We've got a ~~was~~ number of waves

1. plane wave
2. spherical wave
3. paraboidal wave
4. gaussian beam

} know how they propagate through

but it would nice to treat a arbitrary wave (which still ~~satisfies~~ satisfies $\left[\nabla^2 + k^2\right] u(r) = 0$

We can do such analysis by decomposes any old wave $u(r) = \sum_n A_n(r) e^{ik_n z}$

then track how each plane wave propagates thru system & add the resulting planes at the end we get the arbitrary wave out (works only for linear system)

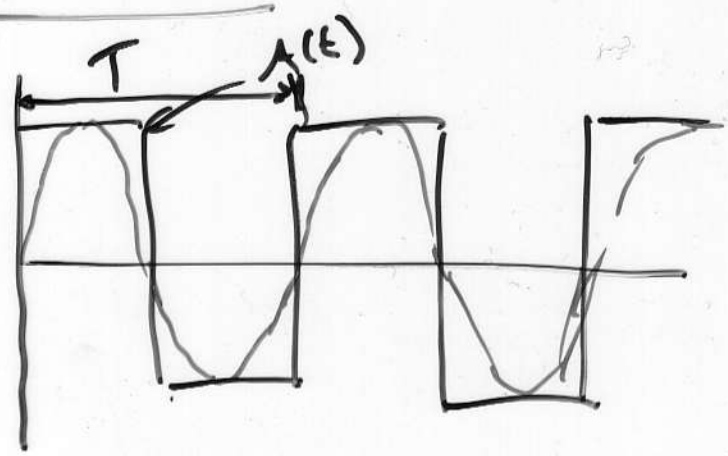


general way
$u(x,y,z)_{in}$

$u_{out}(x,y,z)$

First a (quick) Review of Fourier Series

Fourier Series



$$\omega = \frac{2\pi}{T}$$

A periodic function $A(t)$ can be represented by a Fourier Series

$$A(t) = \frac{1}{2} a_0 + 2\sum_{n=1}^{\infty} a_n \cos(n\omega t) + 2\sum_{n=1}^{\infty} b_n \sin(n\omega t)$$

$$A(t) = \sum_{n=-\infty}^{\infty} c_n e^{-in\omega t}$$

to get $c_n = \frac{1}{T}\int_0^T e^{in\omega t} A(t)\, dt$

So what happens when $A(t)$ is not periodic

we can still use harmonic functions $e^{i n \omega t}$ but now instead of discrete harmonics, the harmonic functions needed form a continuum

in this case the "~~period~~ period" of $A(t)$

$T \to \infty$ The spacing b/w frequency harmonics

$$\omega_{n+1} - \omega_n = (n+1)\omega - n\omega$$

$$= \Delta\omega \to 0 \text{ b/c } T \to \infty$$

$$A(t) = \sum_{n=-\infty}^{\infty} c_n e^{-in\omega t} \xrightarrow[T\to\infty]{} \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{A}(\omega) e^{-i\omega t} d\omega \qquad T = \frac{2\pi}{\Delta\omega}$$

where $\frac{\tilde{A}(\omega)}{2\pi}$ is the amplitude (& phase) of the frequency component @ ω that is needed to make $A(t)$

$\tilde{A}(\omega)$ is calculated the same way as c_n

$$c_n = \frac{1}{T} \int_0^T e^{in\omega t} A(t) dt \to \tilde{A}(\omega) = \int_{-\infty}^{\infty} A(t) e^{i\omega t} dt$$

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{f}(\omega) e^{-i\omega t}\, d\omega = \mathcal{F}^{-1}\left[\tilde{f}(\omega)\right]$$ Inverse Fourier Transform

$$\tilde{f}(\omega) = \int_{-\infty}^{\infty} f(t) e^{i\omega t}\, dt \equiv \mathcal{F}\left[f(t)\right]$$ Fourier Transform

so if you have a function $f(t)$ & you take the FT you will get the frequency content of that function $\tilde{f}(\omega)$

$t \longleftrightarrow \omega$ are the fourier conjugate variables

in the book $t \longleftrightarrow f$ where $2\pi f = \omega$

In Fourier Optics we don't work w/ time & frequency. Rather we work space & spatial frequency

$$x \longleftrightarrow k_x$$
$$y \longleftrightarrow k_y$$

in the book $x \longleftrightarrow \nu_x$ w/ $k_x = 2\pi \nu_x$

b/c engineers

$$f(x) = \frac{1}{2\pi}\int_{-\omega}^{\omega} \tilde{f}(k_x)\, e^{+ik_x x}\, dk_x \qquad \text{IFT}$$

$$\tilde{f}(k_x) = \int_{-\omega}^{\omega} f(x)\, e^{-ik_x x}\, dx \qquad \text{FT}$$

for fourier Optics, this is our FT pair

Dirac Delta function

$$\int_{-\omega}^{\omega} \delta(x)\, dx = 1 \quad ; \quad \delta(x) \begin{cases} = 1 & x = 0 \\ = 0 & \text{else} \end{cases}$$

lets do an example $f(x) = \cos k_0 x$

$$\tilde{f}(k_x) = \int_{-\omega}^{\omega} \cos k_0 x\, e^{ik_x x}\, dx$$

$$= \int_{-\omega}^{\omega} \frac{1}{2}\left(e^{ik_0 x} + e^{-ik_0 x}\right) e^{ik_x x}\, dx$$

$$= \frac{1}{2}\int_{-\omega}^{\omega} e^{i(k_0 + k_x)x} + e^{+i(k_x - k_0)x}\, dx$$

$$= \frac{1}{2}\left[\, \delta(k_x + k_0) + \delta(k_x - k_0)\,\right]$$

using $\int f(k_x)\,\delta(k_x - k_0)\,dk = f(k_0)$ w/ $f(k_x) = e^{ik_x x}$

