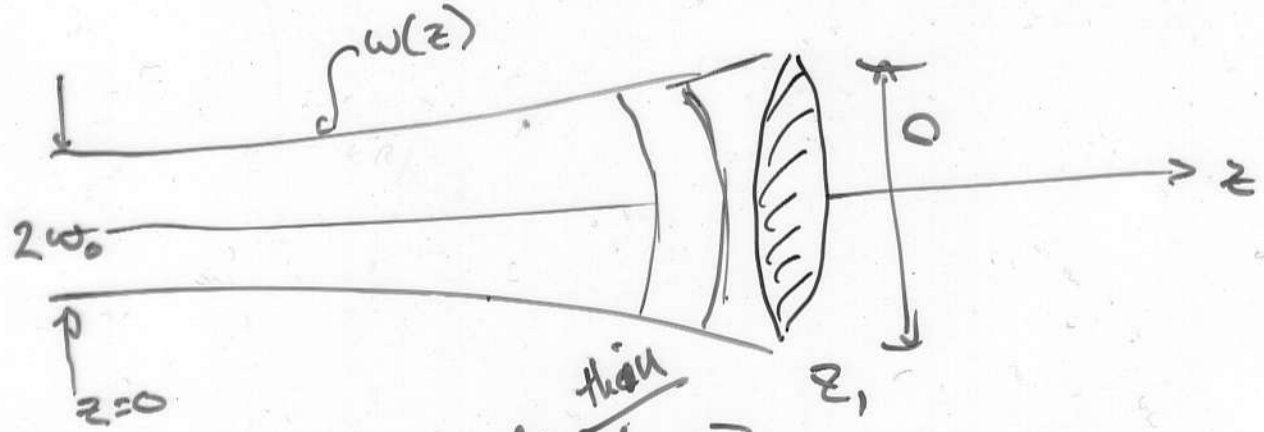
* Midterm in class, 50 min, closed book, Feb 28
I'll post formula sheet beforehand.

* For thursday lecture that if you need Fourier review Read Steck online text 3 chpters ~~3 §1~~
3 §1,2
11 §1,2,3

Gaussian Beams through optical components



what happens to beam after lens?

set $z=0$ @ w_0

(c) $u(r) = A_0 \frac{w_0}{w(z)} e^{-\rho^2/w^2(z)} e^{ik\rho^2/2R(z)} e^{ikz} e^{-i\zeta(z)}$

$$w(z) = w_0\left[1+\left(\frac{z}{z_0}\right)^2\right]^{1/2} ; \quad z_0 = \frac{w_0^2\pi}{\lambda}$$

$$R(z) = z\left[1+\left(\frac{z_0}{z}\right)^2\right]$$

~~test~~ lens $z = z_1$ ...

amplitude right before lens $z = z_1^-$

$$|u(r)| = A_0 \frac{w_0}{w(z_1^-)} e^{-\rho^2/w^2(z_1^-)}$$

what about amplitude right after lens? $z = z_1^+$

if lens is thin (thickness $\ll$ D)

$w(z_1^-) = w(z_1^+)$ ← waist is unchanged as beam goes through lens

so amplitude of beam is unchanged

... what about phase? → as long as lens thin ; paraxial waves

$$|u(r)|_{z=z_1^+} = |t(x,y)\, u(r)|_{z=z_1^-} \quad ; \quad t(x,y) = e^{ink_0 d} e^{-ik_0\left[\frac{\rho^2}{2f}\right]}$$

so indeed the lens does change the phase... so after lens phase

$$\phi|_{z=z_1^+} = \underbrace{kz_1^+ - \phi(z_1^+) + k\frac{\rho^2}{2R(z_1^+)}}_{} + nk_0 d - k\frac{\rho^2}{2f}$$

$$= \underbrace{+ \frac{k\rho^2}{2}\left[\frac{1}{R(z_1)} - \frac{1}{f}\right]}_{\text{new radial phase of beam after it exits lens}} + \underbrace{n_0 kd}_{\text{some constant (uninteresting phase term)}}$$

