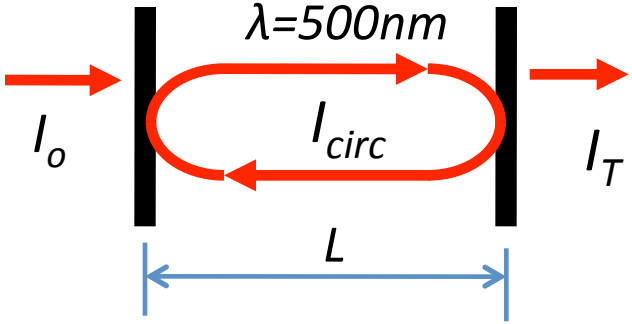
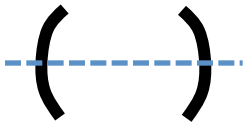
Below are various statements regarding two-mirror cavities. Which of these is incorrect?



(A) After each round trip in the cavity, the beam returns to its original shape (i.e. wavefront curvature.

(B) Only concave mirrors can lead to a stable 2 mirror optical cavity:

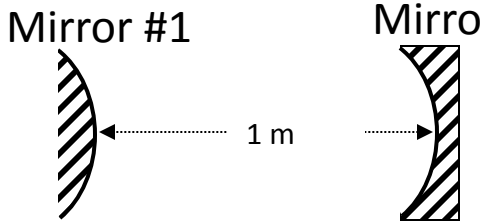


(C) At t=0, immediately after the incoming laser is aimed at a high Finesse cavity, most of the light does not go into the cavity, most of it is reflected.

(D) When on resonance and in steady state, the circulating optical power inside a cavity is always larger than the incoming optical power

(E) More than one statement is not true

The resonator below with two spherical mirrors that each have a field reflectivity of 0.9 and a radius of curvature of 2 m. A field with wavelength of 1 micron is resonant inside the cavity.



$$\frac{1}{q(z)} = \frac{1}{R(z)} + i\frac{\lambda}{\pi W^2(z)}$$

$$q(z) = z - iz_o$$

$$w_0 = \sqrt{\frac{\lambda z_0}{\pi}}$$

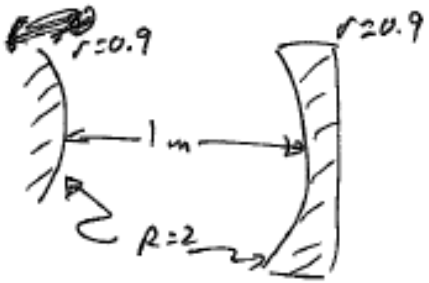
$$R = z\left[1 + \left(\frac{z_0}{z}\right)^2\right] \qquad w = w_0\sqrt{1 + \left(\frac{z}{z_0}\right)^2}$$

a)
Considering only a fundamental Gaussian beam as a mode of the resonator, where is location of the minimum waist? (To the left of mirror #1, inside the resonator, or to the right of mirror #2?) You must justify your answer

b)
If the location of minimum waist is 0.5 m from the nearest mirror, find the size of the minimum waist.

c)
Calculate the waist at mirror #2.

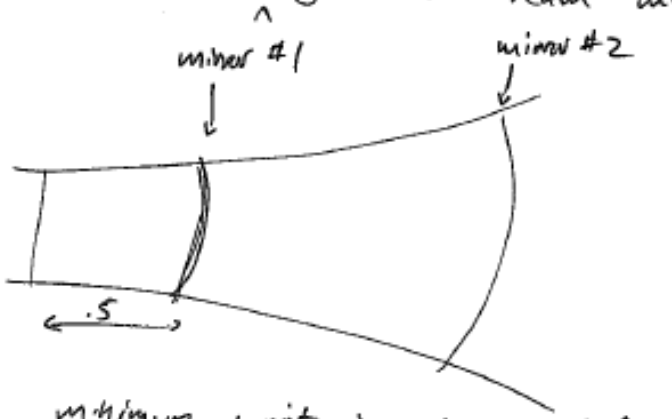
d)
Give an upper and lower bound on the radius of curvature of the Gaussian beam when it is inside the resonator. You must justify your answer to receive any marks



