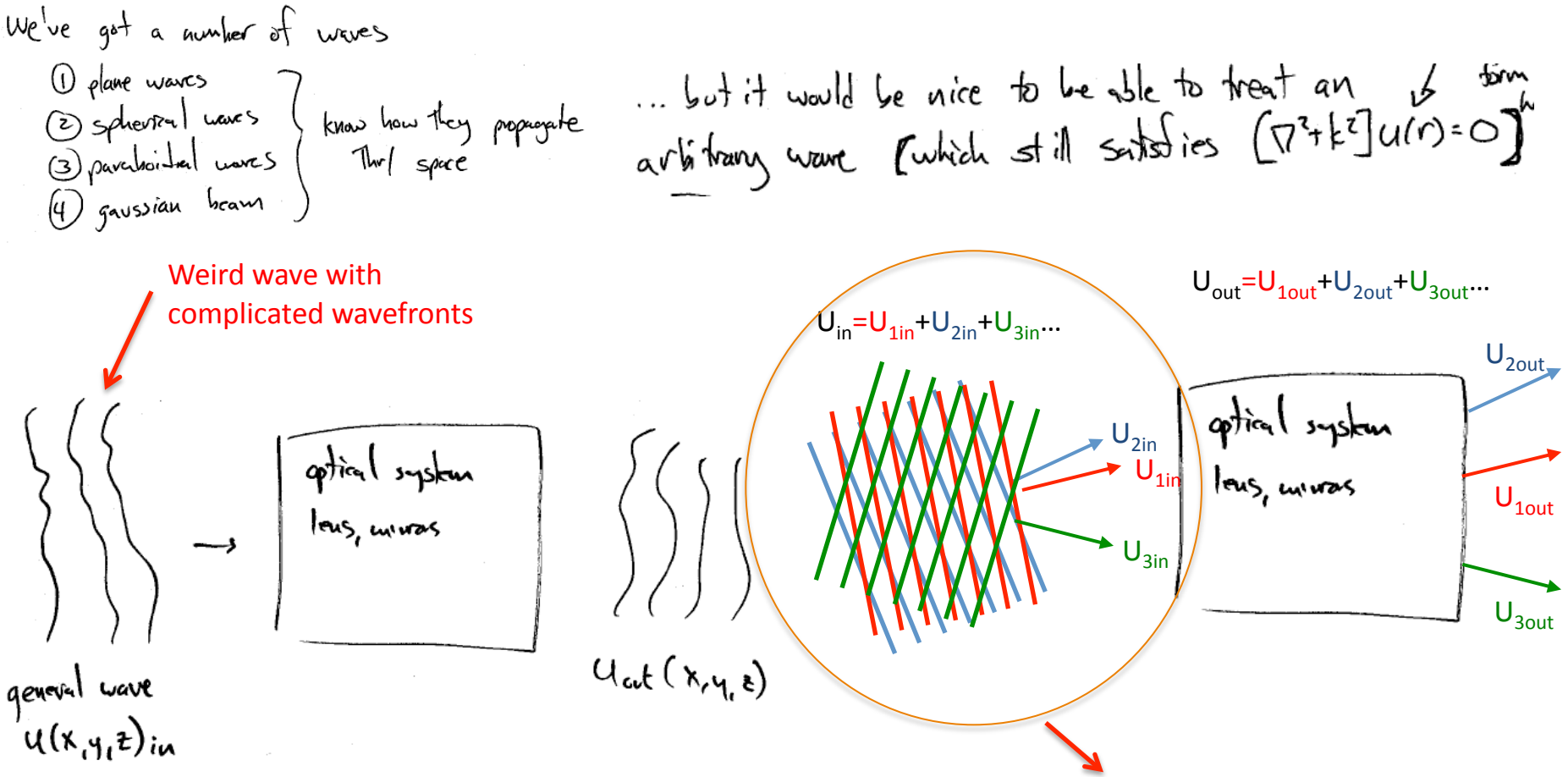
The BIG picture for Fourier Optics:



1- As we discussed on Tuesday, the first step is to decompose the input wave at a fixed z (z=0). This is called the input image, f(x,y), and is a function of x,y.