In words

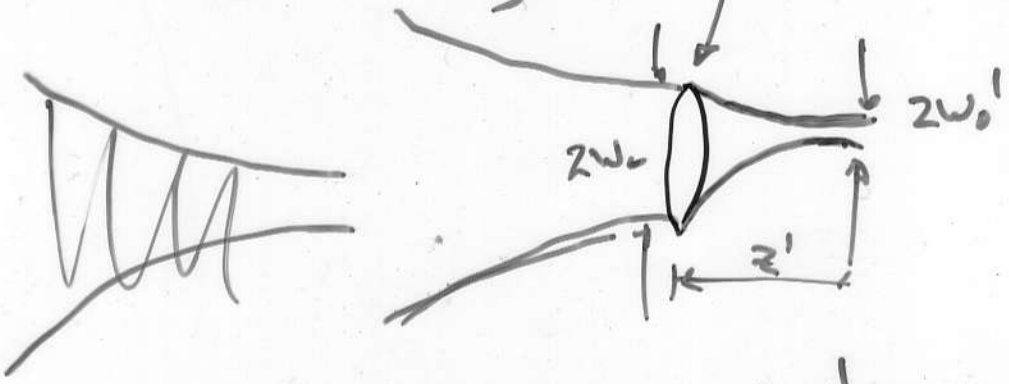
the effect of lens is to multiply the gaussian by radial phase shift

lets look @ some limiting cases

→ in many applications you want smallest spot size

looking @ eqn (1) for smallest w_0' produced w should be the smallest (no matter what lens)

⇒ so we should place lens @ minimum waist of incoming beam in (1) $z=0$



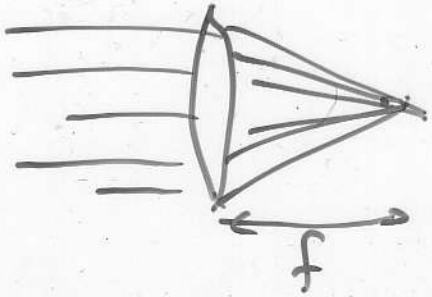
so if $z=0$, $w(z)=w_0$ & $\frac{1}{R}=0$

$$\Rightarrow \frac{1}{R'} = -\frac{1}{f}$$

so (1) & (2)

$$w_0' = \frac{w_0}{\left[1+\left(\frac{\pi w_0}{\lambda f}\right)^2\right]^{1/2}} \quad \& \quad z' = \frac{f}{1+\left[\frac{\lambda f}{\pi w_0^2}\right]^2}$$

⇒ so that means that gaussian beam isn't focussed @ $z'=f$ where as in ray optics

