- let me know of your lab partner by next Thursday (Jan 15th) & I will assign you both a group number
- After Jan 15th I will assign remaining people to partners/groups

- I will post Homework #1 tomorrow
  ⇒ due Jan 16th @ 5 PM
  user: me
  pwd: Iamhappy
- labs will start Jan 19th

Note that the text uses j for a complex number $\sqrt{-1}$ (bc they are engineers). In lecture & homework I will use i ... moreover $j \rightarrow -i$

example: $E_0 e^{j\omega t} \rightarrow E_0 e^{-i\omega t}$

Review of Electromagnetic fields

An electromagnetic field (E-M field) is described by two related vector fields

electric field $\vec{\mathcal{E}}(r,t)$
magnetic field $\vec{\mathcal{H}}(r,t)$
$\Rightarrow$ these are the "real" ie. physical fields of the E-M wave

These two fields are related thru a set of DEQs which Maxwells equations....

look @ a source free case: $\vec{J} = 0$, $\rho = 0$

in this case, Maxwell Eqn. are:

(1) $\nabla \times \vec{\mathcal{H}} = \epsilon_0 \frac{\partial \vec{\mathcal{E}}}{\partial t}$

(2) $\nabla \times \vec{\mathcal{E}} = -\mu_0 \frac{\partial \vec{\mathcal{H}}}{\partial t}$

(3) $\vec{\nabla} \cdot \vec{\mathcal{E}} = 0$

(4) $\vec{\nabla} \cdot \vec{\mathcal{H}} = 0$

$\epsilon_0 \simeq 8.85 \times 10^{-12} \left[\frac{F}{m}\right]$ electric permittivity

$\mu_0 = 4\pi \times 10^{-7} \left[\frac{H}{m}\right]$ magnetic permeability

→ M.E. in free space (vacuum), source free.

Some things to note:

1) $\vec{\mathcal{H}}$ & $\vec{\mathcal{E}}$ are functions time & space
2) vector qualities.

In this class we will work w/ a derived eqn from M.E.
the E-M wave equation

to get E-M wave eqn. operate $\nabla\times$ on (2) w/ $\nabla\times$ & The using vector identities

§5.1 text

$$\nabla^2\vec{\mathcal{E}} - \frac{1}{c_0^2}\frac{\partial^2\vec{\mathcal{E}}}{\partial t^2} = 0 \quad \text{w/} \quad c_0^2 = \frac{1}{\epsilon_0\mu_0} \qquad (5)$$

This is The fundamental eqn of optics: an $\vec{\mathcal{E}}$-field represents an E-M wave traveling @ speed c_0 if (5) is satisfied.

Note: if we carry out a similar procedure ie.

$\nabla\times$ on (1)

$$\nabla^2\vec{\mathcal{H}} - \frac{1}{c_0^2}\frac{\partial^2\vec{\mathcal{H}}}{\partial t} = 0$$

Now what about an E-M wave in a medium?

still no sources ① no current density $\vec{J}=0$

② no charge $\rho=0$

w/ ~~correct~~ condition That The medium is lossless we can derive a new set of M.E.... in order to do This we need to define two additional vector fields

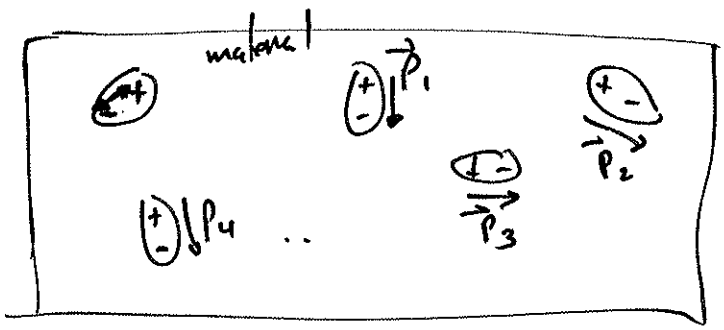
$\vec{\mathcal{D}}$: electric flux density

$\vec{\mathcal{B}}$ magnetic " "