$\lambda = 10^{-6}$

gaussian

This means the resonant gaussian beam must



a) For a mode to be resonate w/ this cavity

The phase fronts must match the mirror curvature so minimum waist is to the left

b) $R = z\left[1+\left(\frac{z_0}{z}\right)^2\right]$

$2 = 0.5\left(1+\left(\frac{z_0}{0.5}\right)^2\right)$

$3 = 4z_0^2$

$\Rightarrow z_0 = \frac{\sqrt{3}}{2}$ $\quad w_0 = \sqrt{\frac{\lambda z_0}{\pi}}$

$= 10^{-3}\frac{(3)^{1/4}}{\sqrt{2\pi}}$

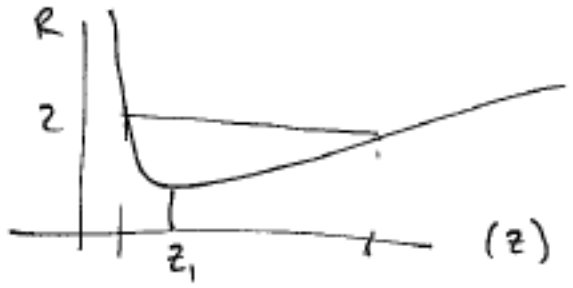
c) $w = w_0\sqrt{1+\left(\frac{z}{z_0}\right)^2}$

$= w_0\sqrt{1+\left(\frac{1.5\cdot 2}{\sqrt{3}}\right)^2}$

$= w_0(1+3)^{1/2}$

$= 2w_0$ w/ w_0

d)



So upper bound is $R=2$. lower bound is e

It turns out $z_1 = z_0$ but you ~~cannot~~ could fi

R is by minimizing $R = z\left[1+\left(\frac{z_0}{z}\right)^2\right]$

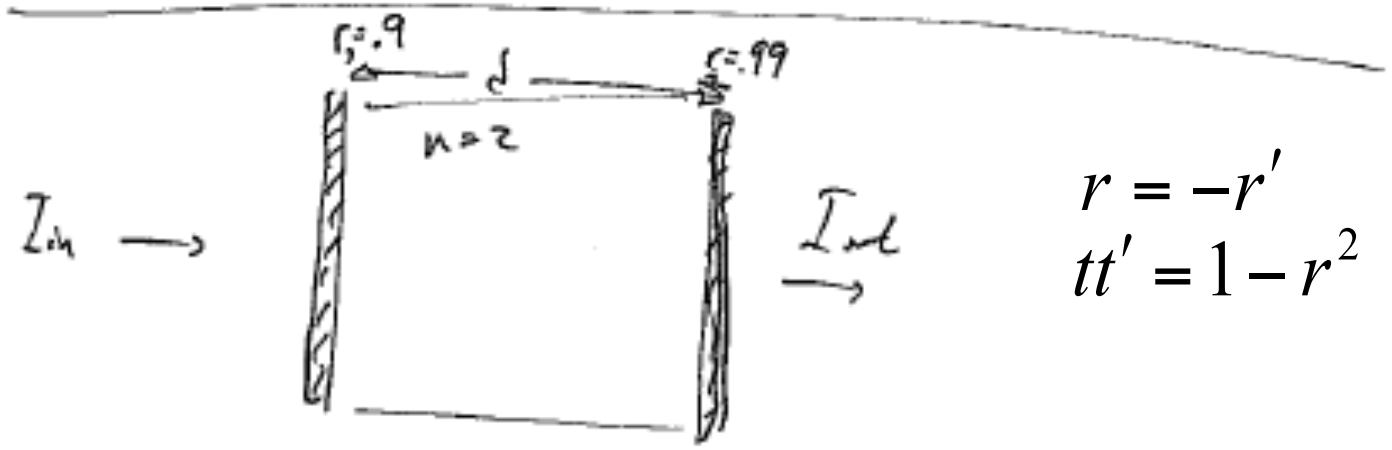
$$\Rightarrow \quad R_{max} = 2$$

$$R_{min} = z\left(1+\left(\frac{z_0}{z}\right)^2\right]\Big|_{z>z_0}$$

$$= z_0 \cdot 2$$

$$= 2 \cdot \frac{\sqrt{3}}{2}$$

$$= \sqrt{3}$$



$$r = -r'$$
$$tt' = 1 - r^2$$

Find an expression for I_{out} & explain how this structure functions as an optical filter

$$I = \frac{I_{\max}}{1 + (2\mathcal{F}/\pi)^2 \sin^2(\varphi/2)}, \qquad I_{\max} = \frac{I_0}{(1 - |r|)^2}.$$

$$\mathcal{F} = \frac{\pi\sqrt{|r|}}{1 - |r|}$$

Bonus: What is the resolution of this optical filter?

