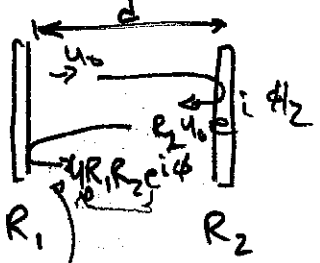
Prelims

- physics web server has been having denial of service attacks... (if you are having trouble downloading PDFs) if you can't get files, email me & all email them to you

- Homework #8 posted, due Wed April 1st @ 5 pm.

- Today we will finish optical resonators & start to treat polarization next week (& the final week)



looking at lossy planar 2 mirror resonator

$U_1 = U_0 r e^{i\phi}$ ← reduction in field

total roundtrip loss = r

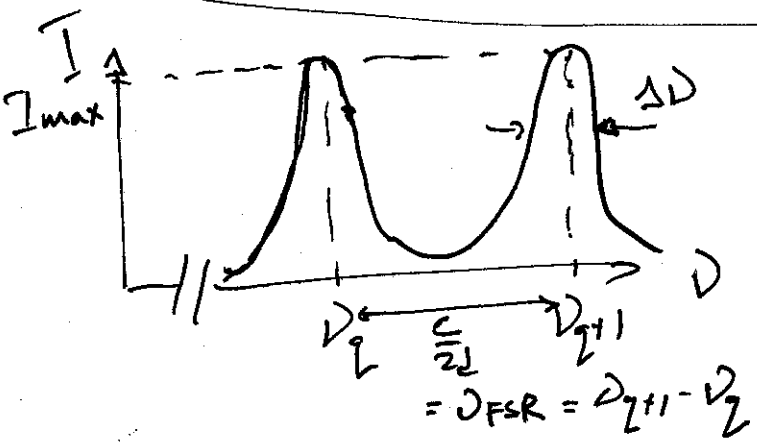
if all loss is due to leakage of mirrors then if the mirrors have a field reflectivity

$R_1, R_2 \Rightarrow r = R_1 R_2$

$\rightarrow U_1 = U_0 R_1 R_2 e^{i\phi}$ total intensity in resonator

$$I = \frac{I_{max}}{1 + \left(\frac{2\mathcal{F}}{\pi}\right)^2 \sin^2\left(\frac{2\pi\nu d}{c}\right)}$$

$I_{max} = \frac{I_0}{(1-r)^2}$; $\mathcal{F} = \frac{\pi\sqrt{r}}{1-r}$; $r < 1$



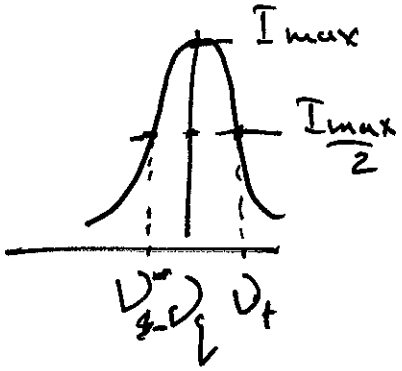
when does $I = I_{max}$?

$\sin^2\left(\frac{2\pi\nu d}{c}\right) = 0 = q\pi$ $\quad q = 0, 1, 2, 3$

$\Rightarrow \nu_q = \frac{c}{2d} q$

↳ same as loss-less case (when $r = 1$), i.e. when cavity in resonant

What is the width of the resonances? $\Delta\nu$



when does the intensity fall by a factor of 2

$\Delta\nu = \nu_+ - \nu_- \cong \Delta\nu_{FWHM}$

$$\left(\frac{2F}{\pi}\right)^2 \sin^2\left(\frac{2\pi\nu d}{c}\right) = 1$$

$$\sin\left(\frac{2\pi\nu_\pm d}{c}\right) \doteq \pm\frac{\pi}{2F} \quad \text{if } F \gg 1$$

