PHYS 408 for March 31th 2011

Why to study polarization? What can it do for us?

-Used to encode information…polarization multiplexing

-Used to make 3-D movies

-Used to make LCD displays

-In anisotropic media, different polarizations (directions of E-field vector) behave differently

-Not surprisingly, when looking at light's interaction with matter, the direction of the electric field can be critical to describing its effect.

Pre-readings questions are important enough that I want to review them

1-If you assume light to have definite polarization state, what pieces of information are required to determine that specific polarization state?

A: One: which basis state, ie linear (x,y), circular (RHCP, LHCP), or ? In most all cases we will use linear (and this is what the Jones vector representation represents). Two: the relative phase between the two components of the basis. Three: the relative amplitude between the two components of the basis.

2-In a few sentences, explain why must we consider time dependence when determining the polarization state of light.

A: Polarization describes the trajectory of the electric field vector tip as the E&M wave propagates. As this is a function of time, we need to use the real electric field to determine the trajectory (and hence polarization)

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Convention for determining handedness of polarization:
-look at fixed z, increment t
-look at wave propagating toward you (look in –z direction)
-in this case CCW=LHCP and CW=RHCP

For the following fields:
a-determine polarization state of these waves (linear, circular, elliptical and handedness)
b-draw them as a function of time (at fixed z) and distance (at a fixed t).
c-write down their Jones vector representation or their complex amplitude

1- $\vec{E} = E_o(\hat{x} + \hat{y})e^{ikz}$
2- $\vec{E} = E_o(4\hat{x}e^{i\pi/2} + 4\hat{y})e^{ikz}$
3- $\vec{E} = E_o(\hat{x}e^{i\pi} - \hat{y})e^{-ikz}$
4- $\vec{E} = (\hat{x} + 5\hat{y}e^{i\pi/2})e^{ikz}$
5- $\vec{J} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

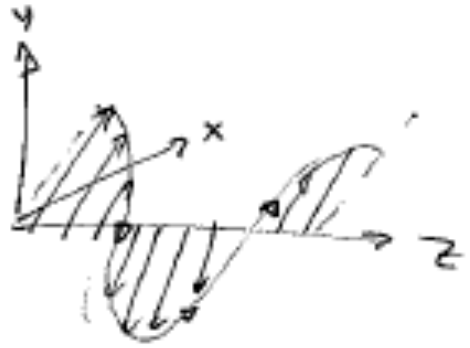
Bonus: Determine the polarization of state of a standing wave made of two counter- propagating plane waves $\vec{E}_1 = E_o\hat{y}e^{-ikz}$ and $\vec{E}_2 = E_o\hat{x}e^{ikz}$

1.) $\vec{E} = E_0(\hat{x}+\hat{y})e^{ikz}$

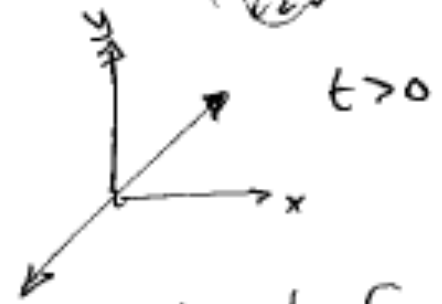
$\vec{\mathcal{E}} = E_0(\hat{x}+\hat{y})\cos(kz-\omega t)$

This is linearly polarized light @ 45° wrt $\hat{x}$ axis.

@ $t=0$



@ $z=0$



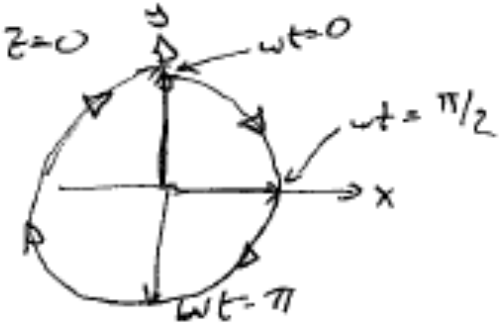
$\vec{J} = E_0\begin{bmatrix}1\\1\end{bmatrix}$ or in normalized form $\vec{J} = \begin{bmatrix}1\\1\end{bmatrix}$

