- Midterm Retake is next Tuesday in class
- Exact same exam
  ⇒ The themes of the 4 ~~prob~~ problems are in email

  ~~One~~ One problem was to determine $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ from $t(x,y)$. Remember ABCD tracks the changes in $q(z)$ via changes in the wavefronts so if $u(x,y,z) = \frac{A}{q(z)} e^{i\frac{x^2+y^2}{2q(z)}} e^{ikz}$

  you need to evaluate changes to $q(z)$ not to "amplitude" $q(z)$

- Homework #6 posted tomorrow & it is due next Friday
  ↳ March 13th

On tuesday we were looking @ image function & the wave ~~the~~ $u(x,y,z)$ that created it.

last example from Fourier lab

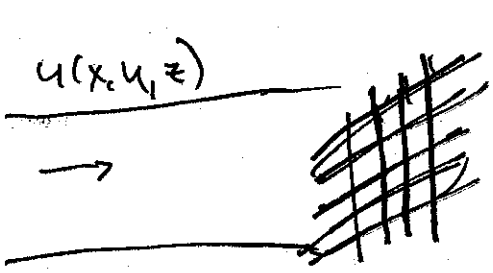


image function

$$f(x,y) = u_{in}(x,y,z)\big|_{z=0} \cdot a(x,y)$$

mesh

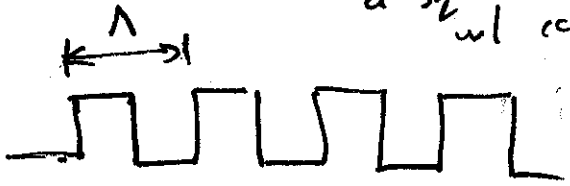
$a(x,y)$ is called the aperature function

$z=0$ i.e. where the mesh is located

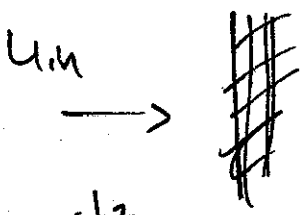
what is $f(x,y)$?

lets look in 1-D?

mesh looks like a square wave w/ constant period Λ



mesh imparts Boundary Conditions on U_{in} which means that U_{out} (outgoing wave) will have harmonics of the mesh [$a(x,y)$] to satisfy these B.C.

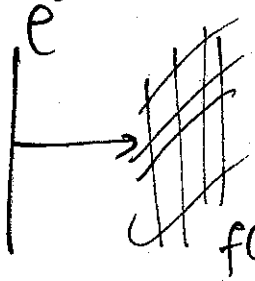


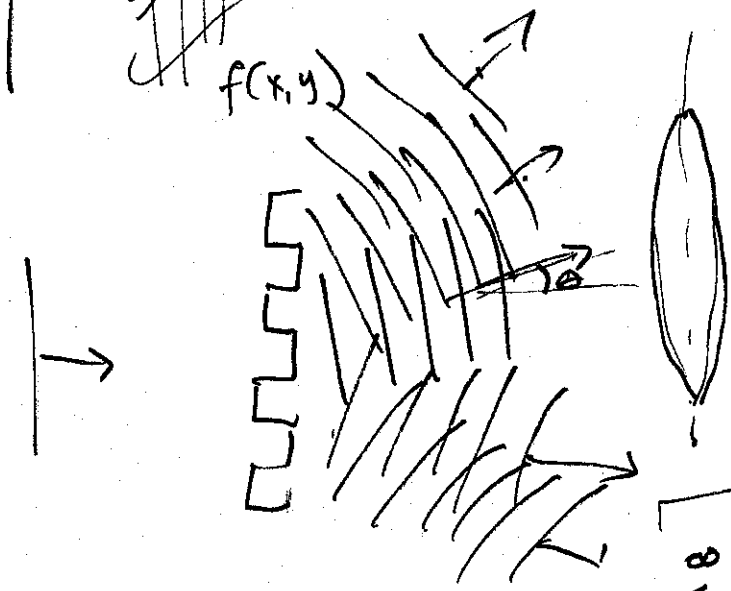
?

