Prelims

- labs start next week
  - ⇒ let me know of your partners by Thurs or I will assign on Thursdays
  - ⇒ Odd # in both sections, so if one person can switch, all will have a partner

- HW #1 posted, due This Friday @ 5 PM in Hennings 328 (slide under door)
  - ⇒ Staple multiple pages (Stapler in main office)
    NO Paper folding

- I don't yet have office hours
  email to setup appoint. or just stop by my lab (Chem/Phys A012)

We left off w/ wave equation

$$\nabla^2 \vec{\mathcal{E}} - \frac{1}{c^2} \frac{\partial^2 \vec{\mathcal{E}}}{\partial t^2} = 0$$

In this class we are going to consider only monochromatic fields that are harmonic functions of time

$$\vec{\mathcal{E}}(r,t) = \vec{A}(r) \cos(\phi(r) - \omega t)$$

- $\vec{A}(r)$ → same amplitude that only has spatial dependence
- $\cos(\phi(r) - \omega t)$ → harmonic function
- ↳ monochromatic

Our life will be much easier if ~~you~~ we go to complex notation

$$\vec{\mathcal{E}}(r,t) = \mathrm{Re}\left[\vec{E}(r) e^{-i\omega t}\right]$$

$\vec{E}(r)$ ← complex amplitude

$\vec{E}(r)$ is the complex amplitude

- only has a spatial ~~resp~~ dependence $[A(r), \phi(r)]$
- → complex # b/c ↑
- → not a measureable electric field

The plan is to do calculation w/ $\vec{E}(r)$. Then when we want the real electric

$$\Rightarrow \vec{\mathcal{E}}(r,t) = \mathrm{Re}\left[\vec{E}(r) e^{-i\omega t}\right]$$

Note That

$$\vec{\mathcal{E}}(r,t) = \frac{1}{2}\left[\vec{E}(r)e^{-i\omega t} + \vec{E}^{*}e^{i\omega t}\right]$$

$$= \frac{1}{2}\left[\vec{E}(r)e^{-i\omega t} + c.c.\right]$$

↳ complex conjugate

. . . - so how do Maxwells look w/ $\vec{E}(r)$

$$\nabla \times \vec{\mathcal{E}} = -\mu_0 \frac{\partial}{\partial t}\vec{\mathcal{H}} \quad ; \quad \vec{\mathcal{E}} = Re\left[\vec{E}(r)e^{-i\omega t}\right]$$

$$\vec{\mathcal{H}} = Re\left[\vec{H}(r)e^{-i\omega t}\right]$$

$$\frac{1}{2}\nabla \times \left[\vec{E}e^{-i\omega t} + c.c.\right] = -\frac{1}{2}\mu_0 \partial_t\left(\vec{H}(r)e^{-i\omega t} + c.c.\right)$$

collect terms that oscillate w/ same frequency

$e^{-i\omega t}$: $\nabla \times \vec{E}(r) = i\omega\mu_0 \vec{H}(r)$

$e^{i\omega t}$: $\nabla \times E^{*}(r) = -i\omega\mu_0 \vec{H}^{*}(r)$ ← no new info b/c it is just c.c. of

Following a similar procedures

$$\nabla \times \vec{H} = -i\omega\epsilon\vec{E}$$
$$\nabla \times \vec{E} = i\omega\mu_0\vec{H}$$
$$\nabla \cdot \vec{E} = 0$$
$$\nabla \cdot \vec{H} = 0$$

→ M.E. in terms of complex amplitude assuming a time harmonic field

$\Rightarrow$ wave equation $\nabla^2\vec{E} + \left(\frac{\omega}{c}\right)^2\vec{E} = 0 \Rightarrow \boxed{\nabla^2\vec{E} + \left(\frac{\omega^2}{c^2}\right)\vec{E} = 0}$

$\nabla^2 \vec{E} + \left(\frac{\omega}{c}\right)^2 \vec{E} = 0$ this is the wave eqn for monochromatic E-M waves that have a time & harmonic ~~&~~ dependence

So plan is to work w/ $\vec{E}, \vec{H}$ then go to real E-field ~~$\mathcal{E}$~~ ($\mathcal{E}$) @ end.