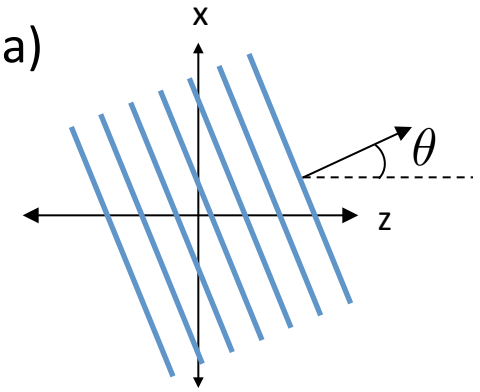
$$U_{in}(x,y,z=0)=f(x,y)=U_1(x,y,z=0)+U_2(x,y,z=0)+U_3(x,y,z=0)$$

PHYS 408 Group Activities for Mar 3 2011

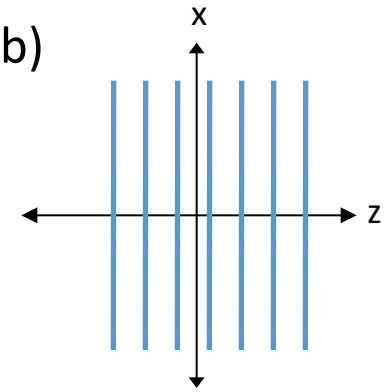
Please complete these exercises without opening the textbook. If you have questions talk with your group mates or raise your hand and David or Ellen will come to help

2- What are the input images for these three cases? Give a mathematical expression and picture of the field image for f(x,y)

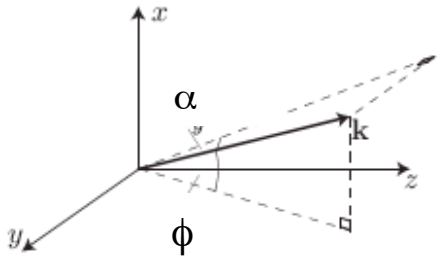
a)



b)



c) A plane wave traveling at an angle α (ϕ) with respect to the y-axis (x-axis)

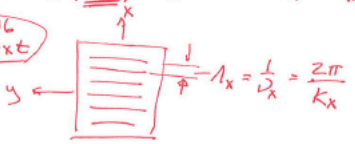


Given the spatial frequency content of an image function f(x,y) explain how you could reconstruct the field U(x,y,z) that created that image?

