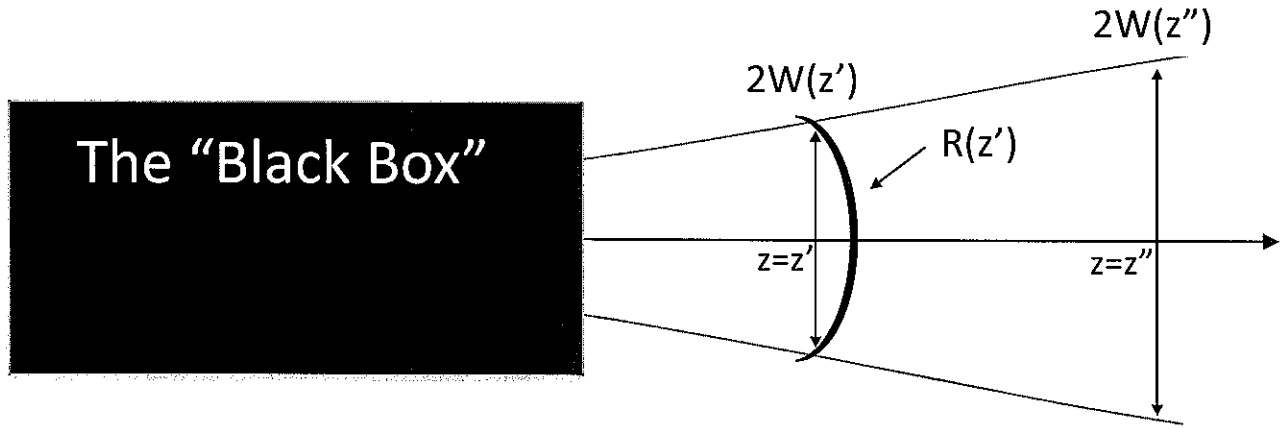
PHYS 408 Group Activities for Oct 19 2010

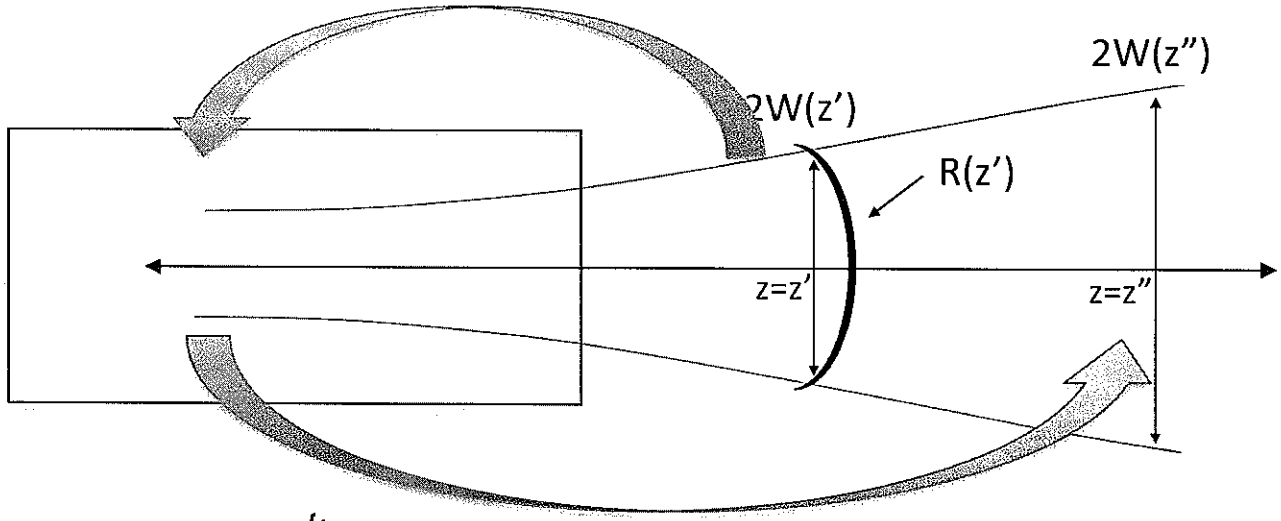
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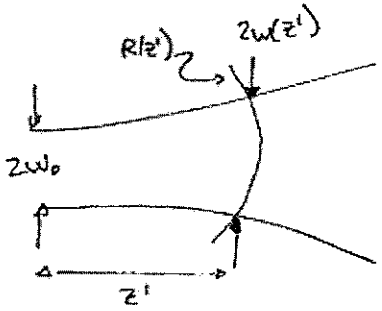


A laser located somewhere in a black box shines a beam out. Say you have measured the beam width W(z') and radius R(z') at z=z' (note that you don't know what z' is).

1-Write a step by step recipe for your lab partner to calculate the (new) beam diameter at z=z". I'm looking for a qualitative recipe here.

$$W_o = \sqrt{\lambda z_o / \pi}$$





$$\begin{cases} w(z') = w_0\left[1 + \left(\frac{z'}{z_0}\right)^2\right]^{1/2} & (1) \\ R(z') = z'\left[1 + \left(\frac{z_0}{z'}\right)^2\right] & (2) \end{cases}$$

Two equations, 2 unknowns [z' & w_0 or z_0]

1-) Plan is to calculate $2w_0$ & z'

$2w_0$: minimum waist; z': location of

2-) Calculate $w(z')$ via (1)

do it by inverting (1) & (2)

$$\Rightarrow \quad z' = \frac{R(z')}{1 + \left[\frac{\lambda R(z')}{\pi w(z')^2}\right]^2} \qquad w_0 = \frac{w(z')}{\left[1 + \left[\frac{\pi w(z')^2}{\lambda R(z')}\right]^2\right]^{1/2}}$$

given $R(z')$ & $w(z')$ find w_0 & its location z'

$$W_o = \sqrt{\lambda z_o / \pi}$$

So to keep track of beam we need to keep track of W(z) and R(z) and to use

$$R(z) = z(1 + z_o^2/z^2)$$

$$W(z) = W_o\sqrt{1 + z^2/z_o^2}$$

$$W_o = \sqrt{\lambda z_o/\pi}$$

to use these eqns we must first find z_o and location of minimum waist...loads of algebra, but there is an easier way...using the "q parameter.