U_M

$$f(x,y) = e^{ikz} \sum_{n=-\infty}^{\infty} c_n e^{i\frac{2\pi}{\Lambda}nx}$$

↳ Fourier Series expansion of the $a(x,y)$, the square wave





$$U_{out} = \sum_{n=-\infty}^{\infty} c_n e^{i\frac{2\pi}{\Lambda}nx} e^{i\sqrt{k^2-\left(\frac{2\pi}{\Lambda}n\right)^2}z}$$

again we can write this b/c we've 1D'd the ~~space~~ specific harmonics (k_x) in $f(x)$

In general you cannot just take on a e^{ikz} to $f(x,y)$ to make $U(x,y,z)$

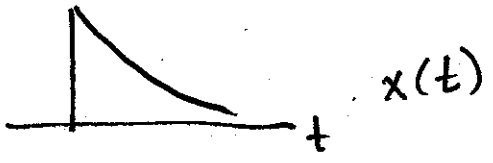
In addition to Fourier ~~=~~ transforms

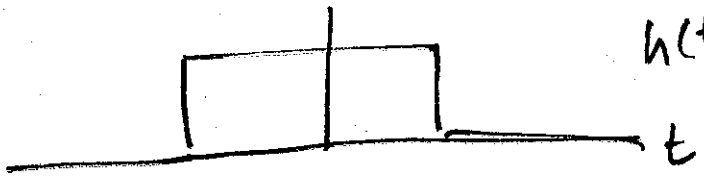
we are also going to us another mathematical convolution

$$y(t) = h(t) \otimes x(t) \equiv \int_{-\infty}^{\infty} h(v)\, x(t-v)\, dv$$

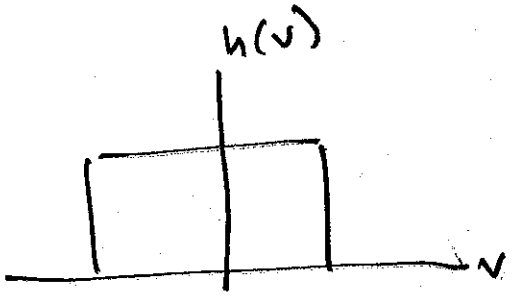
$\otimes$ ← convolution operation

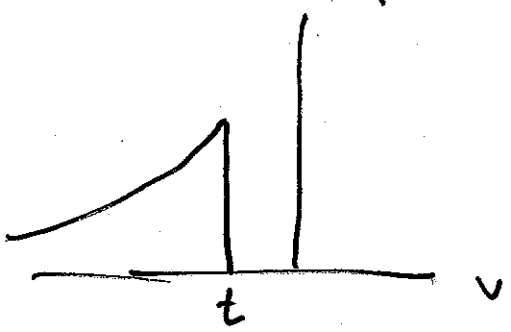
looks terrible... but actually not that bad.





given these two... how to convolve?





see applets...

Why convolution?

We will use the convolution theorem

if $q(x) = b(x) \otimes c(x)$ Then $\mathcal{F}[q(x)] = \mathcal{F}[b(x)] \cdot \mathcal{F}[c(x)]$

is. convolution helps use complete FTs.

but more importantly it helps us by providing a different method to finding the output image given an input image.

$$\Rightarrow \quad g(x,y) = F^{-1}\left[\tilde{h}(k_x,k_y)\cdot F(f(x,y))\right]$$

lots of steps.. F & F^{-1}

but we can use the convolution theorem

$$\underbrace{\tilde{h}(k_x,k_y)}_{\text{x-fer function}} = F\left[\underbrace{h(x,y)}_{\text{impulse response function}}\right]$$

$$g(x,y) = F^{-1}\left[F(h(x,y)\cdot F(f(x,y))\right]$$

$$= F^{-1}\left[F(h(x,y)\circledast f(x,y))\right.$$

$$g(x,y) = h(x,y)\circledast f(x,y)$$

$$\boxed{g(x,y) = \iint f(x',y')\,h(x-x',y-y')\,dx'dy' \quad (1)}$$

- requires that you know $h(x,y)$
- must do convolution

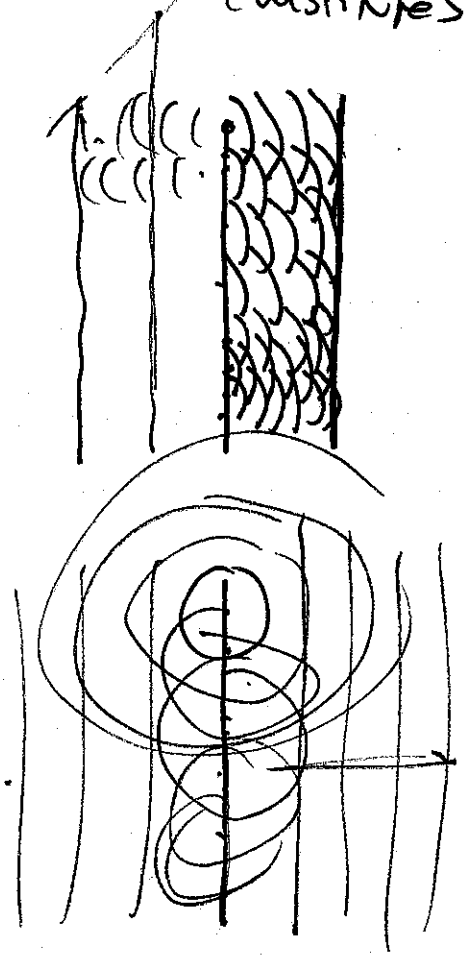
lets ID the physics of this expression.

