27-6 LASER PROPAGATION THROUGH ARBITRARY OPTICAL SYSTEMS

With the basic law of propagation for the complex radius of curvature $q(z)$ in Eq. (27-14) and the defining equations for the real radius of curvature $R(z)$ and beam width $w(z)$ in Eqs. (27-19) and (27-22), we are able to characterize the beam parameters for laser propagation in any homogeneous medium of refractive index n. We now wish to address the question of how the beam changes when it is modified by an arbitrary optical system, one that contains lenses, mirrors, prisms, and so forth.

It is helpful to note the similarity between the behavior of ordinary spherical waves encountered in geometrical optics and Gaussian spherical waves encountered here. In Figure 27-8, the basic law of propagation for each type of wave and the effect of a lens on reshaping the propagating wavefront is illustrated. One is struck with the correspondence between $R(z)$ for ordinary spherical waves and $q(z)$ for Gaussian spherical waves in the defining equations. For example, the law of propagation of ordinary spherical waves, along the z-axis, is given by

$$R_2 = R_1 + (z_2 - z_1) \tag{27-35}$$

Similarly, the basic law of propagation for the laser beam follows from Eq. (27-14) and is given by

$$q_2 = q_1 + (z_2 - z_1) \tag{27-36}$$

an equation identical to Eq. (27-35) except that R has been replaced by q.

Seizing the initiative, we claim that this correspondence[4] can be extended to other basic laws, for example, the simple lens law. For ordinary spherical waves, a common form of the law is

$$\frac{1}{R_2} = \frac{1}{R_1} - \frac{1}{f} \tag{27-37}$$

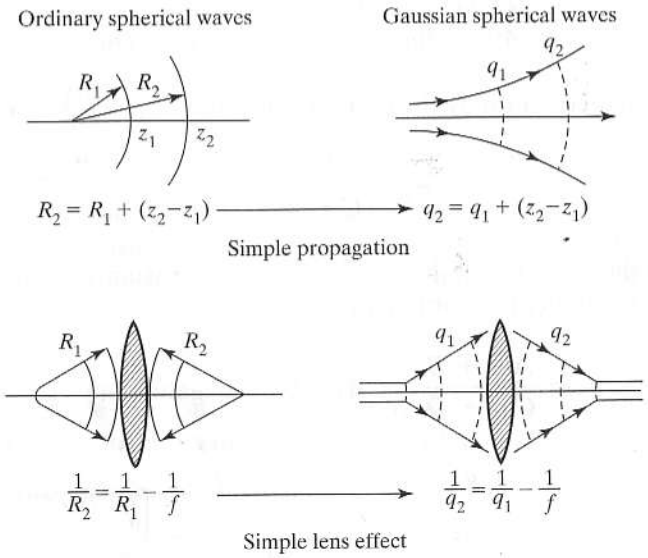


Figure 27-8 Correspondence between ordinary spherical waves and Gaussian spherical waves. If one knows the law for ordinary spherical waves, one can infer a similar law for Gaussian spherical waves by simply replacing $R(z)$ with $q(z)$.

[4]The correspondence we refer to here can be shown to be rigorous, not just analogous.

where R_1 and R_2 are the radii of curvature at incidence and refraction, respectively, and f is the focal length (see Figure 27-8). Replacing R by q for the laser beam, we obtain at once

$$\frac{1}{q_2} = \frac{1}{q_1} - \frac{1}{f} \tag{27-38}$$

This relationship predicts accurately the reshaping of the incident laser beam after refraction by a simple thin lens.

ABCD Law

With the correspondence between $R(z)$ and $q(z)$ established, we can, with the help of the matrix methods outlined in Chapter 18, develop a simple, yet powerful, recipe for laser-beam propagation through an arbitrary optical system. Consider Figure 27-9. Recall from Chapter 18 that, at a given plane, a ray can be described by its height y and slope angle α relative to the optical axis.

Consider a ray with parameters (y_1, α_1) incident on the entrance plane of an arbitrary optical system described by the overall system matrix

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

Upon emerging from the system, the same ray has parameters (y_2, α_2). The radius of curvature R of an ordinary spherical wave can be related to its appropriate paraxial ray parameters y and α by

$$R = \frac{y}{\alpha} \tag{27-39}$$

We know from matrix optics that when ray 1 is changed into ray 2 by an optical system, the change can be represented by the $ABCD$ system matrix as follows:

$$\begin{bmatrix} y_2 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} y_1 \\ \alpha_1 \end{bmatrix} \tag{27-40}$$

Then it must be true that

$$y_2 = Ay_1 + B\alpha_1 \quad \text{and} \quad \alpha_2 = Cy_1 + D\alpha_1 \tag{27-41}$$

Dividing the first equation by the second, and using Eq. (27-39), we obtain

$$R_2 = \frac{AR_1 + B}{CR_1 + D} \tag{27-42}$$

Generalizing the basic result in Eq. (27-42) to a Gaussian spherical wave, by replacing $R(z)$ with $q(z)$, we obtain at once

$$q_2 = \frac{Aq_1 + B}{Cq_1 + D} \qquad ABCD \text{ propagation law} \tag{27-43}$$

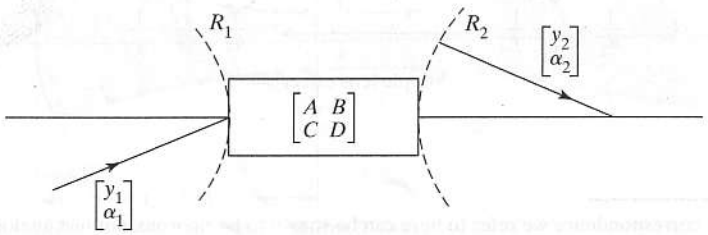


Figure 27-9 Propagation of ordinary spherical waves through an arbitrary optical system via matrix formulation.