In many cases we want to calculate the intensity $\left[\frac{W}{m^2}\right]$ ~ I (time averaged Poynting vector) of an E-M field

$$I := |\langle \mathcal{S} \rangle| = |\langle \vec{\mathcal{E}} \times \vec{\mathcal{H}} \rangle|$$

(real fields)

is there an expression for $\mathcal{S}$, $\langle \mathcal{S} \rangle$, & I in terms of $\vec{E}, \vec{H}$ (not just $\vec{\mathcal{E}}, \vec{\mathcal{H}}$)

$$\mathcal{S} = \vec{\mathcal{E}} \times \vec{\mathcal{H}}$$

$$= \tfrac{1}{2}\left(\vec{E} e^{-i\omega t} + c.c.\right) \times \tfrac{1}{2}\left(\vec{H} e^{-i\omega t} + c.c.\right)$$

$$= \tfrac{1}{4}\left(\vec{E} \times \vec{H}^* + \vec{E}^* \times \vec{H} + \vec{E} \times \vec{H} e^{-i2\omega t} + \vec{E}^* \times \vec{H}^* e^{i2\omega t}\right)$$

$$\langle \mathcal{S} \rangle = \tfrac{1}{4}\left(\vec{E} \times \vec{H}^* + \vec{E}^* \times \vec{H}\right)$$ ← in my homework

This form of $\langle \mathcal{S} \rangle$ motivates us to define a complex poynting vector

$$\vec{S} := \tfrac{1}{2}\left(\vec{E} \times \vec{H}^*\right)$$

$$\langle \not{S} \rangle = \frac{1}{2}(\vec{S} + S^*)$$

$$= \mathrm{Re}[\vec{S}]$$

$$\Rightarrow I = |\langle \not{S} \rangle| = |\mathrm{Re}(\vec{S})|$$

$$\boxed{I = |\mathrm{Re}(\tfrac{1}{2}\vec{E} \times H^*)|}$$

so this is nice b/c can calculated time averaged intensity w/o any need for $\vec{E}, \vec{H}, \vec{J}$; Then integrate

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W/ this E-M review we can start to look @ specific forms of E-M waves ; do optics!

(1) $\nabla^2 \vec{E}(r) + \left(\frac{\omega}{c}\right)^2 \vec{E} = 0$ w/ $\& c^2 = \frac{1}{\epsilon \mu_0}$

our task is to solve this PDE ; still a bit difficult

...so consider ~~simple~~ simple case ; construct a test solutions

$\vec{E}(r) = E_x(r)\,\hat{x}$ ← E field is only pointed in one direct $\hat{x}$ direction

so what if $E_x(r) = E_0$

$\vec{E}(r) = E_0\,\hat{x}$: plug into ~~W.E~~ W.E (1) trivial solution only true if $E_0 = 0$ ... so ~~useful~~

... assume $E_x(r)$ has only a z dependence

$\Rightarrow \vec{E}(r) = E_x(z)\,\hat{x}$ ← test solution

& That test solution makes (1)

$$\frac{\partial^2}{\partial z^2} E_x + \frac{\omega^2}{c^2} E_x = 0 \quad ; \quad E_x(z)$$

now let solve for E_x, one solution is

$$E_x = E_0 e^{ikz} \quad ; \quad k \text{ is a constant } \underline{\text{TBD}}$$

$$(ik)(ik) E_0 e^{ikz} + \frac{\omega^2}{c^2} E_0 e^{ikz} = 0$$

$$\Rightarrow \left[-k^2 + \frac{\omega^2}{c^2} \right] E_0 e^{ikz} = 0$$

for a non-trivial solt $E_0 \neq 0$

$$\Rightarrow k^2 = \frac{\omega^2}{c^2} \quad \ldots \text{ we'll see what } k \text{ is physically in a second.}$$

So what does the real field look like?

$$\vec{E}(r,t) = \text{Re}\left[\vec{E} e^{-i\omega t}\right]$$

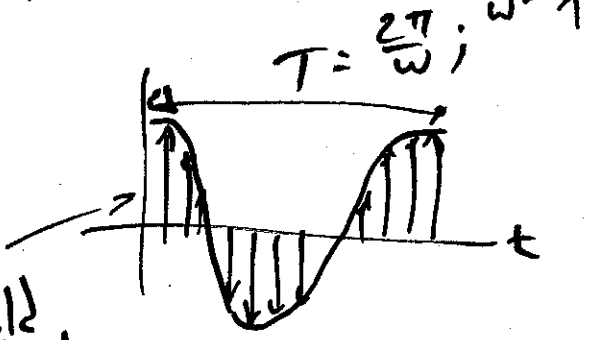
$$= \text{Re}\left[\hat{x} E_0 e^{i(kz-\omega t)}\right] \quad ; \quad k^2 = \frac{\omega^2}{c^2}$$