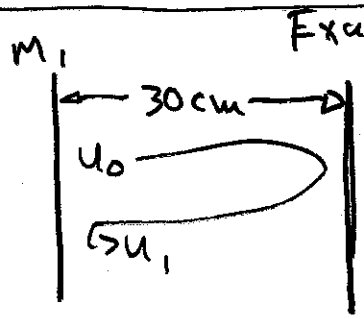
$$\frac{2\pi\nu_\pm d}{c} = \pm\frac{\pi}{2F}$$

$$\nu_\pm = \pm\frac{c}{2d}\frac{1}{2F}\,; \quad \nu_{FSR} = \frac{\nu_{FSR}}{2F}$$

$$\Rightarrow \Delta\nu_{FWHM} = \nu_+ - \nu_- = \frac{\nu_{FSR}}{F}$$

($F \gg 1$)

Example



M_1 has a power reflecting of 90%, M_2 has a power reflecting of 95%

$u_0 = u_0 r e^{i\phi}$

What is the FSR?

What is the $\Delta\nu_{FWHM}$?

$\Rightarrow u_T = u_0 + u_1 + u_2 + \ldots$

$= \frac{u_0}{1 - re^{i\phi}}$

$$\nu_{FSR} = \frac{c}{2d} = \frac{3\times10^8}{2(.3)} = 0.5\times10^9 = 0.5\ GHz$$

to calculate $\Delta\nu$, need $\mathcal{F}$

$$\mathcal{F} = \frac{\pi\sqrt{r}}{1-r} \quad \ldots \text{ what is } r = ?$$

$r = \sqrt{.90}\sqrt{.95}$ ← b/c quote #s are in power, not field

$r = 0.925$ ← reduction in field after one R.T.

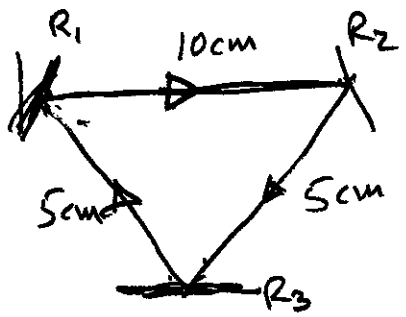
$$\mathcal{F} = \frac{\pi\sqrt{0.925}}{\not\approx 0.075} \simeq 40$$

w/ $\mathcal{F} = 40 \gg 1$ ∴ we can approx $\Delta\nu = \frac{\nu_{FSR}}{\mathcal{F}} = \frac{0.5\,GHz}{40}$

$$\boxed{\Delta\nu \triangleq 12.5\ MHz}$$

---

another example



$R_1 = R_1 = R_3 = 0.9$ power reflection

what $r = ?$

$r = \sqrt{.9}\sqrt{.9}\sqrt{.9}$

What is $\nu_{FSR} = ?$

$\nu_{FSR} = \nu_{q+1} - \nu_q$

$$= \frac{c}{2(10+5+5)\,cm}$$

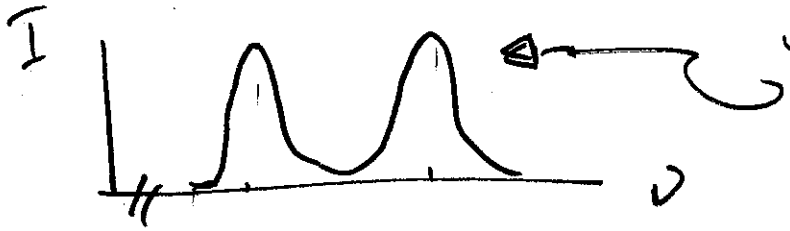
$$= \frac{c}{2(10)} + \frac{c}{2(5)} + \frac{c}{2(5)}$$