The circulating power in the cavity is:

$$I_{cav} = \frac{I_{max}}{1 + \left(\frac{2\mathcal{F}}{\pi}\right)^2 \sin\left(\pi \frac{D}{D_{FSR}}\right)} \quad ; \quad I_{max} = \frac{I_0}{(1-r_1 r_2)^2} = \frac{(1-r_1^2)\, I_{in}}{(1-r_1 r_2)^2}$$

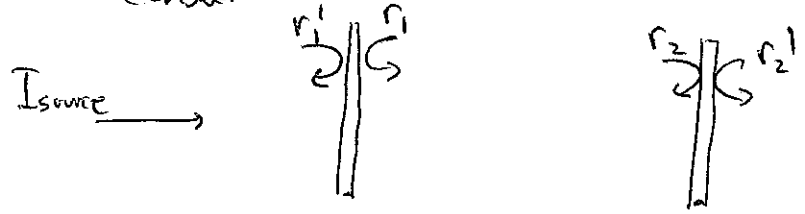
(t_1^2)

$$\therefore \quad I_{out} = t_2^2 = \frac{(1-r_2^2)(1-r_1^2)}{(1-r_1 r_2)^2} I_{in} \; \frac{1}{1 + \left(\frac{2\mathcal{F}}{\pi}\right)^2 \sin\left(\pi \frac{D}{D_{FSR}}\right)}$$

This acts like a filter as only certain D (frequencies) ~~transmit~~ transmit through.

An interesting question is what happens if mirror #1 & #2 are exchanged? Does that effect I_{out}? Why or why not?

In class we were discussing if the power (intensity) inside a cavity is always larger. On resonance this is true. Consider



stokes: $r_1 = -r_1'$

$t_1 t_1' = 1 - r_1^2$

power conservation $\rightarrow$ $r_1^2 + t_1^2 = 1$

$\Rightarrow t_1 = t_1'$

$$I_{cav} = \frac{I_0}{(1-r_1 r_2)^2} \frac{1}{1 + \left(\frac{2F}{\pi}\right)\sin\left(\pi \frac{D}{\lambda_{ESR}}\right)} \quad \text{w/} \quad F = \frac{\pi\sqrt{r_1 r_2}}{1 - r_1 r_2}$$

$\&$ $I_0 = I_{source} t_1^2$

$= I_{source}(1 - r_1^2)$

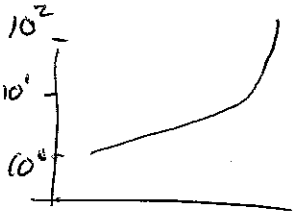
$$I_{cav} = \frac{I_{source}(1-r_1^2)}{(1-r_1 r_2)^2} \frac{1}{1 + \left(\frac{2F}{\pi}\right)\sin\left(\frac{\phi}{2}\right)}$$

if $\frac{I_{cav}}{I_{source}} = \frac{1-r_1^2}{(1-r_1 r_2)^2} \frac{1}{1 + \frac{2F}{\pi}\sin\left(\frac{\phi}{2}\right)} > 1$ Then power in cavity is larger Then input

