PHYS 408 Group Activities for Mar 15 2011

From our work on last week you derived from physics principles a different way to calculate the output image g(x,y). In doing so, you "derived" the Famous Huygens-Fresnel integral.

$$g(x,y) = \frac{i}{\lambda}\iint_{-\infty}^{\infty} f(x',y') \frac{e^{ikz} e^{i\frac{k}{2z}((x-x')^2+(y-y')^2)}}{z} dx'dy'$$

Amplitude/phase of point source or spherical wavelet

Impulse response function h(x,y) of free space: Fresnel approx of spherical wave at (x,y, z=d) from point source, impulse at (x', y' , z=0)

Then, we GREATLY simplified this expression by assuming we are in the far field $k\frac{x'^2+y'^2}{2d} \ll \pi$ (i.e. $exp\left\{i\frac{k}{2z}(x'^2+y'^2)\right\} \to 1$). The Fraunhofer approximation is also famous in its own right.

$$g(x,y) = \frac{i}{\lambda z} e^{ikz} e^{i\frac{k}{2z}(x^2+y^2)} \iint_{-\infty}^{\infty} f(x',y') e^{-i\frac{k}{z}[xx'+yy']} dx'dy'$$

Phase term in the observation plane (x,y)
Unimportant when calculating intensity pattern

Impulse response function h(x,y) of free space are now PLANE WAVES!!! Instead of paraboloidal waves.

5- From the above expression, show that:

$$g(x,y) = h_0 e^{i\frac{k}{2z}(x^2+y^2)} \mathcal{F}\{f(x',y')\}_{\substack{k_x=kx/z \\ k_y=ky/z}}$$

and then explain in words what the far-field image is in terms of the input image.

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$$g(x,y) = \frac{i}{\lambda z} e^{ikz} e^{i\frac{k}{2z}(x^2+y^2)} \iint_{-\infty}^{\infty} f(x',y') e^{-i\frac{k}{z}[xx'+yy']}\, dx'dy' \qquad k\frac{x'^2+y'^2}{2d} \ll \pi$$

$$g(x,y) = h_0 e^{i\frac{k}{2z}(x^2+y^2)} \iint_{-\infty}^{\infty} f(x',y') e^{-i\frac{k}{z}[xx'+yy']}\, dx'dy'$$

$$g(x,y) = h_0 e^{i\frac{k}{2z}(x^2+y^2)} \mathcal{F}\{f(x',y')\}_{\substack{k_x=kx/z \\ k_y=ky/z}} \quad \& \quad I(x,y) \sim |\mathcal{F}\{f(x',y')\}|^2_{\substack{k_x=kx/z \\ k_y=ky/z}}$$

1-In Saleh and Teich and Steck they go further….what additional restrictions are needed to make the output image EQUAL (with/in a constant) to the Fourier transform of the input image? Show it on a sketch.

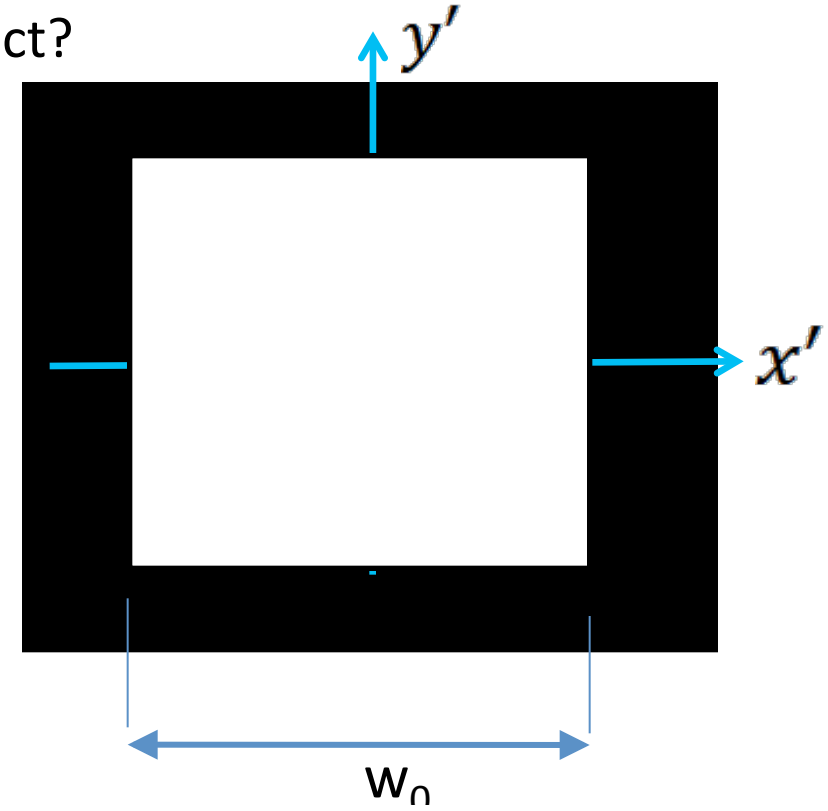
BONUS: Show that if you have satisfied this further condition, then you've automatically satisfied the Fresnel condition