$$\vec{E}(r,t) = \hat{x} E_0 \cos(kz - \omega t)$$

↳ a function of 2 variables, z & t

fix $z = 0$

$$\vec{E}(z=0,t) = \hat{x} E_0 \cos(\omega t)$$

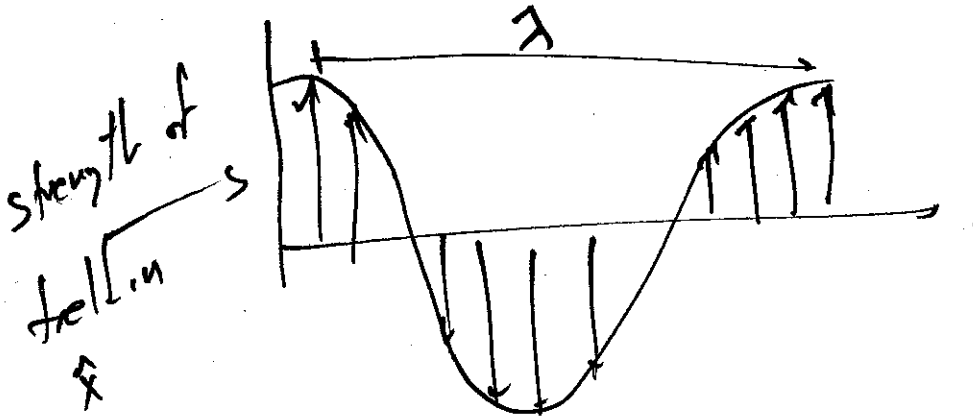


strength of the field in x direction

now lets look @ variations in space

set $t=0$

$$\vec{E}(z, t=0) = \hat{x} E_0 \cos(kz)$$



$kz = 2\pi$ / $z = \lambda$

$$k = \frac{2\pi}{\lambda}$$

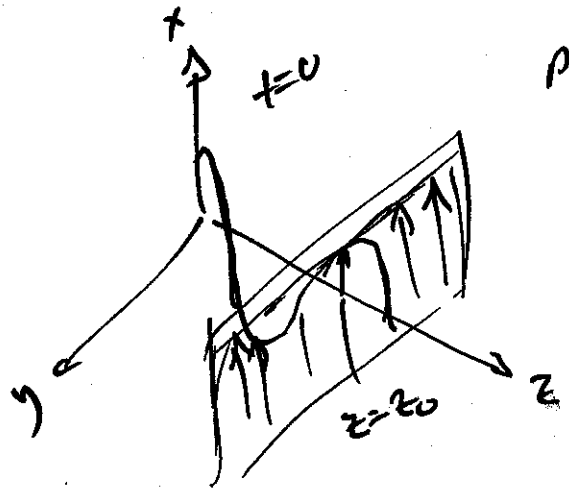
k is spatial frequency of the wave

what determines value of k?

$$k^2 = \frac{\omega^2}{c^2}$$ ← frequency

- This type of E-M wave is called a plane wave... b/c it has wavefronts (surfaces of constant phase) that are planes.

$$\vec{E}(r,t) = E_0 \hat{x} \cos(kz - \omega t)$$



plane $\perp \hat{z}$ @ $z = z_0$

What about $\vec{H}$?

$$\vec{E} = \hat{x} E_0 e^{ikz}$$

$$\nabla \times \vec{E} = i\omega\mu_0 \vec{H}$$

$$ik\hat{y} E_0 e^{ikz} = i\omega\mu_0 \vec{H}$$

$$\Rightarrow \vec{H} = \frac{k}{\omega\mu_0} E_0 e^{ikz} \hat{y}$$

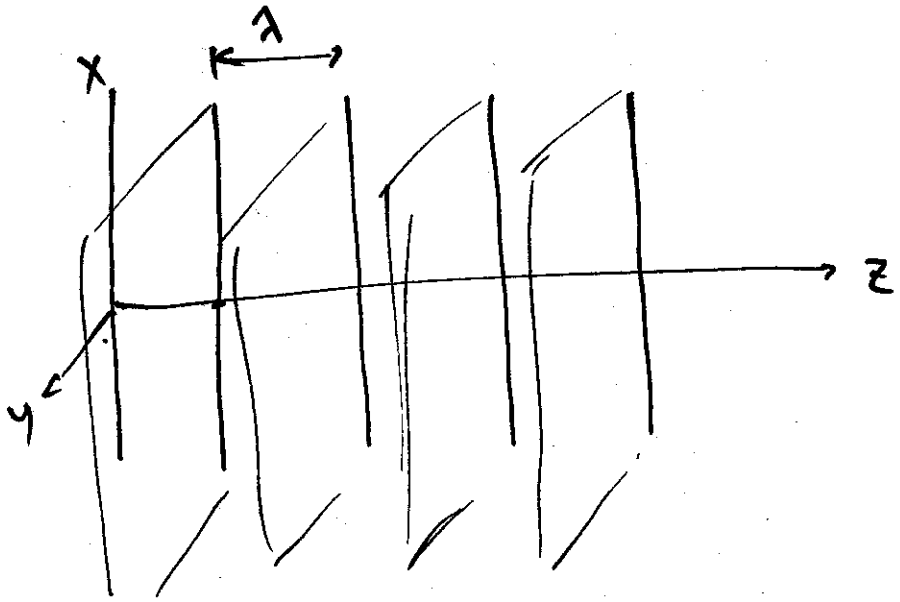
$$= H_0 e^{ikz} \hat{y}$$

$$\vec{\mathcal{H}} = Re\left[H_0 e^{ikz} \hat{y} e^{-i\omega t}\right] = H_0 \cos(kz - \omega t)\hat{y}$$

