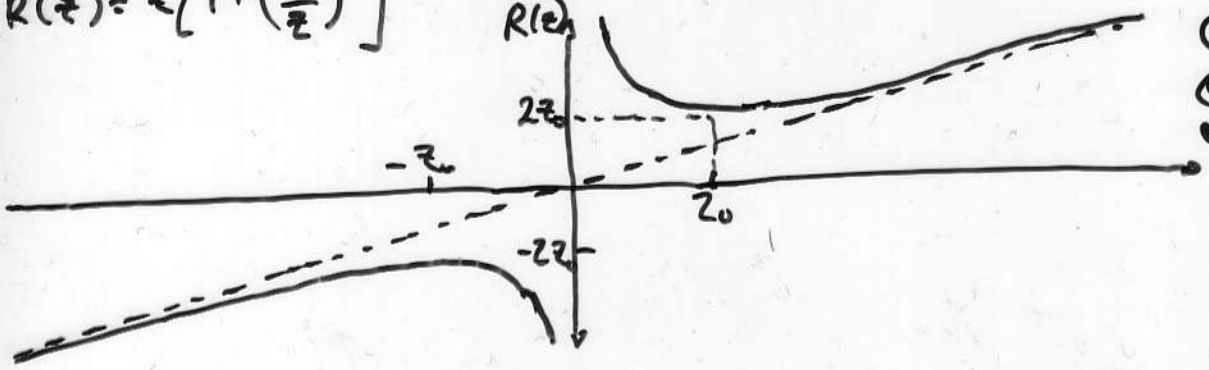
Gaussian Beam: $u(r) = A \frac{w_0}{w(z)} e^{-\rho^2/w^2(z)} e^{\pm ik\rho^2/2R(z)} e^{ikz} e^{-i\zeta(z)}$

had incorrect sign in previous notes, should be +

Phase terms: $\underbrace{kz - \zeta(z)}_{\text{longitudinal phase}} + \underbrace{\frac{k\rho^2}{2R(z)}}_{\text{transverse radial phase}}$

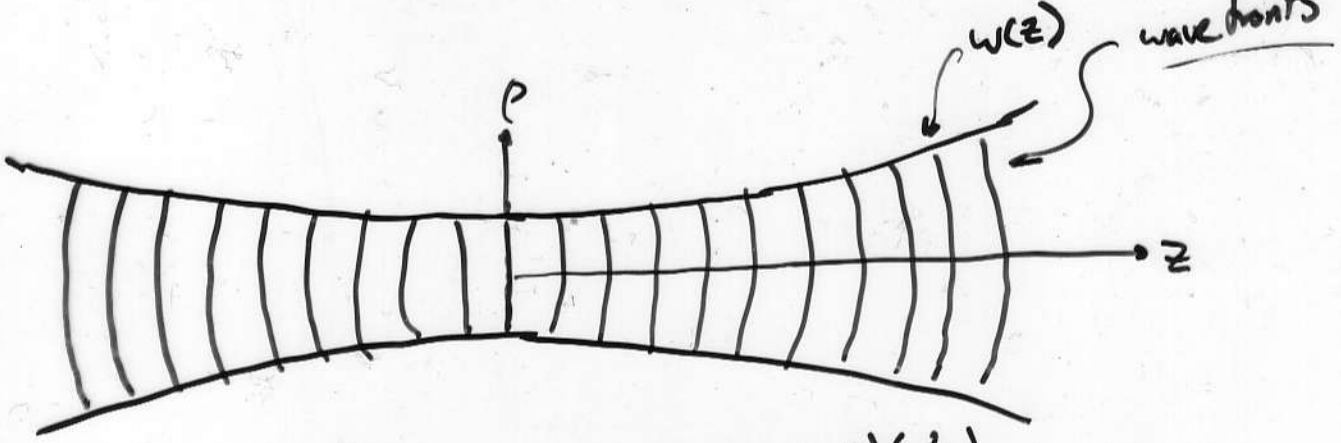
$R(z) = z\left[1 + \left(\frac{z_0}{z}\right)^2\right]$