2- A plane wave is sent through the mask below (x', y' plane).
The mask is completely opaque except for square holes at the center (mask has infinite extent).

(a) Write down an expression for image function at the mask

(b) What is the Fraunhofer field and intensity distribution a distance z away? Plot the intensity distribution.

(c) Comment on how the intensity distribution change as a function of aperture size w_0. Is this what you expect?

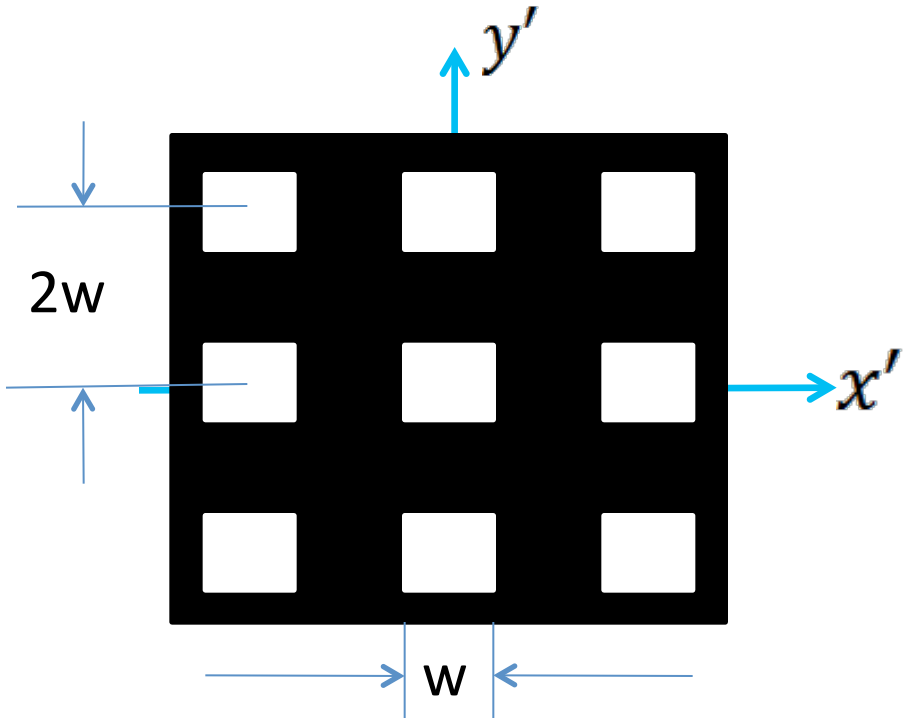


3- A plane wave is sent through the mask below (x', y' plane).
The mask is completely opaque except for square holes placed at various locations in the plane (mask has infinite extent).

(a) Write down an expression for image function at the mask

(b) What is the Fraunhofer field and intensity distribution a distance z away?
Plot the intensity distribution.

(c) What is the central spot size a distance z away.