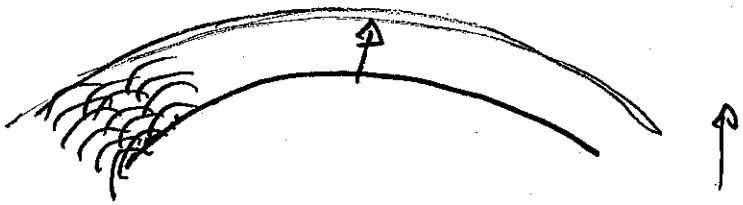
First, need to ~~remind~~ remind you of

Huygen-Fresnel Principle

$\Rightarrow$ states that each point on a wave front generates a spherical wavelet @ every instant

The envelope of these "secondary" wavelets constitutes the overall wavefronts

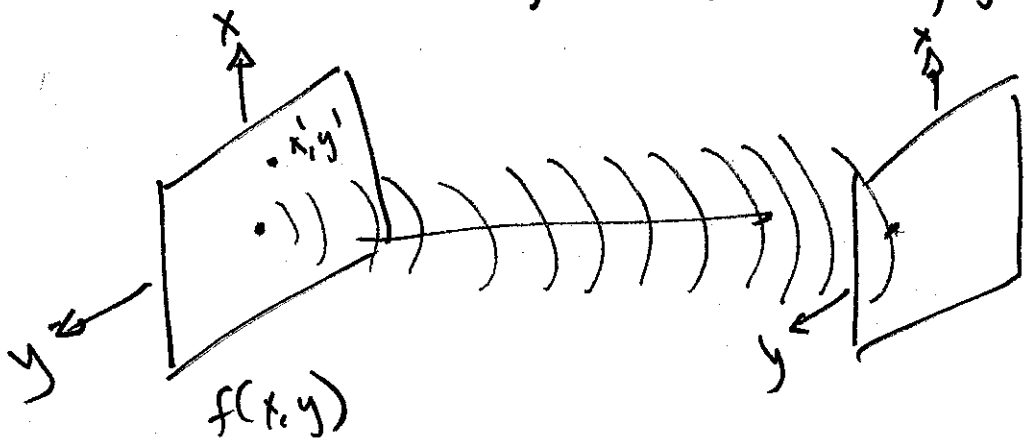




see applet

Physical interp of

$$g(x,y) = \iint f(x',y')\, h(x-x', y-y')\, dx'dy' \quad (1)$$



Suppose that you have a spherical wave ~~ge~~ centered @ the origin, $x, y, z=0$ in the input plane...

what would $f(x,y) \stackrel{?}{=} \delta(x)\delta(y)$

under the paraxial approx?

from (1)

$$g(x,y) = \iint_{-\infty}^{\infty} \delta(x')\delta(y')\, h(x-x', y-y')\, dx'dy'$$

$$g(x,y) = h(x,y) \quad \longleftarrow \text{ hence the name impulse response function for } h(x,y)$$

only true for $f(x,y) = \delta(x)\delta(y)$

under the paraxial approx, what does a spherical wave look like @ $z=d$?

$\Rightarrow$ a ~~parabolic~~ paraboloidal wave

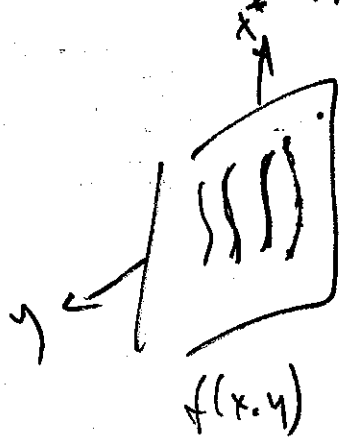
$$h(x,y) \sim e^{ik\frac{x^2+y^2}{2z}}\Big|_{z=d} = \frac{-ik}{2\pi d} e^{ikd} e^{ik\frac{x^2+y^2}{2z}}\Big|_{z=d}$$

now what if the impulse is located @ a point (x', y') ?