2-) $\vec{E} = \tilde{E}_0 4\left(\hat{x}e^{i\pi/2} + \hat{y}\right)e^{ikz}$

$\mathcal{E}_x = 4E_0 \cos(kz - \omega t + \pi/2)$

$\mathcal{E}_y = 4E_0 \cos(kz - \omega t)$





CW: RHCP

Rotates CW w/ $t>0$ @ $z=0$ looking in $-\hat{z}$ direction
=> Right hand

$|\mathcal{E}_x| = |\mathcal{E}_y|$
=> circular polarized

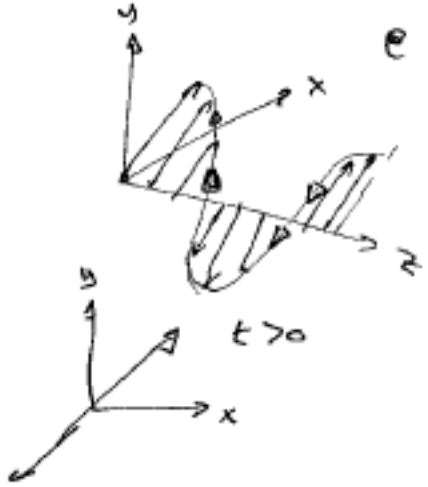
$\vec{J} = 4E_0\begin{bmatrix} i \\ 1 \end{bmatrix}$ or in normalized form $\vec{J} = \begin{bmatrix} 1 \\ -i \end{bmatrix}$

usually put phase in $\hat{y}$ component

3.) $\vec{E} = E_0\left(\hat{x}e^{i\pi} - \hat{y}\right)e^{-ikz}$

$\vec{E} = -E_0\left(\hat{x} + \hat{y}\right)e^{-ikz}$

This is linear polarized light w/ graphs nearly the same as 1).



you might want to write the $\vec{J}$ as $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ but that would mean it is the same wave as 1) (which it isn't).

Jones vectors assume you are propagating in $+\hat{z}$ direction w/ right-handed coordinate system

this means $-z \rightarrow +z'$ (to propagate in $+z$

$x \rightarrow -x'$ } (to maintain RH coordinate)

$y \rightarrow y'$ }

$\Rightarrow \vec{E} = E_0\left(-\hat{x}' + \hat{y}'\right)e^{ikz'}$ ➡ $\vec{J} = E_0\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ in normalized form $\vec{J} = \frac{E_0}{\sqrt{2}}\begin{bmatrix} -1 \\ 1 \end{bmatrix}$

4) $\vec{E} = (\hat{x} + 5\hat{y}e^{i\pi/2})e^{ikz}$

$\mathcal{E}_x = \cos(kz - \omega t)$

$\mathcal{E}_y = 5\cos(kz - \omega t + \pi/2)$

$= -5\sin(kz - \omega t)$

Rotates CCW w/ $t > 0$ @ $z = 0$

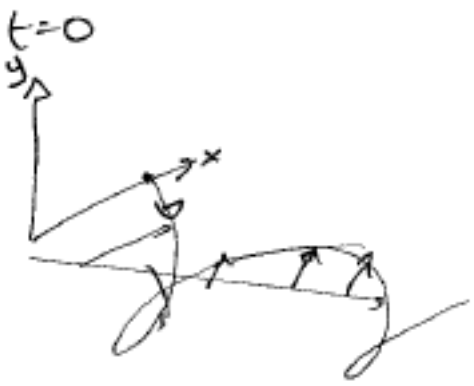
$\rightarrow$ LH

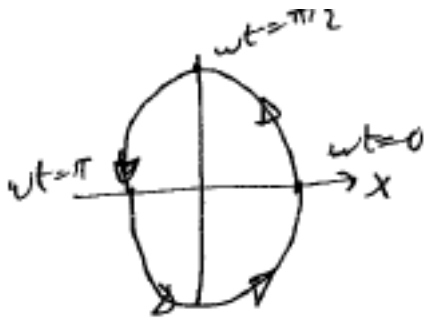
$|\mathcal{E}_x| \neq |\mathcal{E}_y|$

$\Rightarrow$ elliptical, left handed

$\vec{J} = \begin{bmatrix} 1 \\ 5i \end{bmatrix}$

a





CCW: LH

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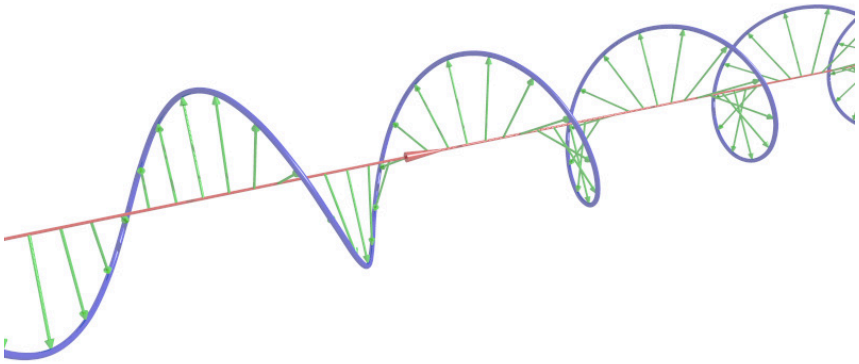
Why this is our handedness convention….(and more to the point how to figure polarization without simple memorization)…

