PHYS 408 Group Activities for Mar 1 2011

Please complete these exercises without opening the textbook. If you have questions talk with your group mates or raise your hand and Jeff or Ellen will come to help

The familiar Fourier Transform pair is:

$$\tilde{f}(\omega) = \int_{-\infty}^{\infty} f(t) e^{i\omega t} dt$$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{f}(\omega) e^{-i\omega t} d\omega$$

1-Focussing on the second equation explain in your own words what the terms $\int_{-\infty}^{\infty}$ $\tilde{f}(\omega)d\omega$ and $e^{i\omega t}$ represent in the context of $f(t)$

If you are having trouble, consider first a Fourier series

In Fourier Optics, instead of time and frequency we will talking about transverse space (x,y) and transverse spatial frequency (k_x,k_y) with this 2-D Fourier Transform pair

$$\tilde{f}(k_x, k_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) e^{-i(k_x x + k_y y)} dx dy$$

$$f(x, y) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{f}(k_x, k_y) e^{i(k_x x + k_y y} dk_x dk_y$$

2-Repeat exercise 1 with this new Fourier transform pair

looking first @ Fourier Series

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{i n \omega_0 t}$$

$\omega_0 = \frac{2\pi}{T}$ where T is period of $f(t)$

- $f(t)$ → function that you build up from harmonics which must be periodic w/ period T
- $\sum_{n=-\infty}^{\infty}$ → sum over all harmonics
- n → amount of harmonics
- $e^{i n \omega_0 t}$ → different harmonics (frequencies)

This should be very familiar to you

In a similar light

$$f(t) = \frac{1}{2\pi} \int_{-\omega}^{\omega} \tilde{f}(\omega) d\omega e^{-i\omega t}$$

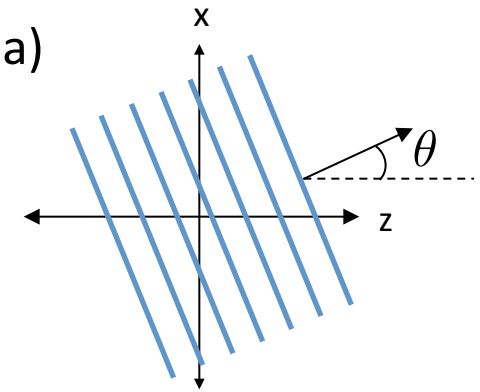
- $f(t)$ → Non periodic function that you are building up w/ many frequencies
- $\int_{-\omega}^{\omega}$ → sum over all frequencies
- $\tilde{f}(\omega) d\omega$ → amount of each ~~frequencies~~ frequency
- $e^{-i\omega t}$ → different frequencies

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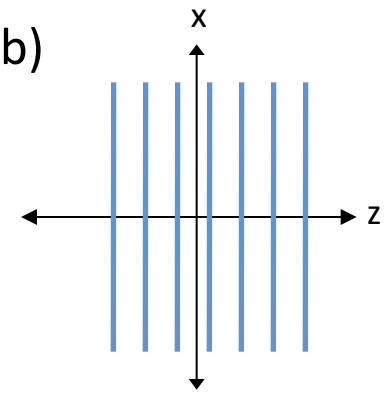
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3- What are transverse spatial frequencies in these three cases? Quantify, locate and label the spatial frequencies in each case

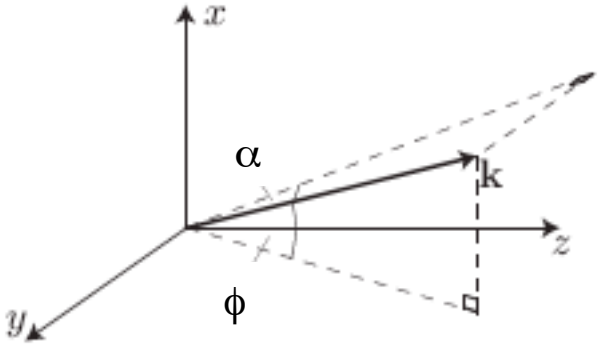
a)



b)

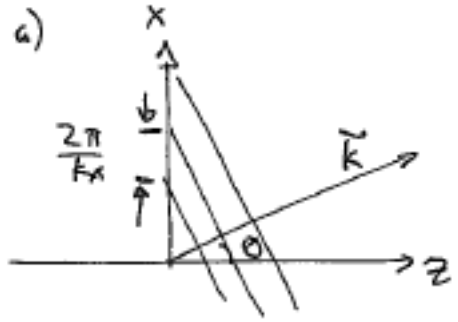


c) A plane wave traveling at an angle α (ϕ) with respect to the xz plane (yz plane)