} → These two fields are needed to model the effect of the medium

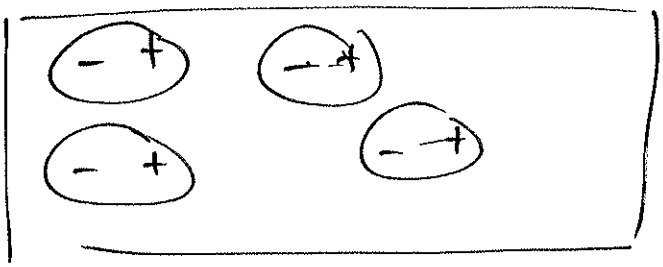
Physically why do we need $\vec{D}$ & $\vec{B}$?

lets consider some "test" medium



- material is made up of molecules that individual dipole moments $\vec{P}_i$

so what happens when $\vec{\mathcal{E}}$ is applied? - if you sum over all $\sum_n \vec{P}_i = 0$ = total polarization density



- all dipoles align to the applied field

$\Rightarrow$ now $\sum_n \vec{P}_i \neq 0$ b/c aligned $= \vec{P}$

So now we must take into account the dipole contributions (material) when finding the total field in the medium

$$\vec{D} = \epsilon_0 \vec{\mathcal{E}} + \vec{P}$$

$\vec{D}$: total field

$\vec{\mathcal{E}}$: applied field (from E.M source)

$\vec{P}$: polarization density — response of the material

In general the polarization density $\vec{P}$ arises from two sources:

(1) a ~~fixed~~ function to $\vec{\mathcal{E}}$ (i.e. it is only exists when $\mathcal{E}$ is present)

(2) not a function of $\vec{\mathcal{E}}$ (a permanent condition of medium)

In this class we will only be ~~a~~ worry about (1)

i.e. $\vec{\mathcal{E}} \rightarrow$ [medium] $\rightarrow \vec{P}$

$\vec{\mathcal{E}}$ applied field

$\vec{P}$ response of material

$\vec{D}$ total field $= \epsilon_0 \vec{\mathcal{E}} + \vec{P}$

$$\vec{B} = \mu_0 \vec{\mathcal{H}} + \vec{M}$$

$\vec{M}$ ← magnetic polarization density

In optics we are only ~~here~~ going to treat dielectric materials $\Rightarrow \vec{M} = 0$

$$\vec{B} = \mu_0 \vec{\mathcal{H}}$$

$$\vec{D} = \epsilon_0 \vec{\mathcal{E}} + \vec{P}(\vec{\mathcal{E}})$$

.. need to ~~see~~ figure out $\vec{P}(\vec{\mathcal{E}})$ to derive wave eqn for field in medium

At first we are only going to consider the simplest dielectric material/medium This means for $\vec{P}$

§5.2 of text

1) Material/Medium is has a $|\vec{P}| \propto |\vec{\mathcal{E}}|$
   $\Rightarrow$ medium is linear
2) medium is non dispersive — $\vec{P}$ is only dependent on the current value of $\vec{\mathcal{E}}$ (i.e. no history)
3) medium is homogeneous — $\vec{P}(\vec{\mathcal{E}})$ is independent of position $\vec{r}$
4) medium isotropic — $\vec{P} \parallel \vec{\mathcal{E}}$

w/ all 4 of these assumptions

$$\vec{P}(r,t) \propto \vec{\mathcal{E}}(r,t)$$

$$\Rightarrow \vec{P}(r,t) = \epsilon_0 \chi \vec{\mathcal{E}}$$

χ ← property of material

w/ this definition of $\vec{P}$ we can write

$$\vec{\mathcal{D}} = \epsilon_0 \vec{\mathcal{E}} + \underbrace{\epsilon_0 \chi \vec{\mathcal{E}}}_{\vec{P}}$$