•If [illegible] then [illegible]

•If [illegible] then [illegible]

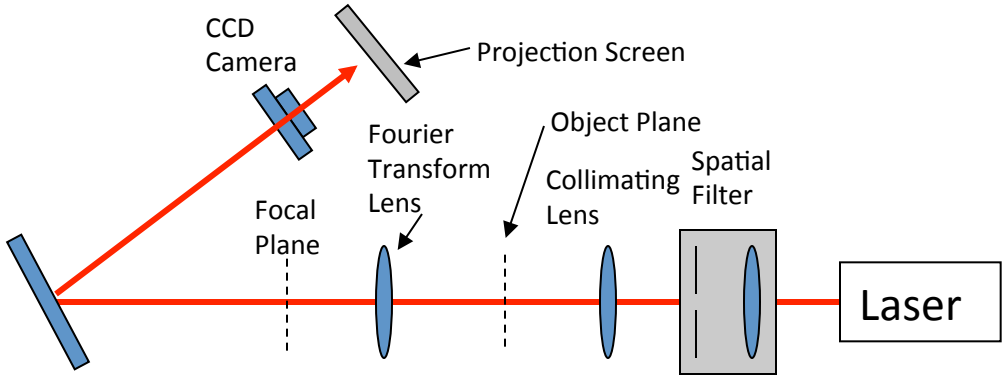
[illegible]

$\delta(x)$	1
$e^{ik_o x}$	$2\pi\delta(k-k_o)$
$\cos(k_o x)$	$\pi[\delta(k-k_o)+\delta(k+k_o)]$
1	$2\pi\delta(k)$
$\exp[-\alpha x^2]$	$\sqrt{\frac{\pi}{\alpha}}\exp\left[-\frac{k^2}{4\alpha}\right]$
$rect\left[\frac{x}{w}\right]$	$w\frac{\sin[kw/2]}{kw/2}$
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]

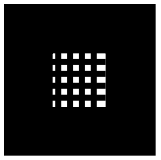
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THIS IS AN OLD EXAM QUESTION

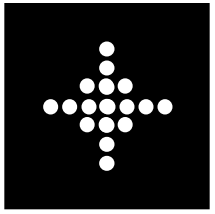
Consider your favorite experiment we did this term: the Fourier Optics lab. A diagram of the setup is shown below.



This mesh grid is inserted in the object plane



Which produces this image in the focal plane

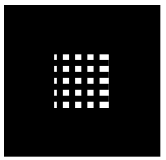


a) In the focal plane, the dots appear brightest in the center and then decrease in amplitude as you move off center. Why?

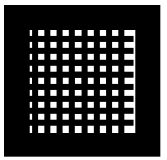
b) Now a new mesh grid is inserted in the object plane. It is almost identical to the old mesh grid (same wire spacing, same wire diameter), however the area of the mesh has increased. As more light makes it through the object plane, the image in the focal plane will change. Name two of the changes and justify your answer. Do not give the trivial answer that the focal plane image appears "brighter"

