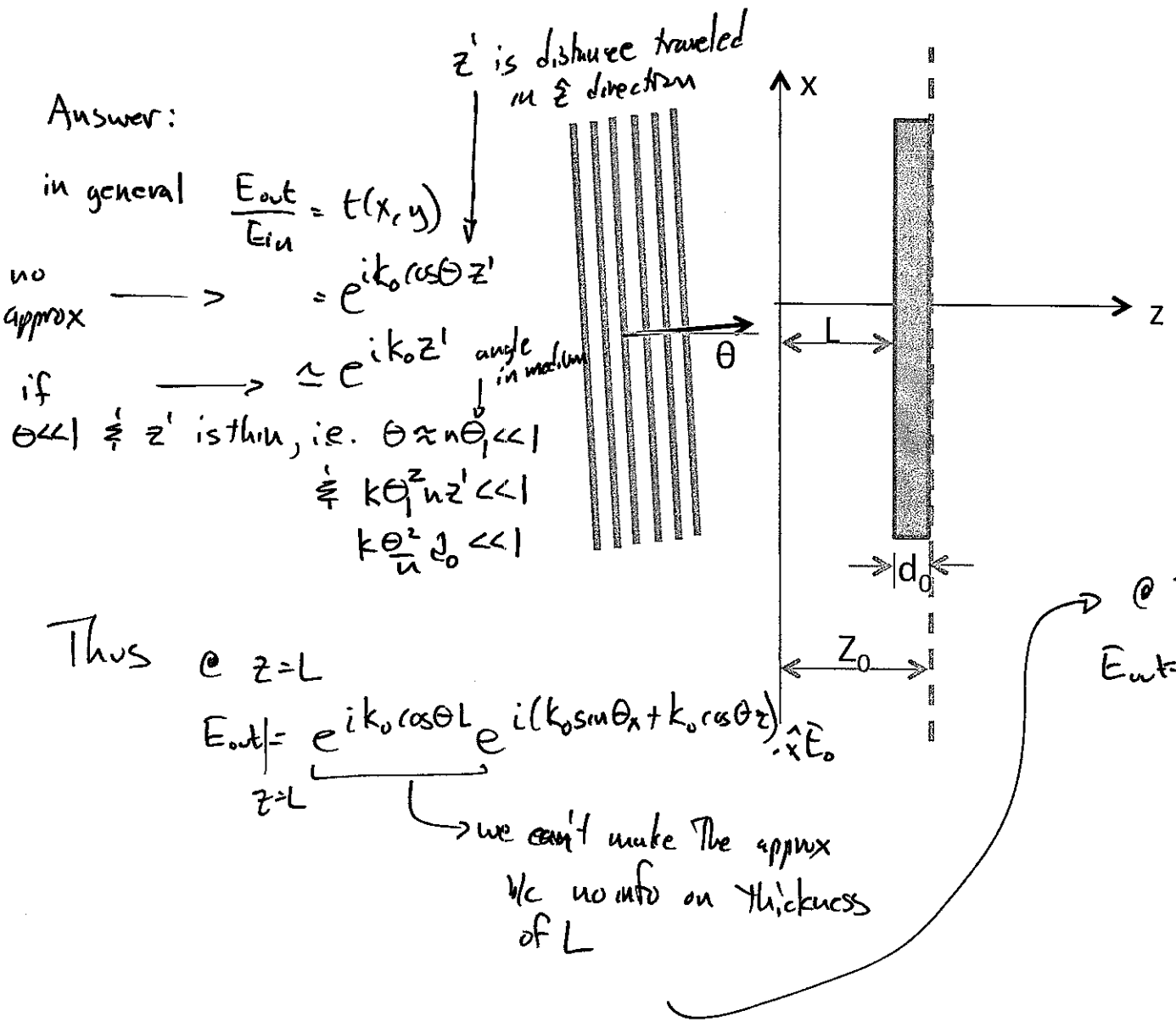
Please complete these exercises without opening the textbook. If you have questions talk with your group mates or raise your hand and Ellen or David will come to help

6- A paraxial E&M plane wave $\vec{E}(r) = E_0\hat{x}e^{i\vec{k}_0\cdot\vec{r}} = E_0\hat{x}e^{i(k_0 sin\theta x + k_0 cos\theta z)}$ with small angle θ w.r.t. the z-axis is incident on this thin piece of glass with index n=2.

What is the complex amplitude of the field (as a function of z) after emerging from the glass?