$$= \underbrace{\epsilon_0 (1+\chi)}_{\epsilon} \vec{\mathcal{E}}$$

$$\vec{\mathcal{D}} = \epsilon \vec{\mathcal{E}}$$

so with this definition of $\vec{\mathcal{D}}$

$$\nabla \times \vec{\mathcal{H}} = \frac{\partial \vec{\mathcal{D}}}{\partial t} = \epsilon \frac{\partial \vec{\mathcal{E}}}{\partial t} = \epsilon_0 \frac{\partial \vec{\mathcal{E}}}{\partial t} + \frac{\partial \vec{P}}{\partial t}$$

$$\nabla \times \vec{\mathcal{E}} = -\mu_0 \frac{\partial \vec{\mathcal{H}}}{\partial t} = -\frac{\partial \vec{\mathcal{B}}}{\partial t}$$

$$\nabla \cdot \vec{\mathcal{E}} = 0 \iff \nabla \cdot \vec{\mathcal{D}} = 0$$

$$\nabla \cdot \vec{\mathcal{H}} = 0 \iff \nabla \cdot \vec{\mathcal{B}} = 0$$

... okay w/ this new set of Maxwells eqns:

a new wave eqn can be found:

$$\boxed{\nabla^2 \vec{\mathcal{E}} - \frac{1}{c^2} \frac{\partial^2 \vec{\mathcal{E}}}{\partial t^2} = 0} \qquad c^2 = \frac{1}{\epsilon \mu_0}$$

in particular: $\frac{c_0}{c} = \frac{\sqrt{\epsilon \mu_0}}{\sqrt{\epsilon_0 \mu_0}} = \sqrt{\frac{\epsilon}{\epsilon_0}}$; $\epsilon = \epsilon_0 (1 + \chi)$

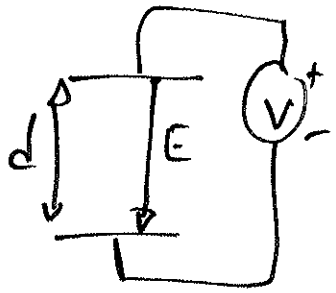
$$= \sqrt{1 + \chi} \equiv n$$

↑ refractive index of material

so effect of material is to slow down the E-M wave by a factor n

The flow of energy of an E-M field is defined in terms of a "Poynting" vector"

"S" $\longrightarrow$ $\vec{\mathcal{S}} := \vec{\mathcal{E}} \times \vec{\mathcal{H}}$



$E = \frac{V}{d}$



$\vec{H} = \frac{I}{2\pi r}$

$\left[\frac{A}{m}\right]$

Things to note:

1) What are units of $\vec{\mathcal{S}}$?

$\Rightarrow \frac{V \cdot A}{m^2} \left[\frac{W}{m^2}\right]$ $\quad \vec{\mathcal{E}} \left[\frac{V}{m}\right]$ $\quad \vec{\mathcal{H}} \left[\frac{A}{m}\right]$

2) $\vec{\mathcal{S}}$ points in the direction of energy flow

3) $\vec{\mathcal{S}} \perp \vec{\mathcal{E}} \perp \vec{\mathcal{H}}$

4) In optics we can't really measure $\vec{\mathcal{E}}$, $\vec{\mathcal{H}}$, $\vec{\mathcal{S}}$ .. The reason is f_ν 300THz too fast for any detector

$\Rightarrow$ rather we measure time-averaged $\vec{\mathcal{S}}$

$I := |\langle \vec{\mathcal{S}} \rangle|$ ← ~~inst~~ intensity

We need to solve our eqn.

$$\nabla^2 \vec{\mathcal{E}} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\vec{\mathcal{E}} = 0$$

to figure out

$\vec{\mathcal{E}}(\ldots \vec{r}, \vec{k}, I)\ldots$

In this class we are only going to consider monochromatic fields... This means $\vec{\mathcal{E}}, \vec{\mathcal{B}}$ are harmonic functions of time & thus have a single frequency.... In math

$$\vec{\mathcal{E}}(\vec{r},t) = \vec{A}(\vec{r}) \cos\left[\phi(\vec{r}) - \omega t\right]$$

- ω: monochromatic single frequency
- $\cos[\phi(\vec{r}) - \omega t]$: harmonic function
- $\vec{A}(\vec{r})$: amplitude