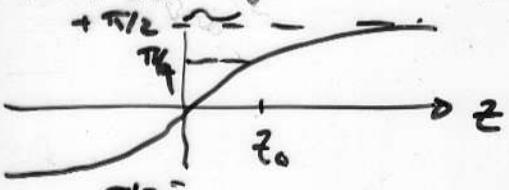
$\rho z = 0 \quad R = \infty$

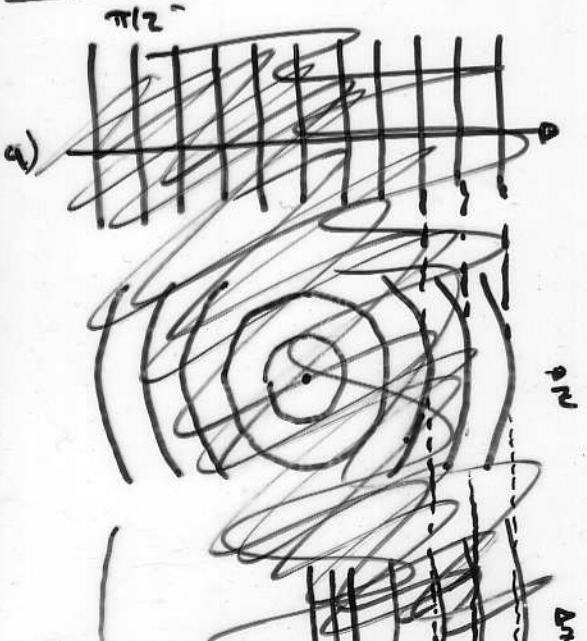
$\rho z = z_0 \quad R = 2z_0$

$\rho z = \infty \quad R = \infty$



What about $\zeta(z)$? $\zeta(z) = +\tan^{-1}\left(\frac{z}{z_0}\right)$





plane wave

$\sim e^{ikz}$

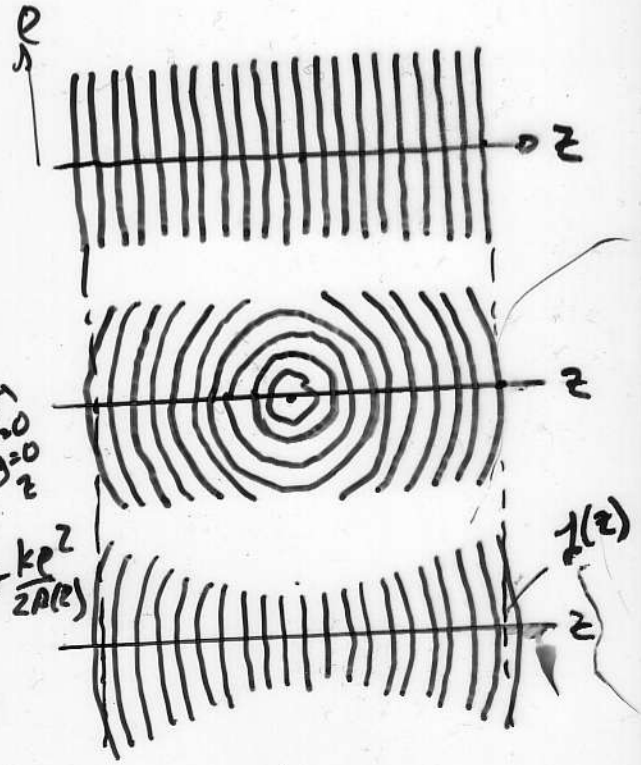
spherical

$\sim e^{ikr}$

$\sim e^{ikz}$ $r = x = 0$, $y = 0$, z

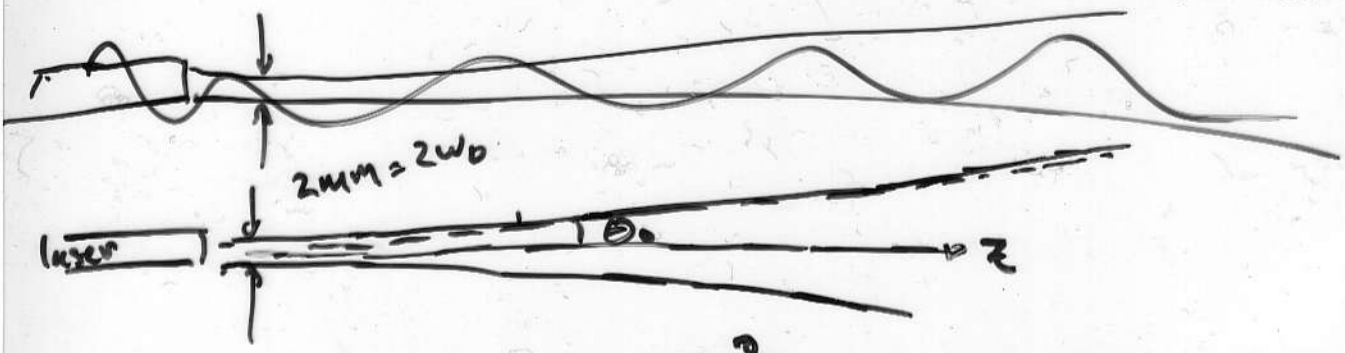
gaussian

$\sim e^{ikz - \zeta(z) + \frac{k\rho^2}{2R(z)}}$



Let's look @ some typical numbers for a beam emitted by a green laser pointer

Nd:YAG laser $\lambda = 532\ nm$, $P = 2\ mW$, $2\omega_0 = 2\ mm$ $\Rightarrow$ have all info to calculate the behavior of the beam as it propagates