A paraxial E&M plane wave $\vec{E}(r) = E_0\hat{x}e^{i\vec{k}_0\cdot\vec{r}} = E_0\hat{x}e^{i(k_0 sin\theta x + k_0 cos\theta z)}$ with small angle θ w.r.t. the z-axis is incident on this thin piece of glass with index n=2. What is the complex amplitude of the field immediately after the piece of glass (at $z=z_0$)?



Answer:

in general $\frac{E_{out}}{E_{in}} = t(x,y)$

no approx ——> $= e^{ik_0 cos\theta z'}$

if ——> $\simeq e^{ik_0 z'}$ (angle in medium)

$\theta \ll 1$ & z' is thin, i.e. $\theta \approx n\theta_1 \ll 1$

$\Leftarrow k\theta_1^2 n z' \ll 1$

$k\frac{\theta^2}{n} d_0 \ll 1$

Thus @ $z=L$

$E_{out}|_{z=L} = e^{ik_0 cos\theta L} e^{i(k_0 sin\theta_x + k_0 cos\theta z)}\cdot\hat{x}E_0$

↳ we can't make the approx b/c no info on thickness of L

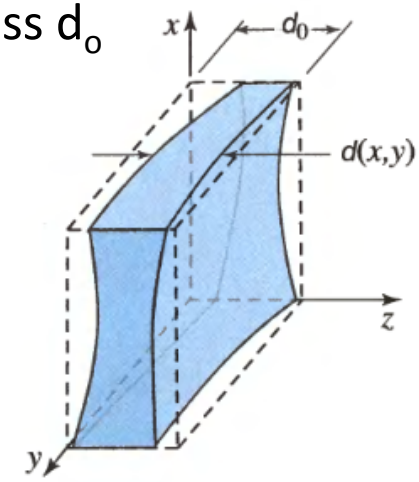
@ $z=z_0$

$E_{out} = E_0\hat{x}e^{ik_0 cos\theta L} e^{ik_0 d_0 n} e^{ik(sin\theta_x + cos\theta z)}$

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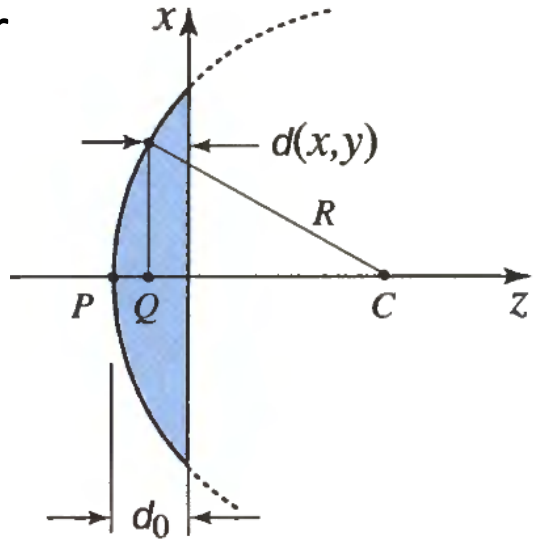
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7-If the thickness now depends on x,y then we can change the outgoing wave by more than just a constant phase shift. Find the transmittance for an lossless piece of glass with arbitrary thickness d(x,y) and maximum thickness d_o (d_o is thin).

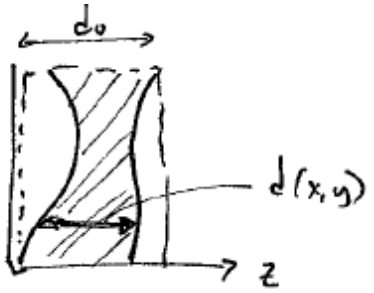


Hint: t(x,y) accounts for phase shift in z direction so you need to account for how much glass and how much air the wave goes through in the z direction as a function of x,y

Bonus: What is t(x,y) for a "thin" spherical lens?



Images from B. Saleh and M Teich, Fundamentals of Photonics, 2nd Edition. (2007) Wiley and Sons



As long as d_0 is thin enough ξ' you have paraxial waves both in & out of material → note this means effectively you are ignoring refraction @ surface

(The same approximations that were covered in the 1st two questions today)

Then the transmittance is:

$$t(x,y) = \underbrace{e^{ik_0[d_0 - d(x,y)]}}_{\text{phase delay in air}} \underbrace{e^{ik_0 n d(x,y)}}_{\text{phase delay in material}} = e^{ik_0 d_0} e^{ik_0(n-1)d(x,y)}$$