2 a) $u(x,y,z) = Ce^{ik(x\sin\theta + z\cos\theta)}$ ← field

$f(x,y) = u(x,y,z=0) = Ce^{ikx\sin\theta}$ ← input image

see fig 4.1-2 page 106 in text



b) $u(x,y,z) = Ce^{ikz}$

$f(x,y) = u(x,y,z=0) = C$



---

2 c) $u(x,y,z) = e^{ik(y\sin\alpha + x\sin\phi)} e^{ik_z z}$

$e^{i(k_x x + k_y y + k_z z)}$

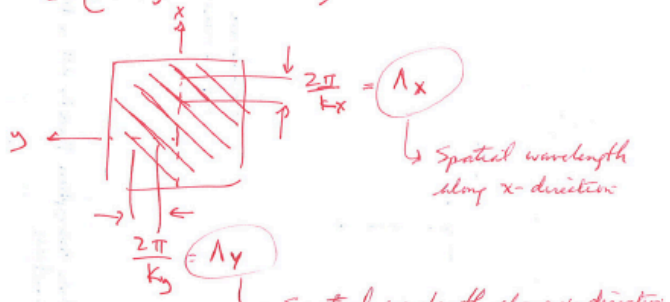
w/ $k_z = \sqrt{k^2 - k_x^2 - k_y^2}$

$k_x = k\sin\phi$

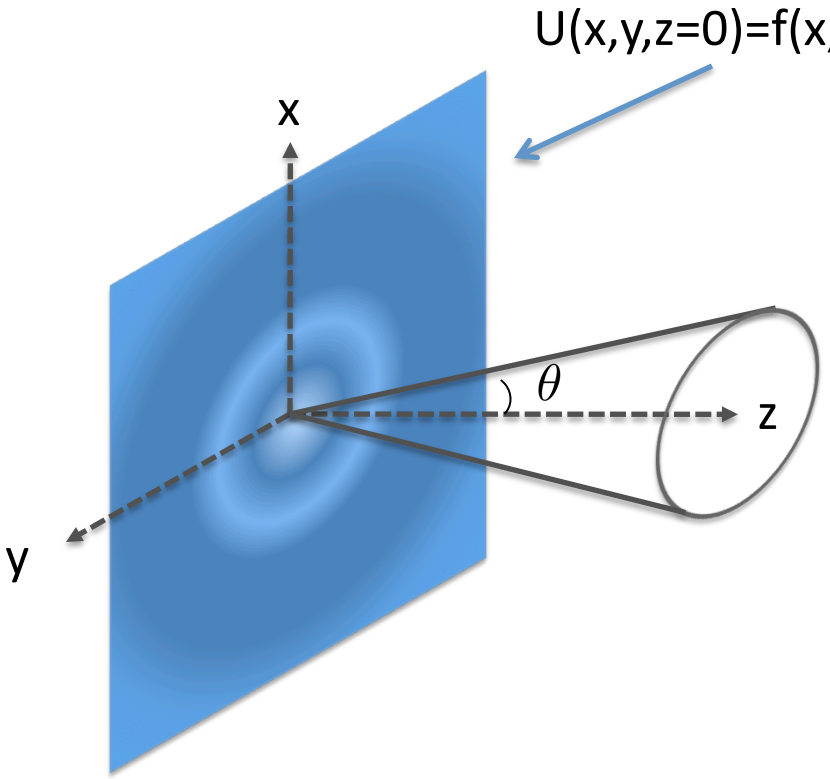
$k_y = k\sin\alpha$

$f(x,y) = u(x,y,z=0)$

$= e^{ik(y\sin\alpha + x\sin\phi)}$



3- If a field U(x,y,z) creates an image at z=0, ie U(x,y,z=0)=f(x,y) that is a circularly symmetric function with a maximum spatial frequency of 200/mm, determine the angle of the cone within which the plane wave components of U(x,y,z) are confined. Assume the wavelength is $\lambda = 2\pi \times 10^{-7}$