$$= \frac{c}{(10+5+5)\,cm}$$

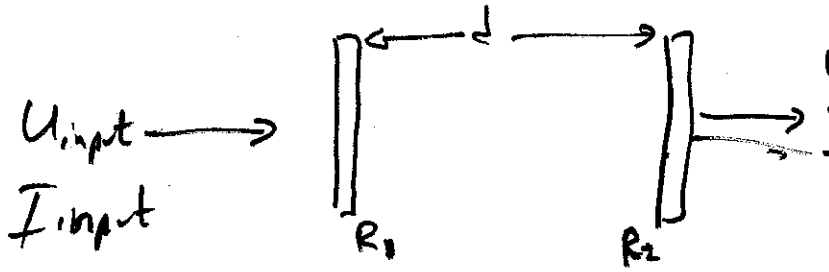
That is $\nu_{FSR} = \frac{c}{\ell}$

ℓ is total path length in one round trip

Applications of Fabry-Perot resonators ← "cavities" ← "etalon"



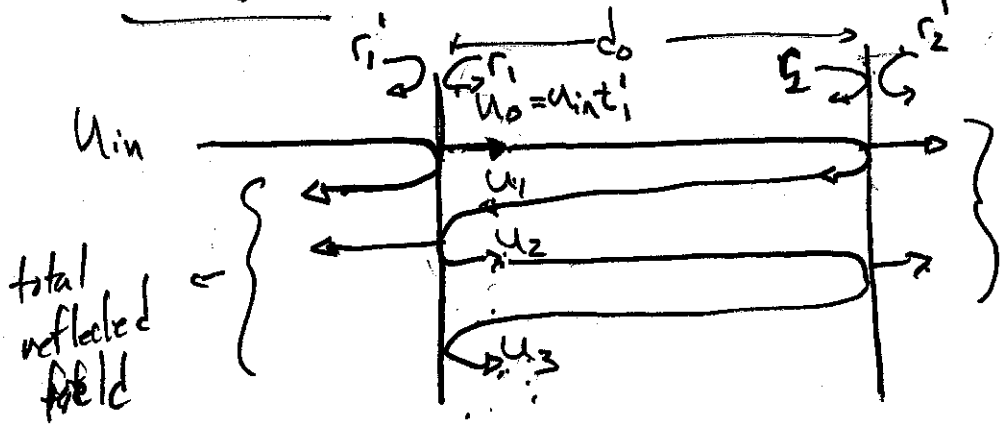
Typically filters are implemented in this configuration:



→ will be a function of frequency due to the action of the resonator

Analysis

$r_2 = -r_2'$
$r_1 = -r_1'$



to avoid book-keeping nightmare we can find total field in resonator

$U_T = U_0 + U_1 + U_2 + \ldots$ given U_0 & total output field is $U_{out} = U_T t_2 e^{ikd}$

$U_{reflected} = U_T t_1 + U_{in} r_1'$ ← not quite correct?

$= U_T - U_{in} t_1'$

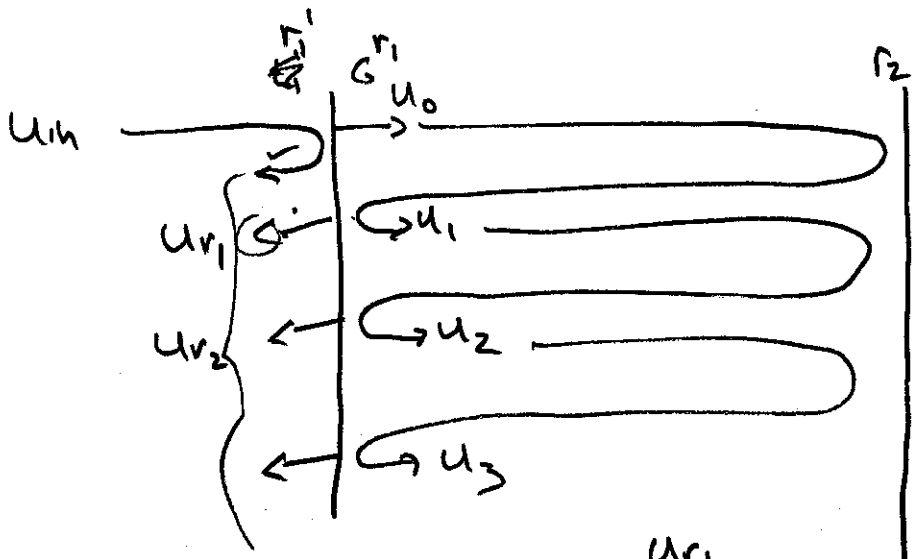
lecture in Jan?