for ease of plotting lets set $r_2 = 1$

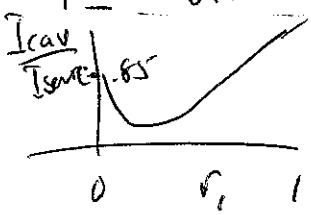
on resonance

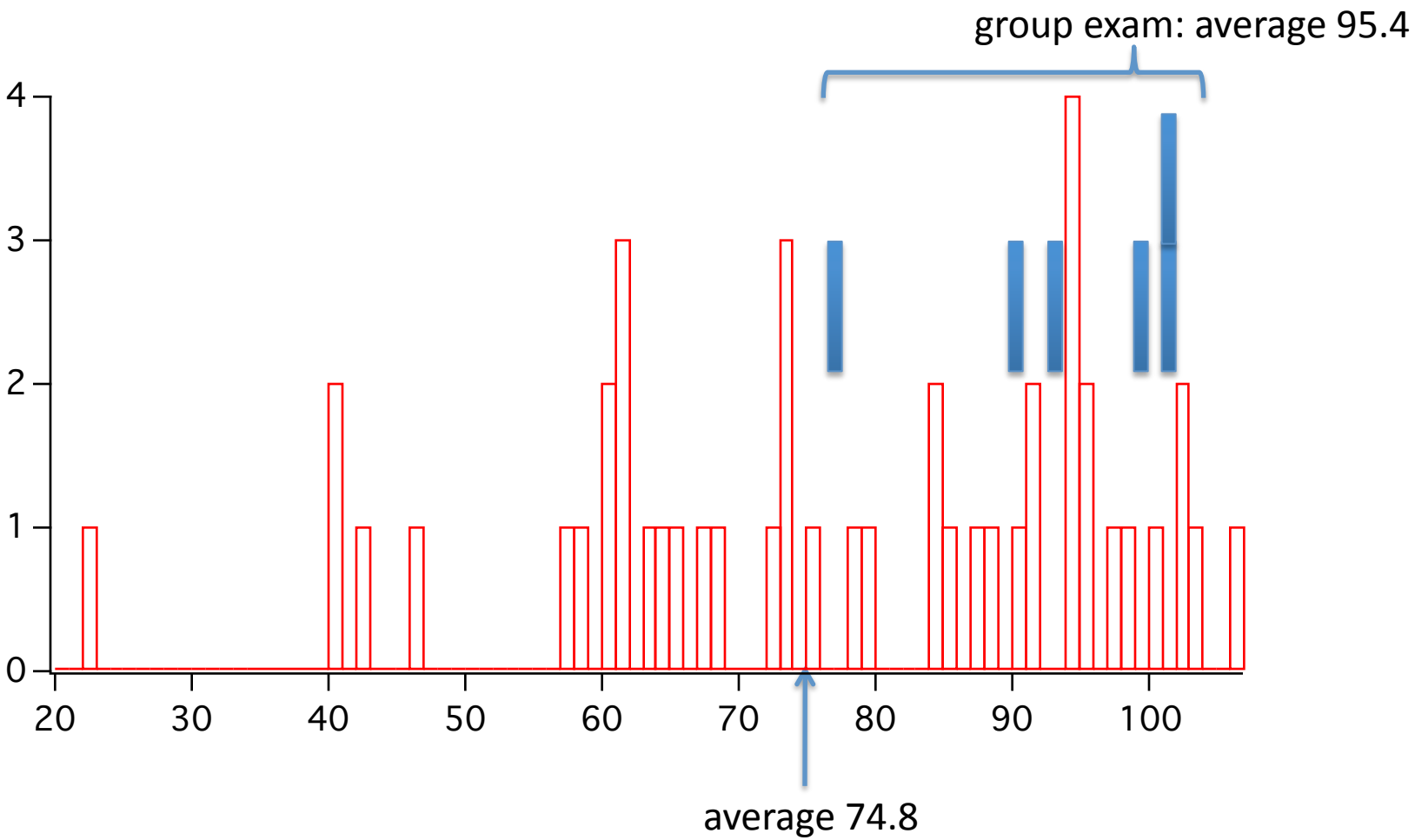
$$\frac{I_{cav}}{I_{source}} = \frac{1-r_1^2}{(1-r_1)^2}$$ which is >1 for $\Leftrightarrow 0 < r_1 < 1$ (I plotted it in Matlab) $\Rightarrow$



when ~~I_{cav}~~ off resonance $\frac{I_{cav}}{I_{source}} = \frac{1-r_1^2}{(1-r_1)^2} \frac{1}{1 + \frac{2\sqrt{r_1}}{1-r_1}}$

$\&$ indeed in This case $\frac{I_{cave}}{I_{sor}} < 1$




group exam: average 95.4
4
3
2
1
0
20
30
40
50
60
70
80
90
100
average 74.8