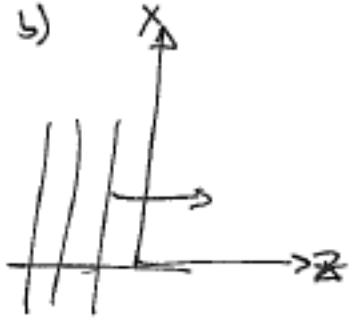
Suppose you have a field that is made up of the three plane waves shown above. Sketch the spectrum (the spatial frequency content) of the field

3.) a)



$k_x = k\sin\theta$ ← transverse spatial frequencies in $\hat{x}$ direction

b)



$k_x = 0$ ← This should make sense. There is no variation in $\hat{x}$ direction, hence $k_x = 0$

c) $k_x = k\sin\phi$

$k_y = k\sin\alpha$

Bonus:



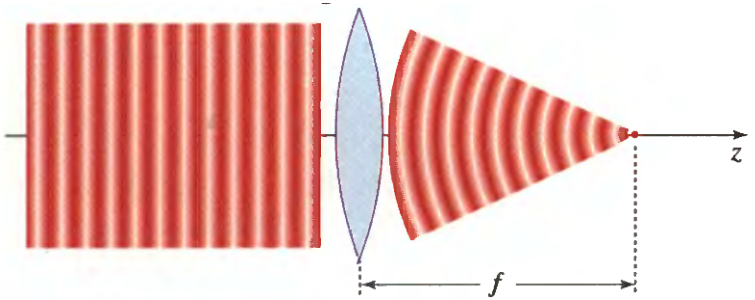


some of you had additional terms from projections but note how the angles are defined…see pages 105 from S&T They are angled with respect to planes not axes

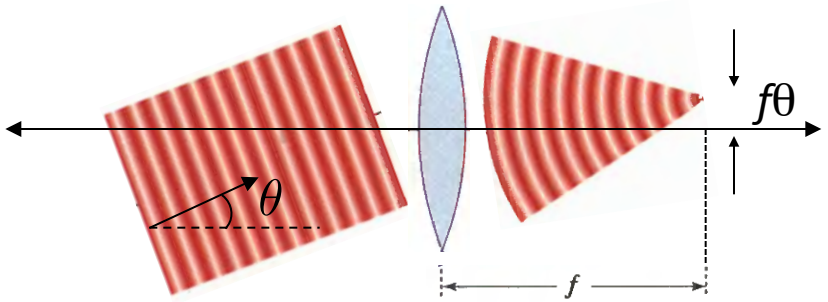
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Within the paraxial approximation, after going through a thin lens, a plane wave gets focussed to a point (x=0,z=*f*)



Within the paraxial approximation, after going through a thin lens, a plane wave propagating an angle θ gets focussed to a point (x=*f*θ,z=*f*)



Images from B. Saleh and M Teich, *Fundamentals of Photonics*, 2nd Edition. (2007) Wiley and Sons

4-These two results prove that the transverse image that appears at z=f is the Fourier transform of the transverse input image to the lens. Explain IN DETAIL and in your own words why this conclusion can be reached.

Prove the result shown in the second picture.

4) As discussed ~~earlier~~ a plane wave w/ angle θ wrt $\hat{x}$ axis get focussed to a point $(x = f\theta,\ z = f)$

⇒ There is also a one-to-one (& linear) correspondance between the angle a plane wave makes w/ the optical axis & its transverse spatial frequencies

i.e. $k_x = k\sin\phi \simeq k\phi$ (paraxial approx)

$k_y = k\sin\alpha \simeq k\alpha$

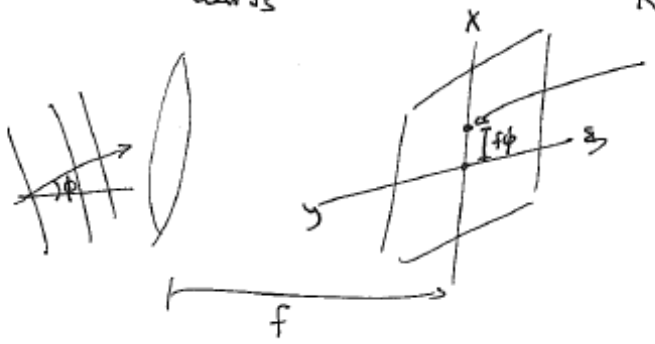
This means that each specific transverse spatial frequency (k_x, k_y) get focussed to a point

$x = f\phi = f\frac{k_x}{k}$

$y = f\alpha = f\frac{k_y}{k}$

@ the focal plane $z = f$

in other words



if there is light here then the transverse spatial frequency $k_x = k\phi$ exists ~~or diff~~ at input of lens