Stokes Relations

$r_{1,2}' = -r_{1,2}$

$t_{1,2}' t_{1,2} = 1 - r_{1,2}^2$

$1 + r_{1,2} = t_{1,2}$



$$u_{reflected} = \overbrace{r_1' u_{in}}^{u_{r_1}} + \overbrace{u_0 r_2 e^{i\phi} t_1'}^{u_{r_2}} + u_0 r_1 r_2^2 e^{i2\phi} t_1'$$

$$= r_1' u_{in} + u_0 r_2 t_1' e^{i\phi}\left(1 + r_1 r_2 e^{i\phi} + (r_1 r_2)^2 e^{i2\phi} + \cdots\right)$$

$$= r_1' u_{in} + u_0 r_2 t_1' e^{i\phi}\left(\frac{1}{1 - r_1 r_2 e^{i\phi}}\right)$$

from earlier in class

$$I = \frac{I_0 \mathcal{F}/(1-r)^2}{1 + \left(\frac{2\mathcal{F}}{\pi}\right)^2 \sin^2\left(\frac{2\pi\nu}{c} d\right)}$$

$\phi = 2kd = 2\cdot\frac{2\pi}{\lambda} d$

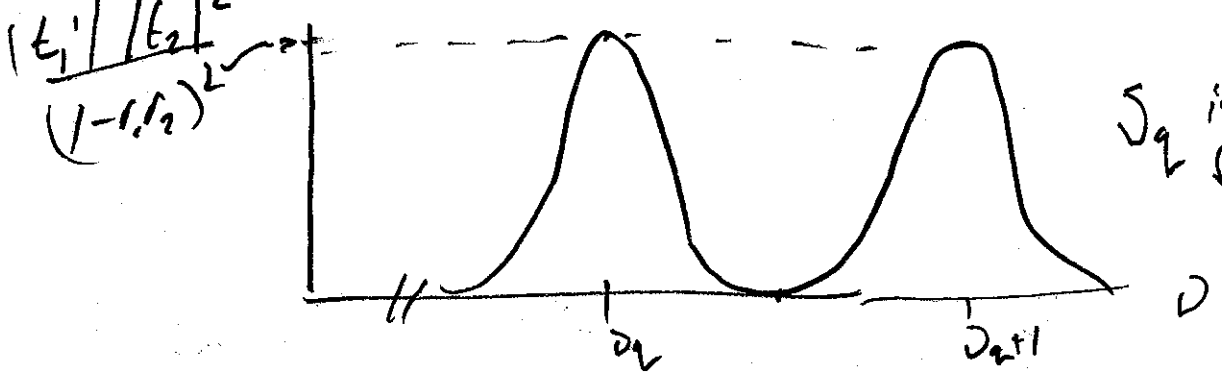
$\mathcal{F} = \frac{\pi}{\delta}$

$\frac{2\pi c}{\nu} \frac{2\pi\nu d}{c}$

$I_0 = |u_0|^2 = |t_1'|^2 I_{in}$

$I_{out} = |t_2|^2 I$ from $u_{out} = t_2 u$

$$\frac{I_{out}}{I_{in}} = \frac{I |t_2|^2 |t_1'|^2}{I_0} = \frac{|t_1'|^2 |t_2|^2}{(1-r_1 r_2)^2} \frac{1}{1 + \left(\frac{2\mathcal{F}}{\pi}\right)^2 \sin^2\left(\frac{2\pi\nu d}{c}\right)}$$