How did we get here?

$$E(x,y,z) = \frac{A}{z} e^{ik\left(\frac{x^2+y^2}{2z}\right)} \xrightarrow{z \to z - iz_o} E(x,y,z) = \frac{A}{z - iz_o} e^{ik\left(\frac{x^2+y^2}{2(z-iz_o)}\right)}$$

$$U(x,y,z) = \frac{A_o}{\sqrt{(z^2 + z_o^2)}} e^{i\tan^{-1}(z/z_o)} e^{ik\left(\frac{x^2+y^2}{2z(1+z_o^2/z^2)}\right)} e^{-\left(\frac{x^2+y^2}{\lambda/\pi(z^2/z_o+z_o)}\right)} e^{ikz}$$

$$U(x,y,z) = \frac{A_o}{\sqrt{(z^2 + z_o^2)}} e^{i\tan^{-1}(z/z_o)} e^{ik\left(\frac{x^2+y^2}{2R(z)}\right)} e^{-\left(\frac{x^2+y^2}{W^2(z)}\right)} e^{ikz}$$

We keep track of the beam via R(z) and W(z)...consider $q(z) = z - iz_o$

$$U(x,y,z) = \frac{A}{q(z)} e^{ik\left(\frac{x^2+y^2}{2q(z)}\right)} e^{ikz}$$

using $z_o = \frac{\pi w_o^2}{\lambda}$

$= \frac{1}{R} + \frac{i\lambda}{\pi w^2}$

3- Express 1/q(z) in terms of R(z) and W(z). $\frac{1}{q} = \frac{1}{z - iz_o} = \frac{z + iz_o}{z^2 + z_o^2} = \frac{z}{z^2 + z_o^2} + \frac{iz_o}{z^2 + z_o^2} = \frac{1}{z\left(1 + \frac{z_o^2}{z^2}\right)} + \frac{i}{z_o\left(1 + \frac{z^2}{z_o^2}\right)}$

4- How many free parameters are in the q(z)? What are they and explain how it is enough to fully describe a Gaussian beam at any location

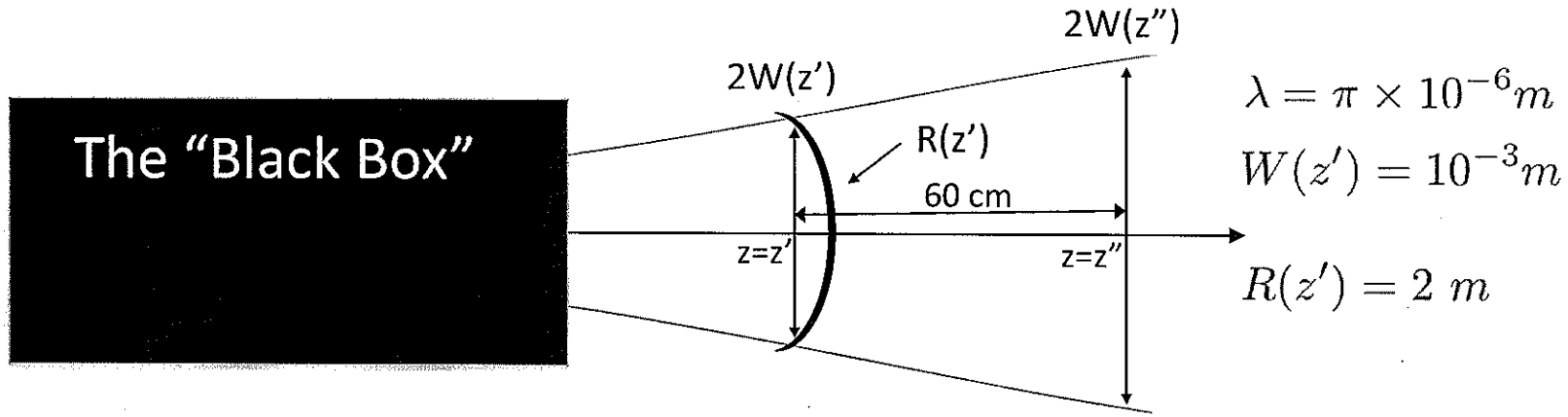
two, z and z_o (or R and W)
z locates waist, z_o used to find w_o or describes waist and divergence

5- Given this definition of q(z), at what position in z does the minimum waist occur? $z = 0$!

$z = 0$

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6-Using the q(z), find the location of the minimum waist (z')

$\frac{1}{q} = \frac{1}{R} + i\frac{\lambda}{\pi w^2} = \frac{1}{2} + i\frac{\pi e-6}{\pi (10^{-3})^2} = 0.5 + i \qquad q = \frac{1}{0.5+i} = \frac{0.5-i}{0.25+1} = 0.4 - 0.8i$

$z' = 0.4m$ (0.4m to the left)

7-Using the q(z), find the beam width 60 cm away [W(z")]

$z'' = 0.4m + 0.6m = 1m \qquad q = 1 - 0.8i \qquad \frac{1}{q} = \frac{1}{1-0.8i} = 0.61 + 0.48i$

z_0 unchanged

$0.48 = \frac{\lambda}{\pi w^2}$

$w = \sqrt{\frac{\lambda}{0.48\pi}} = 1.4e-3m$