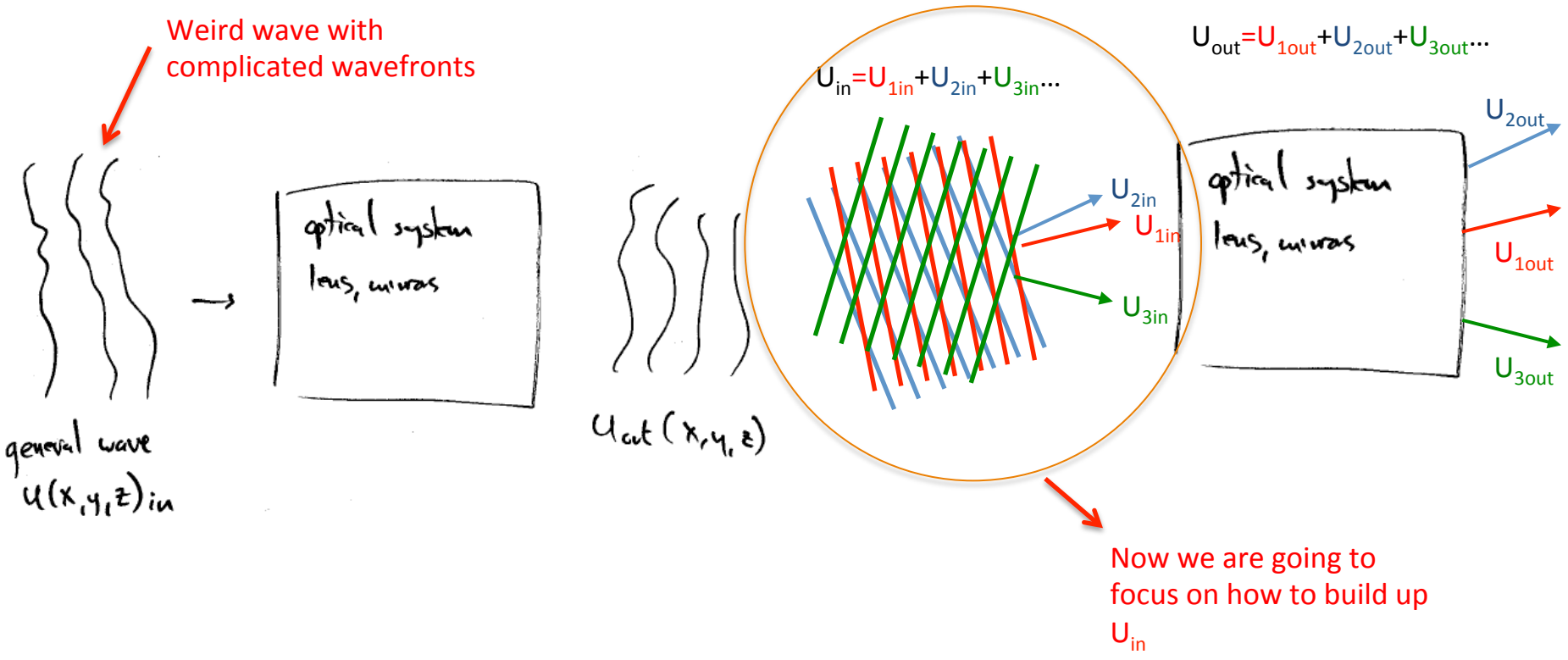
The BIG picture for Fourier Optics:

We've got a number of waves

1. plane waves
2. spherical waves
3. parabolidal waves
4. gaussian beam

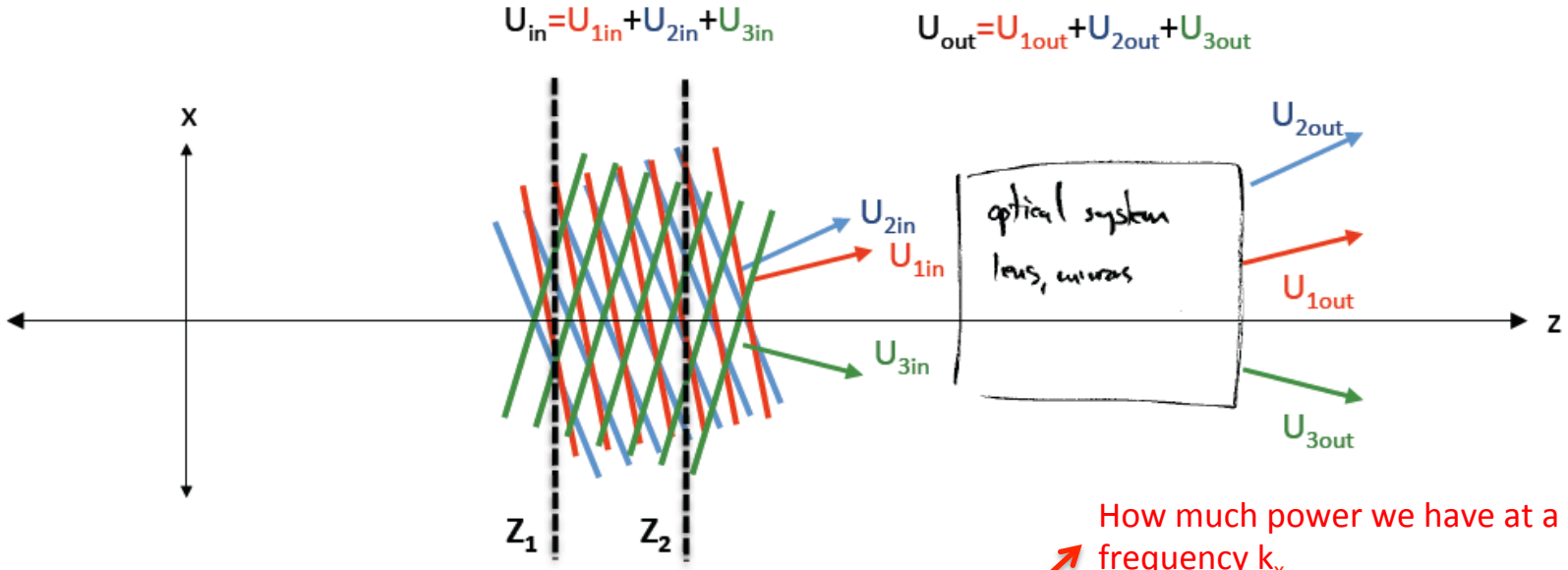
} know how they propagate thru space

... but it would be nice to be able to treat an arbitrary wave [which still satisfies $[\nabla^2 + k^2]U(r) = 0$ form]



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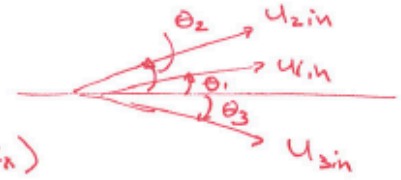
How much power we have at a certain frequency k_x

5- The overall field U_{in} at z_1 and z_2 is composed of the **same** 3 plane waves (U_{1in},U_{2in}, and U_{3in}) **(a)** Draw the power spectrum $S(k_x)$ Vs k_x at z_1 and z_2 , and **(b)** explain why (or why not) the overall field U_{in} is the same at z_1 and z_2.

6- In order for the input wave $U(x,y,z)_{in}$ to be broken up into a sum of plane waves it must be evaluated at some point fixed point z_o along the z-axis to form an "input image" $U(x,y,z=z_o)_{in}=f(x,y)$. Explain why this is the case.

5

$U_{in} = U_{1in} + U_{2in} + U_{3in}$



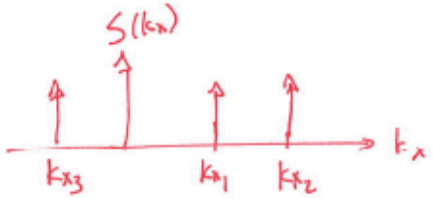
a) The power spectrum $S(k_x)$ is just how much power you have @ a specific k_x

for U_{1in} $k_{x_1} = k\sin\theta_1$

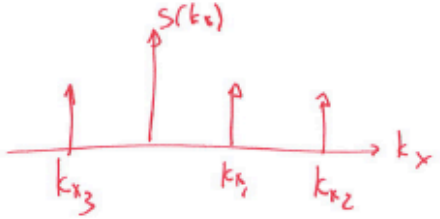
U_{2in} $k_{x_2} = k\sin\theta_2$

U_{3in} $k_{x_3} = k\sin\theta_3$

@ $z = z_1$



@ $z = z_2$ we have the same three plane waves so $S(k_x)$ will be the same



b) Even though the spectral content is the same @ $z = z_1$ & $z = z_2$. The field $U(x, y, z = z_1) \neq U(x, y, z = z_2)$

The reason is that U_{1in}, U_{2in} & U_{3in} accumulate different amounts of phase as they travel from $z_1 \rightarrow z_2$

So when these plane waves interfer @ z_1 & z_2 they will create a different <u>total</u> field

of course looking @ it from the perspective of just U_{in} (→ the overall field), of course $U_{in}|_{z=z_1} \neq U_{in}|_{z=z_2}$

6

This was answered in 6b. We must ~~fixed z~~ fix z in order to get a unique set of plane waves ~~to build up~~ (both how much of each plane wave but also <u>their phases</u>) to build up $U(x, y, z)$