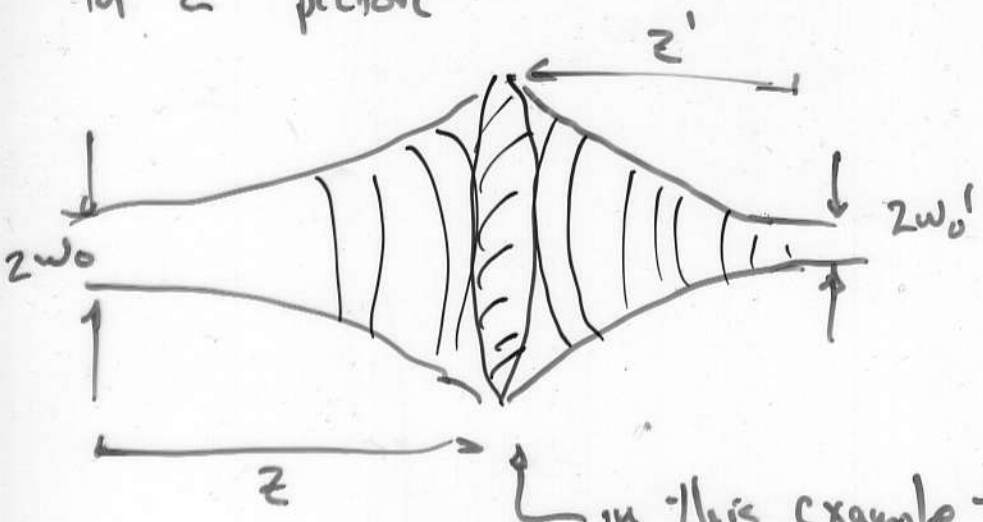


... how do we connect these two pictures

what is produced? ⇒ a new gaussian beam w/ same waist (@ lens) but a new radial phase

$$\frac{1}{R'} = \frac{1}{R} - \frac{1}{f}$$

In a picture



↳ in this example the sign & magnitude of f relative to R causes the new radial phase (R') to flip sign

so to find the new w_0' & z' (new minimum waist & its location)

we can use eqn from last

Thus lecture

$$w_0' = \frac{w(z)}{\left[1 + \left[\frac{\pi w(z)^2}{\lambda R(z)}\right]^2\right]^{1/2}} \quad (1)$$

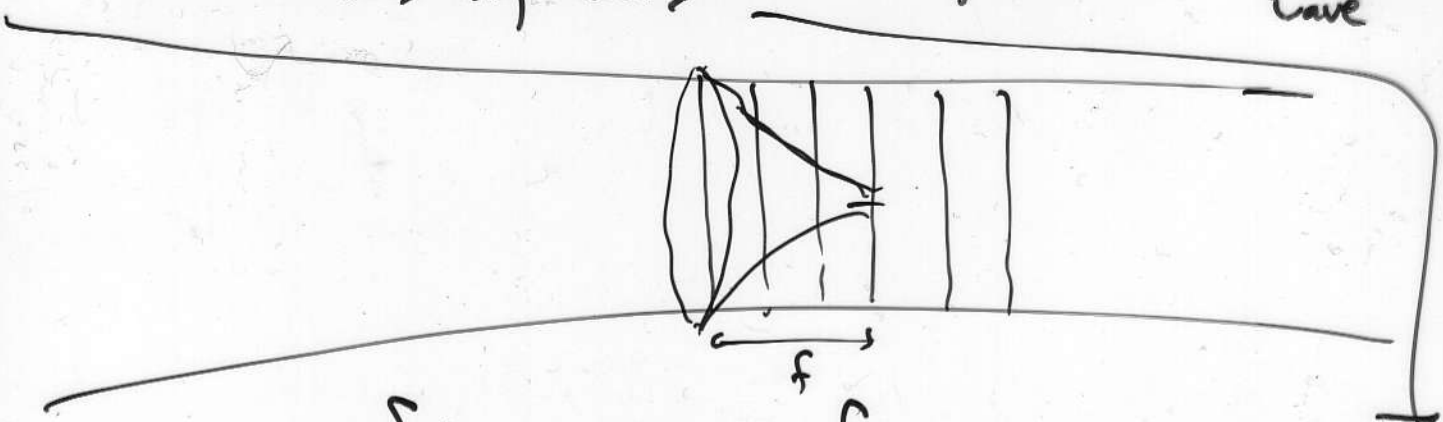
[for sign convention R(z) is specified to the left of w_0 location)

$$z' = \frac{R'}{1 + \left[\frac{\lambda R'}{\pi w^2}\right]^2} \quad \text{w/ } \frac{1}{R'} = \frac{1}{R} - \frac{1}{f} \quad (2)$$

to connect make z_0 big

what does big mean?