(A) There isn't enough info

(B) Approximately 0.06 rad

(C) Approximately 0.6 rad

(D) Approximately 0.006 rad

(E) Smaller than 0.006 rad



$\sin\theta_x = \frac{k_x}{k}$; $\nu_x = \frac{200}{10^{-3}}$

$= \frac{4\pi \cdot 10^5 \cdot \lambda}{2\pi}$ $= 2\times10^5$

$\Rightarrow k_x = 2\pi \nu_x$

$= 2 \cdot 10^5 \cdot 2\pi \cdot 10^{-7}$ $= 4\pi\times10^5$

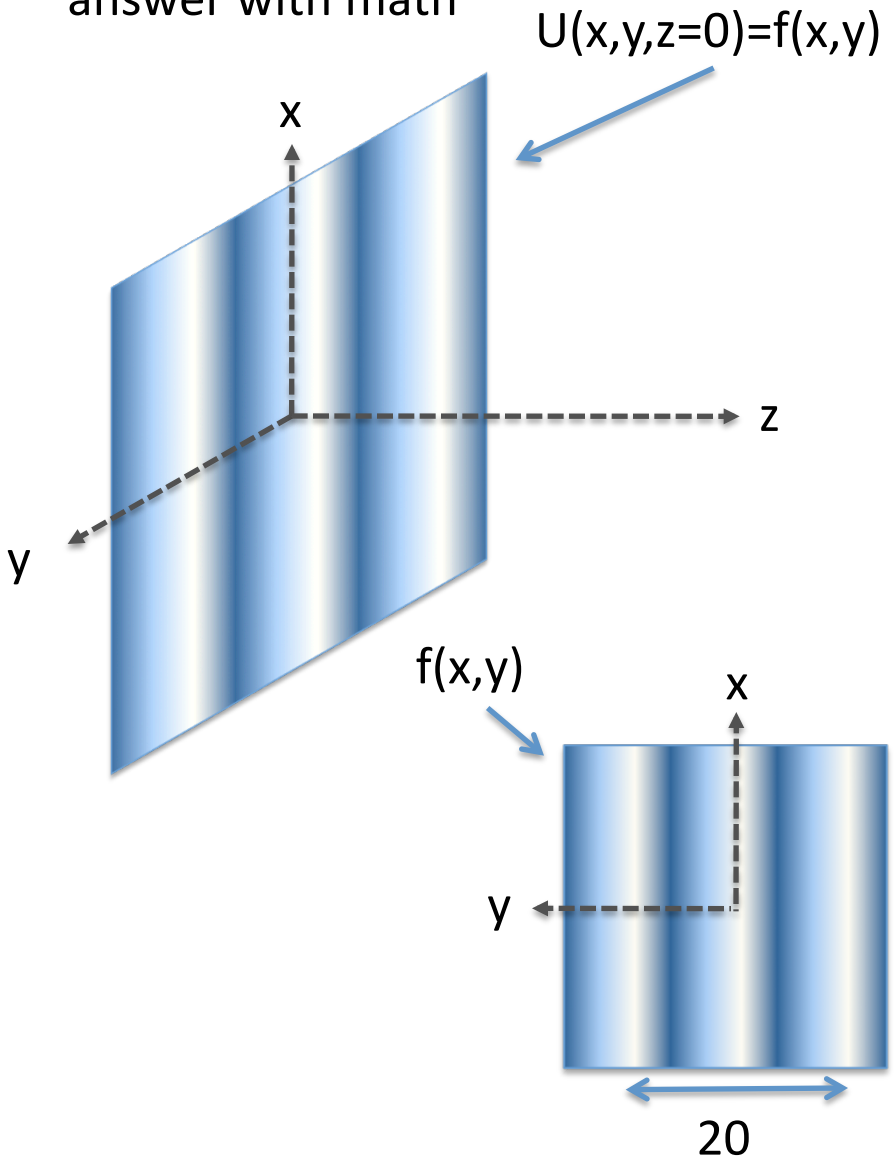
$= 4\pi \cdot 10^{-2}$

$\Rightarrow \sin\theta_x \approx \theta_x = 4\pi \cdot 10^{-2}$

$= 0.12$ rad

so closest answer is (B)

4-If a field U(x,y,z=0)=f(x,y) is shown below as the amplitude (maxima are dark, minima are light), pick the best f(x,y). Justify your answer with math



(A) $A\cos(k_{yo}y)$

(B) $A\cos(k_{xo}x)\cos(k_{yo}y)$

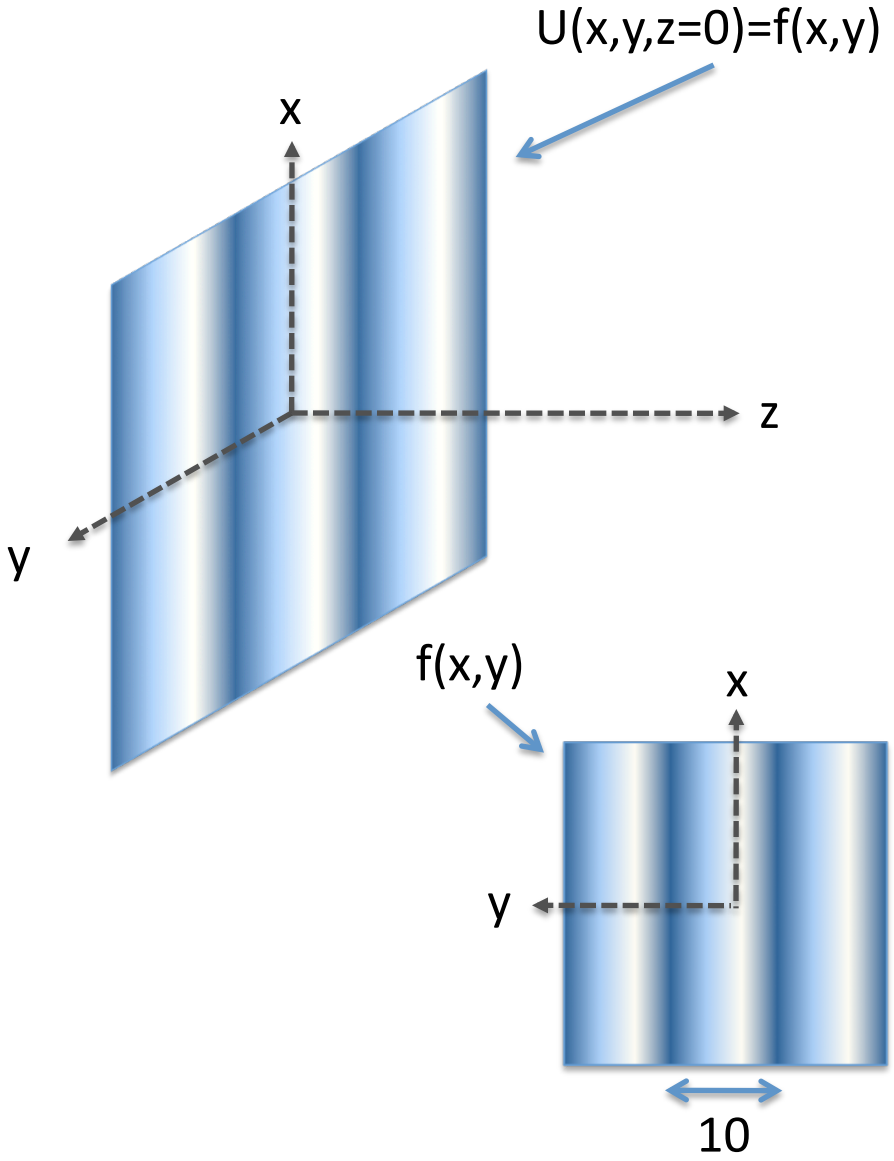
(C) $A\cos(2\pi y/20)$

(D) $A\cos(2\pi x/10)$

(E) $A\cos(2\pi y/10)$

No x dependence, so B & D are out. Period is 10, so C is out. Both A & E are correct answers, but E is the better answer as it has the specific period

5- If f(x,y)=Acos(2πy/10) how many plane wave(s) have made up U(x,y,z)? Justify your answer with math.