$$\vec{E} = (\hat{x} + 5\hat{y}e^{i\pi/2})e^{ikz}$$

$$\vec{\mathcal{E}} = \hat{x}E_o cos(kz - \omega t) + 5\hat{y}E_o cos(kz - \omega t + \pi/2)$$

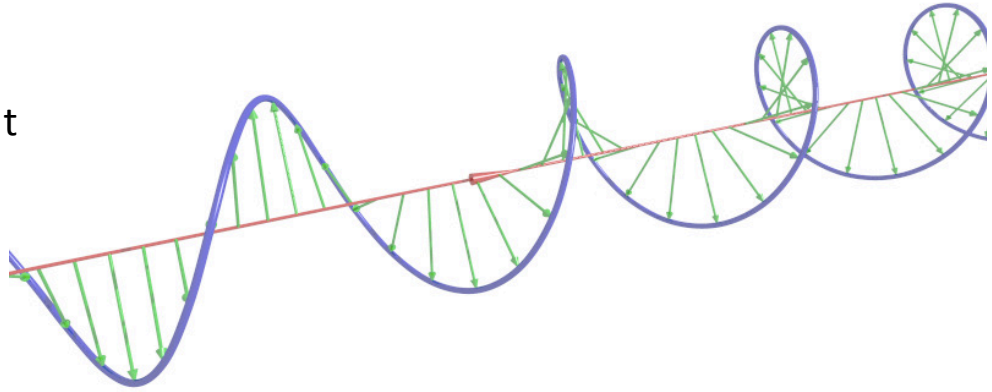
$$\vec{\mathcal{E}} = \hat{x}E_o cos(kz - \omega t) - 5\hat{y}E_o sin(kz - \omega t)$$

-look at fixed z, increment t



-look at fixed t , increment z

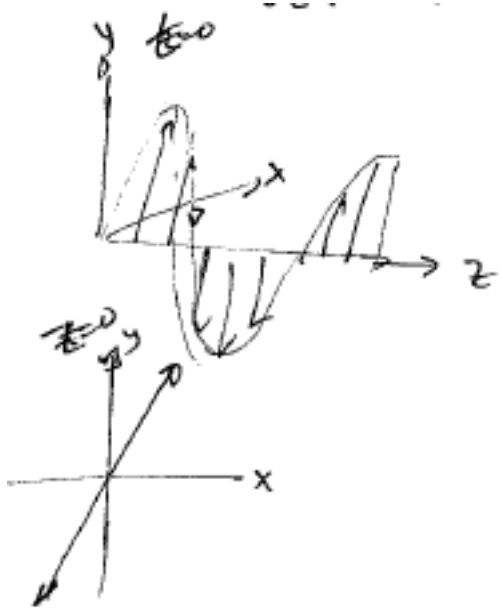
t



5) $\vec{J} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

$\vec{E} = (\hat{x} + 3\hat{y})e^{ikz}$

linearly polarized @ an angle $\theta = \tan^{-1}\left(\frac{3}{1}\right)$ wrt to $\hat{x}$ axis



Below are various statements regarding our representation of a complex amplitude of an electric field with a Jones vector. Which of the statements below is incorrect (not true)?

$$\vec{E} = e^{i\phi_x}\left[A_x\hat{x} + A_y e^{i(\phi_y-\phi_x)}\right]e^{ikz} \quad \Rightarrow \quad \vec{J} = \begin{bmatrix} A_x \\ A_y e^{i(\phi_y-\phi_x)} \end{bmatrix}$$

(A) You have assumed a plane wave.