$z_0 \to \infty$ phase front of gaussian look like plane wave



$$z' = \frac{f}{1 + \left[\frac{\lambda f}{\pi w_0^2}\right]^2} = \frac{f}{1 + \left\{\frac{f}{z_0}\right\}^2}$$ so if $z_0 >> f$

$$\cong f$$ ← ray optics picture recovered

in this limit what is the produced spot size?

$$w_0' = \frac{w_0}{\left(1 + \left(\frac{z_0}{f}\right)^2\right)^{1/2}} \cong \frac{w_0 f}{z_0}$$

for $\frac{z_0}{f} >> 1$

$$\therefore w_0' = \frac{\lambda}{\pi w_0} f$$

smallest spot $f \downarrow \lambda \downarrow$ $w_0 \uparrow$

for real life applications w/o much trouble you can get a spot size realistically of $w_0' \cdot \frac{4}{3}$

so we have a gaussian beam changes through a lens, propagates a distance ... it seems to be another Rayleigh & book-keeping nightmare ... there's better way

$$u(r) = \frac{A}{z - iz_0} e^{ik \frac{\rho^2}{2(z-iz_0)}} e^{ikz}$$

} gaussian form before splitting up $u(r)$ into phase & amplitude terms

$q(z) = z - iz_0$ & relate $q(z)$

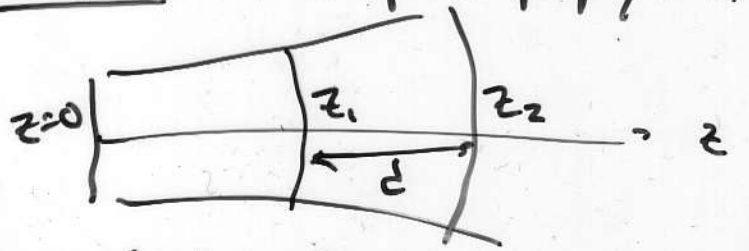
$$\frac{1}{q(z)} = \frac{1}{R(z)} - i \frac{\lambda}{\pi w^2(z)}$$

$$u(r) = \frac{A}{q(z)} e^{ik \frac{\rho^2}{2q(z)}} e^{ikz}$$

<u>key</u> - $q(z)$ completely defines the gaussian beam through (z_0)

- so if we can track changes of $q(z)$ then we know how the beam changes ... so how to track $q(z)$? §3.2D on S&T
§6.5 of Steck online text

<u>1st case</u> free space propagation



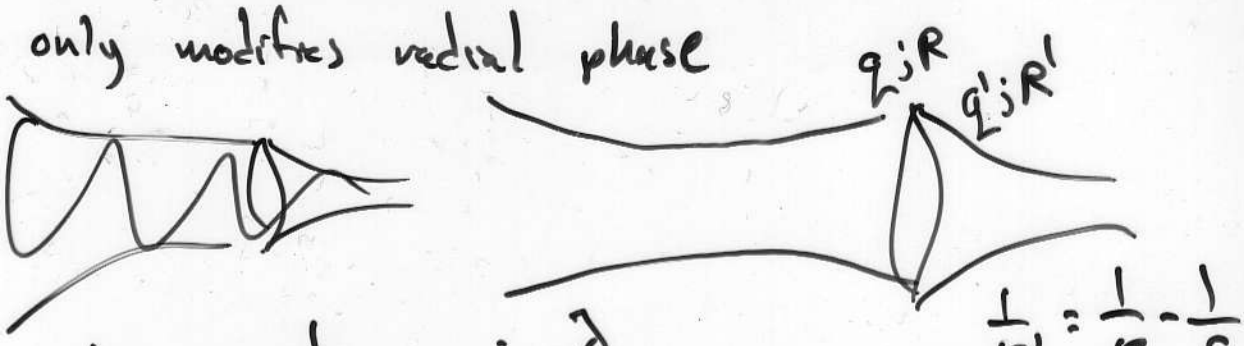
@ z_1 $q(z_1) = z_1 - iz_0$

@ z_2 $q(z_2) = z_2 - iz_0$

so $q(z_1 + d) = z_1 + d - iz_0$
$= q(z_2)$

case two lens

from earlier in lecture we saw that lens only modifies radial phase



$$\frac{1}{R'} = \frac{1}{R} - \frac{1}{f}$$

so $\frac{1}{q(z)} = \frac{1}{R(z)} - i\frac{\lambda}{\pi w^2(z)}$ ← before lens

$\frac{1}{q'(z)} = \frac{1}{R(z)} - \frac{1}{f} - i\frac{\lambda}{\pi w^2(z)}$ after lens