Something to note

$\vec{E} \perp \vec{H}$ from $\nabla \times \vec{E}$

$\Rightarrow \vec{E} \perp \vec{H} \perp$ direction propagation of the wave



$$\vec{E} = \hat{x} E_0 e^{ikz}$$

wavefronts are surfaces (or lines in 2-D) of constant phase

The most general plane wave

$$\vec{E} = \left[E_x \hat{x} + E_y \hat{y} + E_z \hat{z}\right] e^{i\left[k_x x + k_y y + k_z z\right]}$$

$\vec{k} = k_x \hat{x} + k_y \hat{y} + k_z \hat{z}$ ← k vector tells you the direction of wave propagation

$$\vec{E} = \left[E_x \hat{x} + E_y \hat{y} + E_z \hat{z}\right) e^{i \vec{k} \cdot \vec{r}}$$

- still must satisfy wave eqn

$$\nabla^2 \vec{E} + \left(\frac{\omega}{c}\right)^2 \vec{E} = 0$$

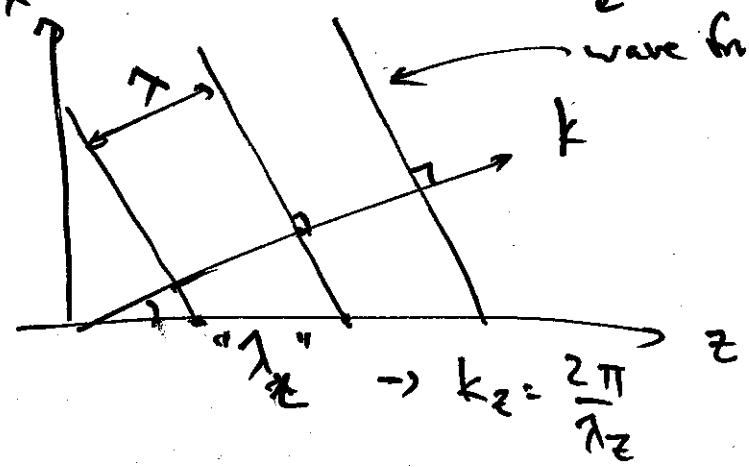
$$k_x^2 + k_y^2 + k_z^2 = \left(\frac{\omega}{c}\right)^2 \equiv |\vec{k}|^2 = k^2 = \left(\frac{2\pi}{\lambda}\right)^2$$

- $\vec{E} \perp \vec{k}$

$E_x \hat{x} + E_y \hat{y} + E_z \hat{z} \perp \vec{k}$

$\Rightarrow$ so for given $\vec{k}$, the relative values of E_x, E_y, E_z must make it $\perp \vec{k}$

for example $k = k_x \hat{x} + k_z \hat{z}$



$$k_x^2 + k_z^2 = k^2$$

$\vec{E} = E_0 \hat{x} e^{ikz}$ $\vec{H} = H_0 \hat{y} e^{ikz}$; $H_0 = \frac{k}{\omega \mu_0} E_0$

... what is the time averaged intensity?

$I = |Re(\vec{S})|$; $\vec{S} = \frac{1}{2} \vec{E} \times \vec{H}^*$ complex poynting vector

$\vec{S} = \frac{1}{2} E_0 \hat{x} e^{ikz} \times H_0^* \hat{y} e^{-ikz}$

$= \frac{1}{2} \frac{k}{\omega \mu_0} E_0 E_0^* \hat{z}$ ← direction of energy flow in $\hat{z}$ direction is the same as wave direction (at least in this class)

$= \frac{1}{2} \frac{k}{\omega \mu_0} |E_0|^2 \hat{z}$

$k = \frac{\omega}{c} = \frac{\omega}{\omega \sqrt{\mu_0 \epsilon}}$ $= \frac{1}{2} \sqrt{\frac{\epsilon}{\mu_0}} |E_0|^2 \hat{z}$

$\Rightarrow I = \frac{1}{2} \sqrt{\frac{\epsilon}{\mu_0}} |E_0|^2$ ... no need to find $\vec{H}$ for calculating intensity