(A) 0
(B) 1
(C) 2
(D) 3

one plane wave would only give you a phase variation

$\Rightarrow U(x,y,z) = Ae^{ik(x\sin\theta + z\cos\theta)}$

$f(x,y) = Ae^{iky\sin\theta}$

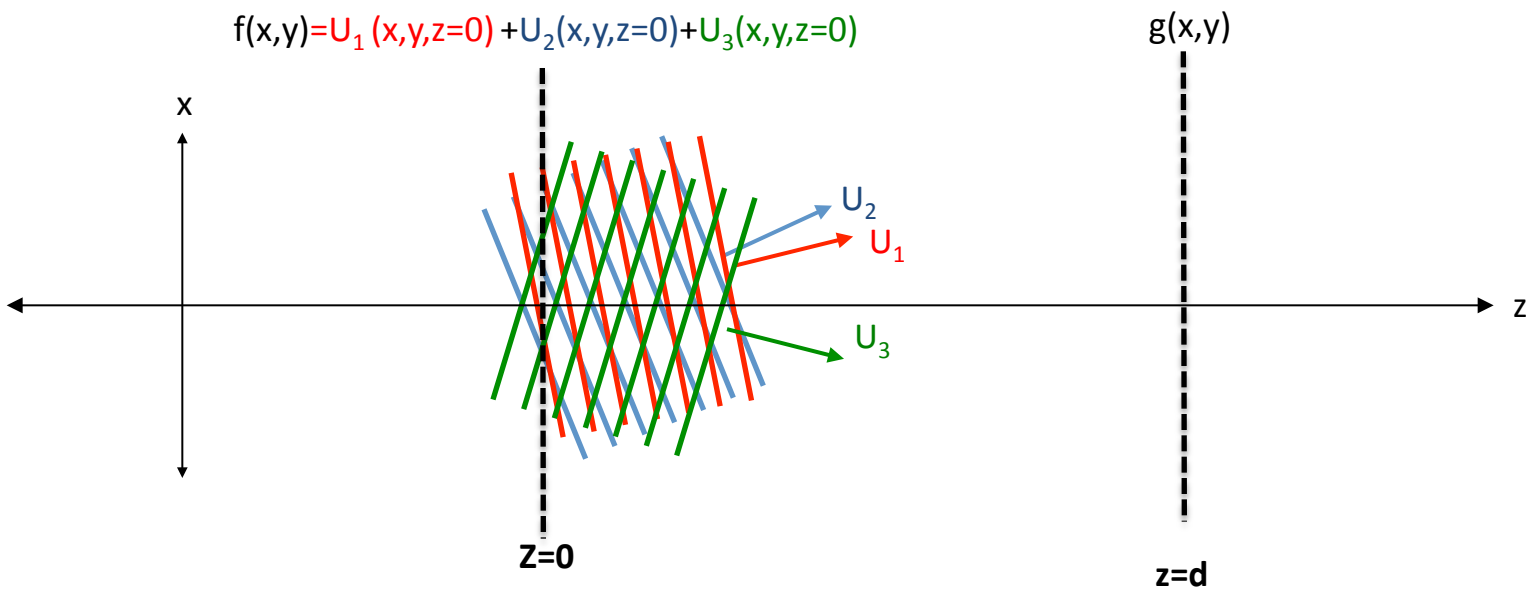
you need two plane wave for an amplitude variation

$\Rightarrow f(x,y) = A\left[e^{i\frac{2\pi}{10}y} + e^{-i\frac{2\pi}{10}y}\right]$

1st 2nd

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6- The field U(x,y,z) at z=0 can be decomposed into 3 plane waves (U_1,U_2, and U_3) at evaluated at z=0. In other words, the input image f(x,y)=U(x,y,z=0) and can be constructed by the interference of 3 plane waves as shown. At z=d find the (new) output image g(x,y)=U(x,y,z=d).

7- In your own words explain what you just did mathematically to get the output image g(x,y).

Repeat 1&2 for an infinite sum of plane waves (which are needed to create the most arbitrary wave)