$$\boxed{\frac{1}{q'(z)} = \frac{1}{q(z)} - \frac{1}{f}}$$ for lens

In general, we can represent the transformation of the q parameter through the following relation

(3) $$q_2 = \frac{Aq_1 + B}{Cq_1 + D}$$ where $A, B, C, \& D$ are constants

$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$

some examples free space

$A=1 \quad B=d$

$C=0 \quad d=1$

from (2) $q_2 = q_1 + d$ ✓ as before

lens

$A=1 \quad B=0$

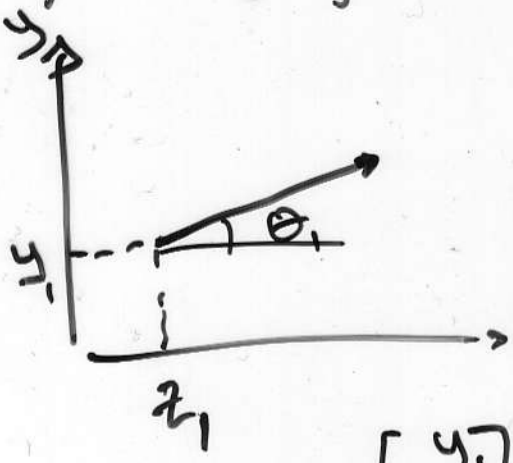
$C=-\frac{1}{f} \quad D=1$

(2) gives $q_2 = \frac{q_1}{-\frac{q_1}{f}+1}$

$\Rightarrow \frac{1}{q_2} = -\frac{1}{f} + \frac{1}{q_1}$ ✓

... okay but what are A, B, C, D physically?

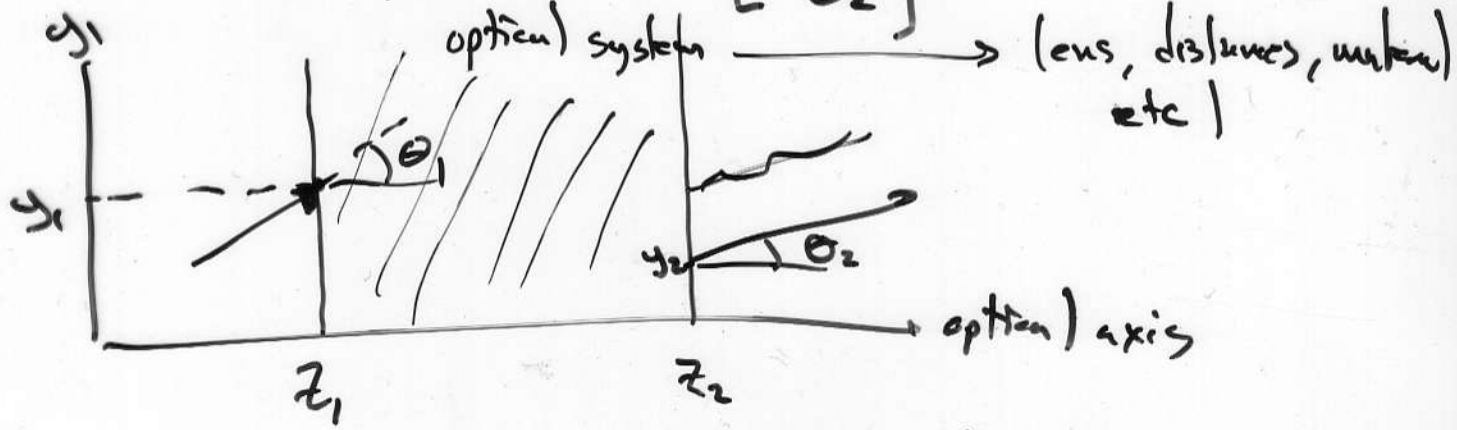
lets take a lovely step back in time when light as rays... life was ~~simple~~ simple



