Fundamental Equation of Optics (& This class)

$$\nabla^2 \vec{E} - \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \qquad \vec{E}(r,t) \qquad (1)$$

$\Rightarrow$ Hard Equ to solve

In this class we are going to consider only ~~monochromatic~~ monochromatic fields that are harmonic functions of time

$$\vec{E}(r,t) = \underbrace{\vec{A}(r)}_{\text{amplitude depends only on space}} \cos\left(\phi(r) - \underbrace{\omega t}_{\text{monochromatic only one frequency } \omega}\right)$$

$$\vec{E}(r,t) = \mathrm{Re}\left[\vec{E}(r)\, e^{-i\omega t}\right]$$

$$\vec{E}(r) = \vec{A}(r)\, e^{i\phi(r)} \Rightarrow \text{complex amplitude}$$

$\vec{E}(r)$ is complex amplitude

(1) only has a spatial dependence

(2) not a measureable field b/c complex #

Plan of attack on eqn (1)

Do calculations w/ $\vec{E}(r)$ then when we want

The real electric field

$$\Rightarrow \vec{\mathcal{E}}(r,t) = \mathrm{Re}\left[\vec{E}(r)e^{-i\omega t}\right]$$

$$= \frac{1}{2}\left(\vec{E}(r)e^{-i\omega t} + \vec{E}^{*}e^{i\omega t}\right)$$

$$= \frac{1}{2}\left(\vec{E}(r)e^{-i\omega t} + c.c.\right)$$

note $\quad j \Leftrightarrow -i$

- book uses j
- I use i

$$\boxed{i = -j}$$

The reason we use complex amplitudes is that the time & spatial dependence become separable $\Rightarrow$ separate-able & these solving eqn (1) becomes much easier to solve