a) $z_0 = \omega_0^2 \frac{\pi}{\lambda} = \frac{(10^{-3})^2 \pi}{532 \times 10^{-9}} \simeq \frac{\pi}{5} \times 10^{+3} \simeq 2 \times 10^2$

$= \frac{10^{6} \pi}{0.5 \times 10^{6}} = \frac{\pi}{\frac{1}{2}} \simeq 2\pi = \boxed{6\ m}$

b) $\Theta = \frac{\omega_0}{z_0}$

$= \frac{\lambda}{\pi \omega_0} = \frac{2\lambda}{2\omega_0 \pi} = \frac{1 \times 10^{-6}}{2 \times 10^{-3} \pi} = \frac{10^{-3}}{2\pi} \sim .15\ mrad$

c) beam diameter @ 18 m?

$2\omega(z) = 2\omega_0 \left[1 + \left(\frac{z}{z_0}\right)^2\right]^{1/2} \Big|_{z=18}$

$= 2 \times 10^{-3} \left[1 + \left(\frac{18}{6}\right)^2\right]^{1/2}$

$= 2 \times 10^{-3} \left[1 + 9\right]^{1/2}$

$2\omega_2 = 2 \times 10^{-3} [3.3] = 6.6\ mm$

d) beam diameter on moon? $z = 3.5 \times 10^8\ m$

$2\omega(z) = 2\omega_0 \left[1 + \left(\frac{z}{z_0}\right)^2\right]^{1/2} \simeq 2\omega_0 \left[\frac{z}{z_0}\right]$ for $z \gg z_0$

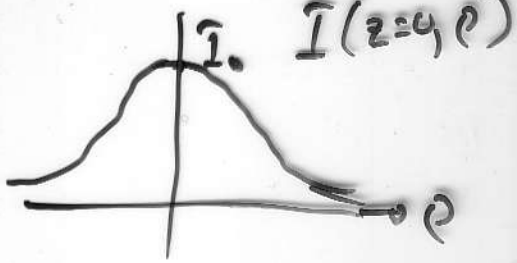
$= 2 \times 10^{-3} \left[\frac{3.5 \times 10^8}{6}\right] \simeq 10^5\ m$

$= 100\ km$

What is the optical intensity (on axis?) $\rho = 0$ $\quad I\left[\frac{W}{m^2}\right]$

$$I = I_0 \left[\frac{w_0}{w(z)}\right]^2 e^{-2\rho^2/w^2(z)}$$

$$I(\rho=0, z) \sim I_0 \left[\frac{w_0}{w(z)}\right]^2$$



... but what about I_0?

$$P = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy\, I_0 \left[\frac{w_0}{w(z)}\right]^2 e^{-2\rho^2/w^2(z)}$$

$$= \int_0^{\infty} d\rho\, 2\pi\rho\, I_0 \left[\frac{w_0}{w(z)}\right]^2 e^{-2\rho^2/w^2(z)}$$

$$P = \frac{1}{2} I_0 \pi w_0^2 \longleftarrow \text{independent of } z!$$

$$\Rightarrow I_0 = \frac{2P}{\pi w_0^2} = \frac{2 \cdot 2\times 10^{-3}}{\pi (10^{-3})^2} = \frac{4}{\pi} 10^3 \left[\frac{W}{m^2}\right]$$

$$\approx 1.3 \text{ kW/m}^2$$

So $\quad I\left[\rho = 0, z = 0\right] = I_0 \left[\frac{w_0}{w(z)}\right]^2$

$$\approx 1.3 \text{ kW/m}^2$$

$$w(z) = w_0\left[1 + \left(\frac{z}{z_0}\right)^2\right]$$

$$I(\rho=0, z=z_0) = I_0 \left[\frac{w_0}{w(z)}\right]^2$$

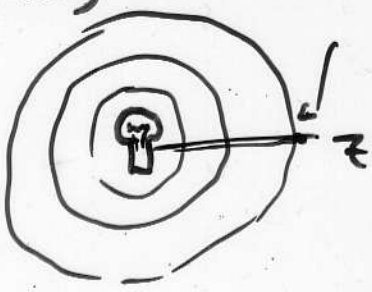
$$= I_0 \left[\frac{1}{2}\right]$$

$$\approx 0.65 \text{ kW/m}^2$$

$$z = z_0 \doteq 6\text{m}$$

lets compare this intensity w/ a light ~~bulb~~ bulb.