Hint: You could use the convolution theorem

Old Mesh Grid

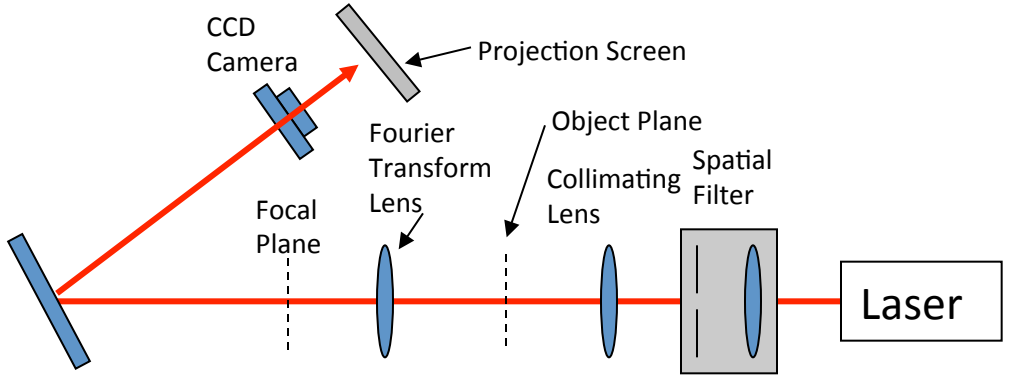


New Mesh Grid

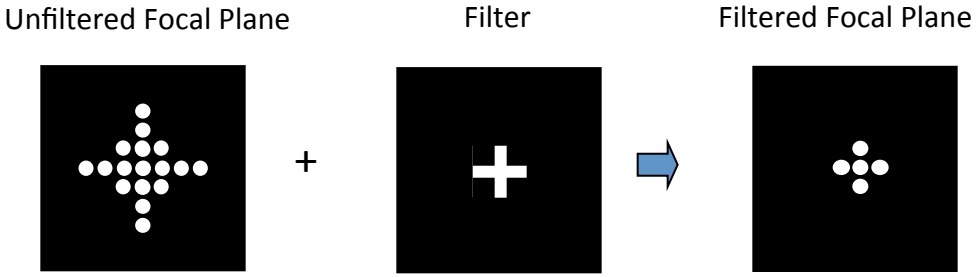


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THIS IS AN OLD EXAM QUESTION

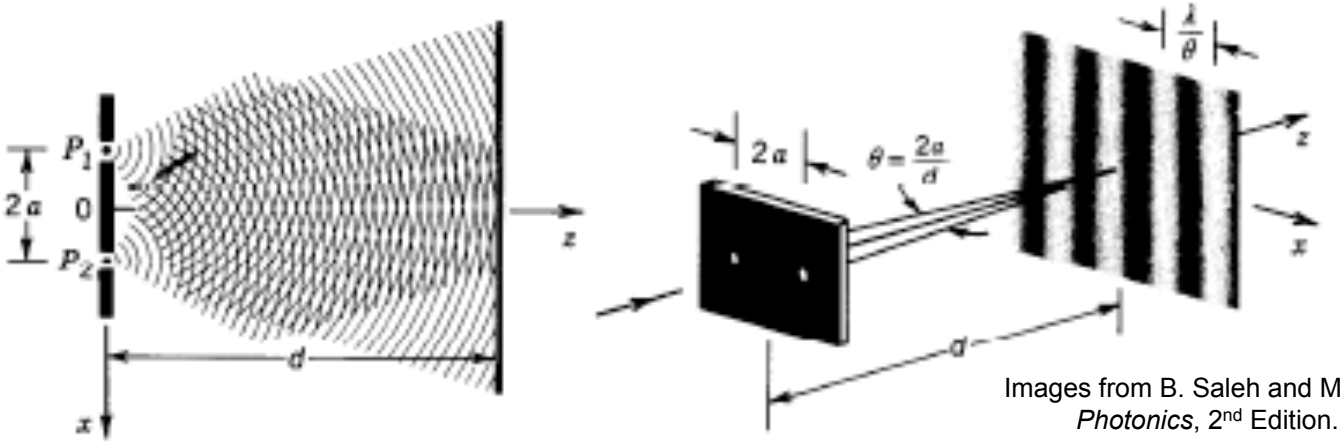


c) The original mesh is now placed back in the object plane. If nothing is placed at the focal plane, an image of the mesh appears on the projection screen. However, if some filters are placed at the focal plane then the image on the projection screen changes. Describe the resulting image on the screen when the focal plane is filtered as shown below. It would be best to use words plus a mathematical expression in your answer.



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In first or second year physics courses you should have been introduced to Young's two slit experiment which is used to demonstrate the wave-particle duality of light.



Images from B. Saleh and M Teich, *Fundamentals of Photonics*, 2nd Edition. (2007) Wiley and Sons

Assume the pinholes P_1 and P_2 are infinitely small and thus produce spherical waves when illuminated with a monochromatic plane wave For a large distance *d* to the observation screen use the paraboloidal approximation for the spherical wave and calculate the interference between the two waves. Work only in the x-z plane and show the answer is: $I(x, z = d) \approx 2I_o[1 + cos(kx\theta)]$

Now we are going to use Fourier Optics and diffraction (Fraunhofer far-field diffraction) to get the same answer (again only in the x-z plane).

First calculate the far-field diffraction pattern of the intensity at a distance z=d from a pair of finite width slits (with width *W*) a distance *2a* apart. There shouldn't be any integrals in your final answer. Now let *W* go to zero and confirm that you recover the expression above