$$f(x,y) = \delta(x-x')\,\delta(y-y')$$

$$\Rightarrow g(x,y) = h(x-x', y-y') \quad \text{from (1)}$$

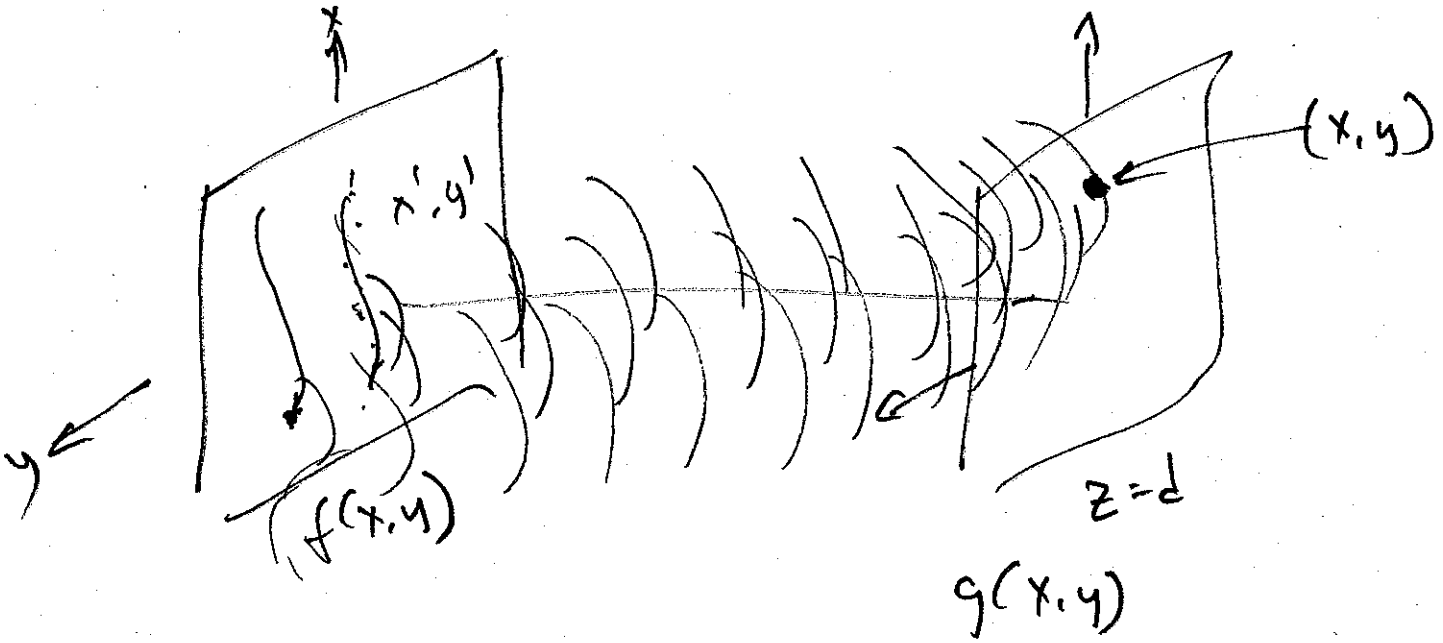
now apply Huygen-Fresnel principle



- each point on the wavefront as it passes through the input plane @ point $(x', y', z=0)$ generates a spherical wave w/ amplitude $f(x', y')$ that contributes to the output $g(x,y)$

→ must sum over all x', y' @ $z=0$ to get total field @ $z=d$ (i.e. $g(x,y)$)

$$\int_{-\infty}^{\infty}\int dx'dy'\, \underbrace{f(x',y')}_{\text{amplitude of wavelet}}\, \underbrace{h(x-x', y-y')}_{\text{response (or illumination) @ point } x,y,z=d \text{ in output plane from that wavelet @ point } x',y',z=0}$$



lets do an example (from tuesday)

$$u(x,y,z) = e^{i\left(\frac{\kappa}{2}x + \frac{\kappa}{3}y + k_z z\right)} \quad @ z=d$$

$$g(x,y) = \iint_{-\infty}^{\infty} f(x',y')\, h(x-x', y-y')\, dx'\, dy'$$

$$= \iint_{-\infty}^{\infty} e^{i\left(\frac{\kappa}{2}x' + \frac{\kappa}{3}y'\right)} e^{ik\frac{(x-x')^2 - (y-y')^2}{2d}} dx'\, dy'$$

... gulp ☹ must couple the square in x'

not fun

Moral: sometimes easier to do FT

" " " convolution