(100 W bulb)



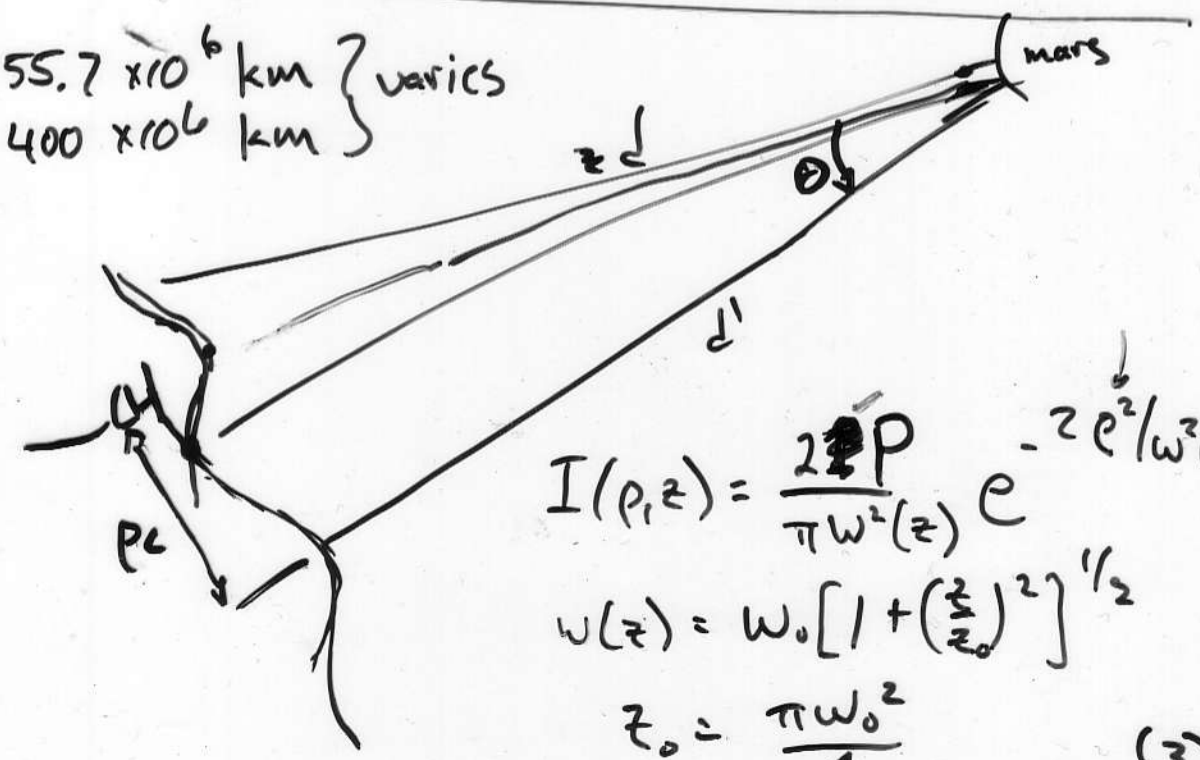
$$I = \frac{P}{4\pi r^2}$$

$$I(x, y=0, z=z_0=6) = \frac{100}{4\pi 36} = 0.2 \text{ W/m}^2$$

so even the laser has a total power of 2mW (a factor of appx 5×10^{-4} smaller compared to the light ~~bulb~~ bulb), it's intensity is 3×10^3 higher @ $z = z_0 = 6$m

---

$d = 55.7 \times 10^6$ km } varies
400×10^6 km }



$$I(\rho, z) = \frac{2P}{\pi w^2(z)} e^{-2\rho^2/w^2(z)} \quad (1)$$

$$w(z) = w_0\left[1 + \left(\frac{z}{z_0}\right)^2\right]^{1/2} \quad (2)$$

$$z_0 = \frac{\pi w_0^2}{\lambda} \quad (3)$$

plug in (3) & (2) into (1)