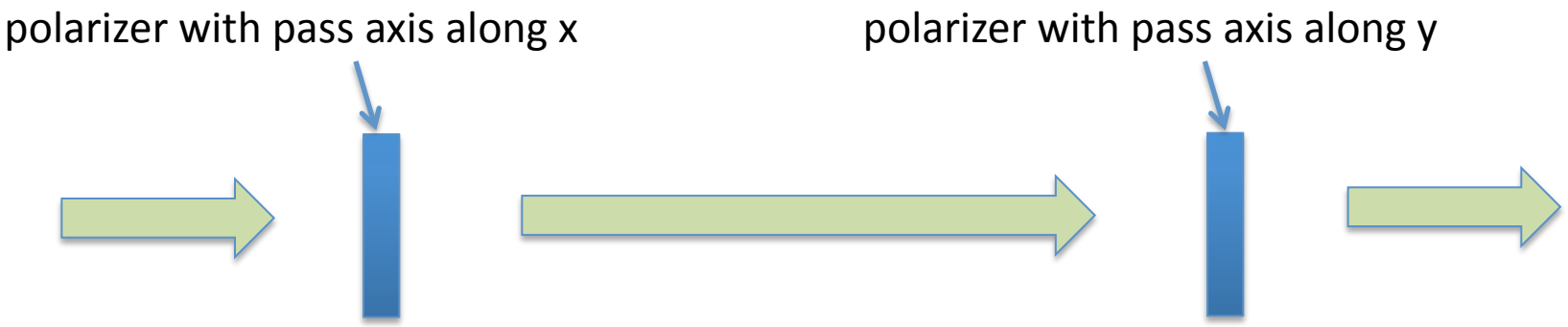
(B) You have assumed a direction of propagation

(C) You have assumed a right-handed coordinate system

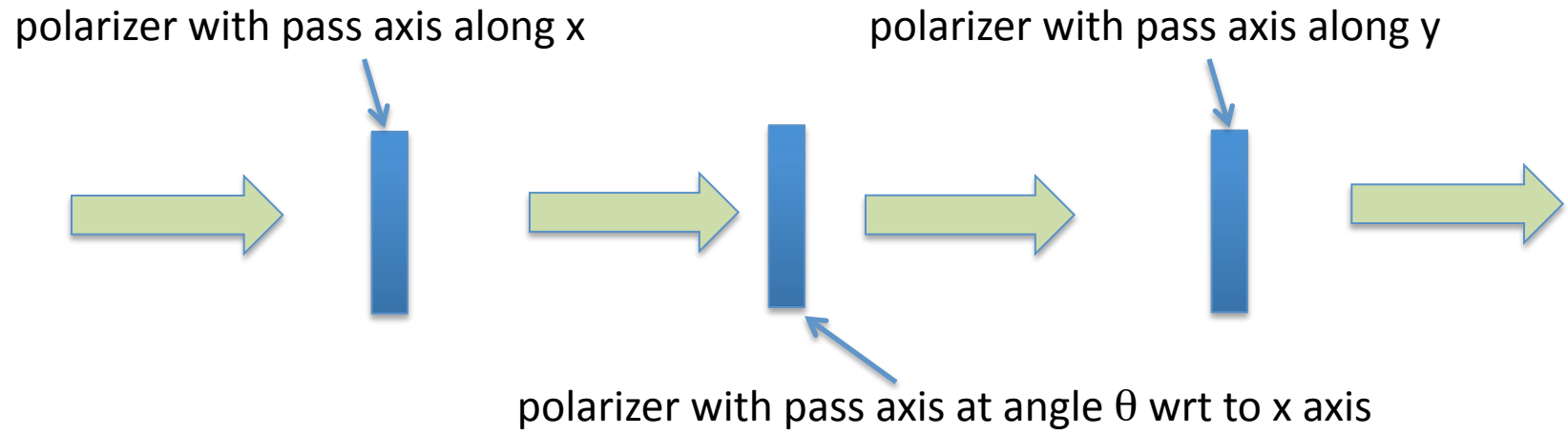
(D) You cannot calculated the intensity of the field with it

(E) More than one statement is not true

1- Use Jones vectors and polarization device matrices to show output intensity is zero:



2- Now insert a third polarizer in between the first two….use Jones vectors and polarization device matrices to show output intensity is now NOT zero:



1) y polarizer, x polarizer, arbitrary input

$$= \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} A \\ Be^{i\phi} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} A \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\Rightarrow$ nothing is transmitted

2) polarizer @ angle θ wrt x axis

$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \bar{\bar{R}}(-\theta) \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \bar{\bar{R}}(\theta) \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} A \\ Be^{i\phi} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix} \begin{bmatrix} A \\ 0 \end{bmatrix}$$

$$R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} A\cos^2\theta \\ A\sin\theta\cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ A\sin\theta\cos\theta \end{bmatrix}$$