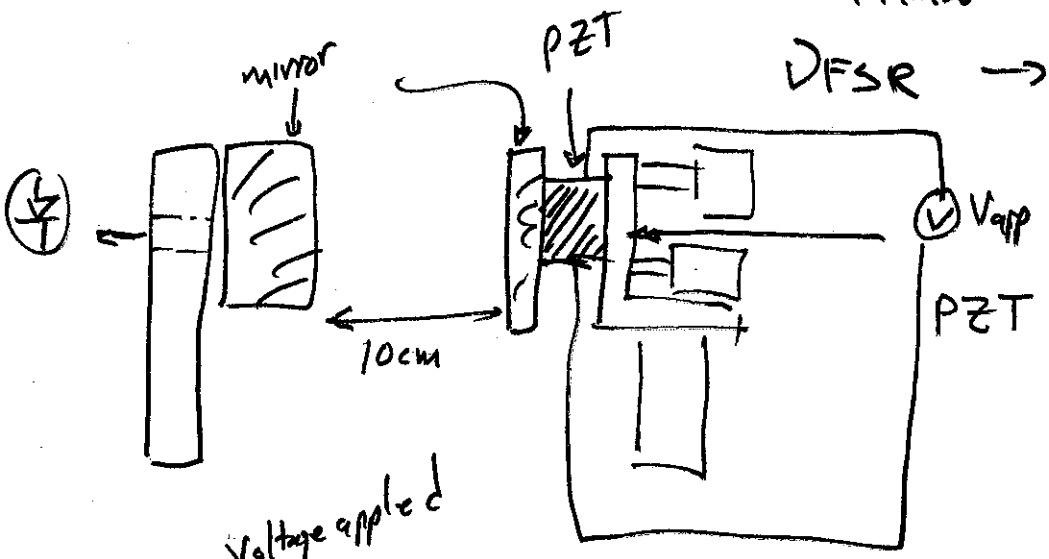
ν_q is where field is resonant

Example of ~~real~~ real device:

scanning fabry-Perot resonator used to measure laser wavelength since it can pass narrow frequencies. (i.e. λ_s)

$\Delta\nu_{FHMW}$ → resolution

ν_{FSR} → further the F-P can scan w/o aliasing

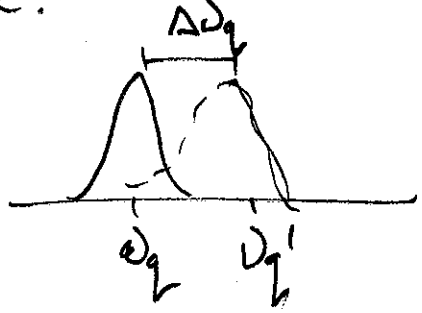


piezo-electric transducer

apply a voltage PZT expands changing mirror ~~separation~~ separation

given a change in d of δd, how much does q^{th} resonance move?

$V_{app} \sim 500\,V$ $\delta d \sim 1\mu m$



$$\Delta\nu_q = \nu_q' - \nu_q$$

$$= q\frac{c}{2(d+\delta d)} - \frac{c}{2d}q$$

$$= \frac{qc\,\delta d}{2d^2\left(1+\frac{\delta d}{d}\right)}; \quad \frac{\delta d}{d} \ll 1$$

$$\simeq -qc\frac{\delta d}{2d^2} \Rightarrow \frac{\Delta\nu_q}{\nu_q} \simeq \frac{\delta d}{d} = \frac{10^{-6}}{10^{-1}} = 10^{-5}$$

so that mean is $\lambda \simeq 1\mu m$, $\nu_q \simeq 300\,THz$

$\Rightarrow \Delta\nu_q \simeq 3\,GHz$