paraxial rays: we can keep track of rays by a vector direction (θ); position (y)

$$\begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix}$$ defines the ray

at some later point z_2 $\begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix}$



so when $\theta \simeq \sin\theta$ (paraxial ray)

general linear transform

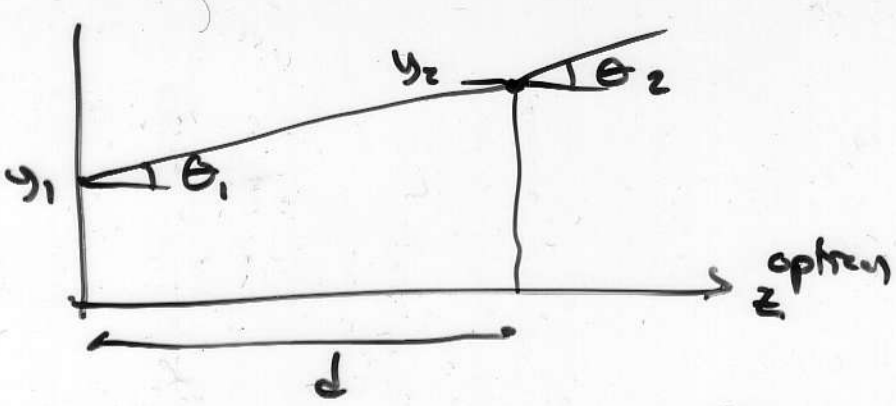
$$\begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix}$$

where ABCD characterise the optical system

$$y_2 = A y_1 + B\theta_1$$
$$\theta_2 = C y_1 + D\theta_1$$

ray matrices for paraxial rays

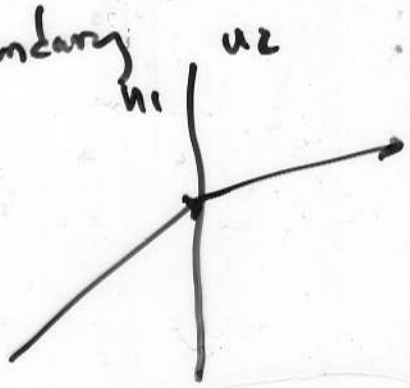
free space



$\theta_1 = \theta_2$ (no diffraction) $\Rightarrow$ $C=0$ $D=1$

$y_2 = y_1 + \theta_1 d \Rightarrow$ $A=1$ $B=d$ $\quad \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$

Refraction at a boundary



$y_1 = y_2 \Rightarrow A=1 \quad B=0$

$\theta_1 = \frac{n_2}{n_1}\theta_2 \Rightarrow C=0 \quad D=\frac{n_1}{n_2}$

$$\begin{bmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{bmatrix}$$

more [illegible] on page 26

the advantage of unitary

d_1 | f | d_2

$$\begin{bmatrix} y_{out} \\ \theta_{out} \end{bmatrix} = \overline{\overline{M}}_{d_2}\,\overline{\overline{M}}_{lens}\,\overline{\overline{M}}_{d_1} \begin{bmatrix} y_{in} \\ y_{out} \end{bmatrix}$$

so it turns that for tracking the q parameter of our gaussian beams we use these matricies in the follow eqn

$$q_2 = \frac{Aq_1 + B}{Cq_1 + D}$$