& set equal to 0.1 pW/m² to solve for ρ

$\Rightarrow \rho_c$ (radius of) minimum intensity for detector

$d\theta = \rho_c$

for minimal approach,

$$\theta_c = \left[\frac{d}{2c}\right]^{-1} = 4.5\ \mu\text{rad}$$

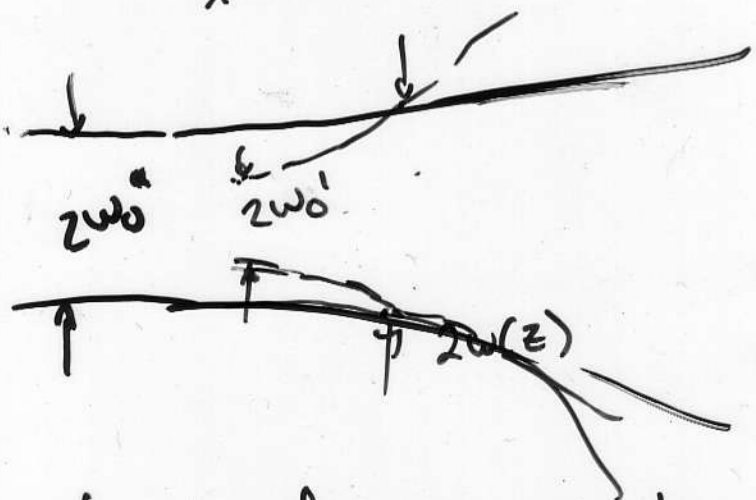
for maximum distance

$$\theta_c = \left[\frac{d}{2c}\right]^{-1} = 3.3\ \mu\text{rad}$$

From these two examples, that there are only two free parameters of a gaussian beam (for a given λ)

- Amplitude (I_0, A, P)
- minimum waist (w_0, z_0)

if instead of being given z_0 (or w_0) we are given $w(z)$ is that even to uniquely specify the gaussian beam

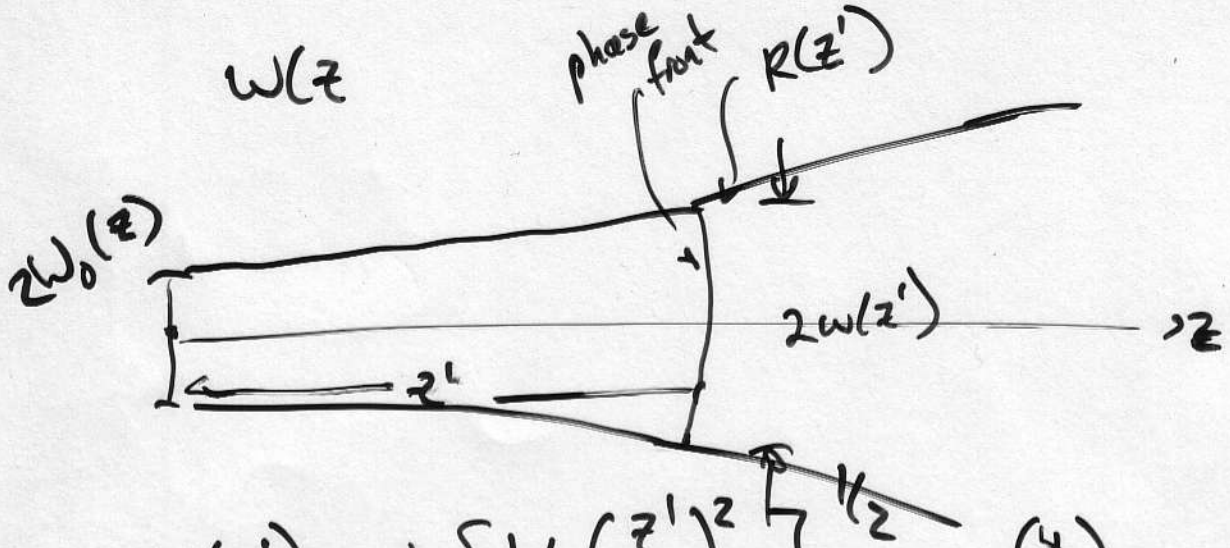


w_0

could represent many different beam each w/ a different w_0 & location of w_0

so to uniquely specify a particular gaussian beam we need one piece of info.

if we know both $w(z')$ & $R(z')$



$$w(z') = w_0\left[1 + \left(\frac{z'}{z_0}\right)^2\right]^{1/2} \quad (4)$$

$$R(z') = z'\left[1 + \left(\frac{z_0}{z'}\right)\right] \quad (5)$$

we have two eqns & two ~~unknowns~~ unknowns

z_0 & z'

take (4) & (5) & invert Then

$$z' = \frac{R(z')}{1 + \left[\frac{\lambda R(z')}{\pi w(z')^2}\right]^2}$$

$$w_0 = \frac{w(z')}{\left[1 + \left[\frac{\pi w(z')^2}{\lambda R(z')}\right]^2\right]^{1/2}}$$