Chapter 1

The Optical Cavity

1.1 Objectives

In this experiment the goal is to explore the spatial and temporal characteristics of the transverse and longitudinal modes a two-mirror optical resonator. This resonator is similar to the HeNe laser resonator (that you have or will investigate in another lab) except that there is no gain element in the cavity and it is therefore a "passive" resonator. The sections at the end of this chapter contain several useful notes and equations for this lab. Specific objectives are:

1. Explore the concepts of resonance, spatial mode matching, and resonator stability using the TEM_{00} transverse mode of a Helium-Neon laser and an external two-mirror optical cavity.

2. Couple higher-order Hermite-Gaussian modes into the cavity and determine the optical frequency spectrum associated with these different transverse modes of a spherical mirror cavity.

3. Compute from first principles the expected transmission function as a function of the length of a Fabry-Perot optical resonator and compare your predictions with the experimentally measured transmission function. Explore the frequency resolution of the Fabry-Perot optical resonator.

1.2 Introduction

One of the most common methods used in the characterization of laser light involves sending the light into an external optical cavity. Because laser light properties such as frequency components and transverse modes are dependent on the optical resonator that is part of the laser, we can think of the laser light as carrying information about the laser resonator. By sending the laser light into a separate optical cavity, the laser light can be analyzed as long as we know the characteristics of the external cavity. In a sense, by sending laser light into an external resonator, we are using one optical cavity to characterize another.

To understand better how this process works, in the first part of this lab you will send a beam of laser light into an external cavity that you will construct. Because external cavity length changes randomly due to mirror vibrations and

temperature fluctuations, the injected laser light will be resonantly coupled to different cavity modes (longitudinal and transverse) at different points in time. Remember that resonant coupling will only occur when at least one of the frequency components of the laser light matches at least one of the mode frequencies of the external cavity (which are changing in time due to mirror vibrations, etc.). Since resonant coupling of light into an external cavity also leads to enhanced transmission of the light through the external cavity, you will be able to see the effects of the time-dependent coupling by looking at the changes in the intensity profile of the beam as it exits the external cavity.

Suppose the laser light is purely monochromatic, and has a TEM_{00} beam profile. The ideal situation for transmitting the laser light through an external cavity would be to have the cavity length stabilized, with the frequency of the laser light resonant with the frequency of one of the modes of the cavity. Furthermore, the transverse mode of the light would ideally match a transverse mode of the cavity: if the laser light propagates in a TEM_{00} mode (by which we mean the fundamental transverse mode of the laser resonator), we would ideally want it to match the fundamental transverse mode (TEM_{00}) of the external cavity. Such mode matching does not occur automatically, since the fundamental external cavity mode will have its own set of Gaussian parameters of spatially varying beam width and radius of curvature, entirely independent of the parameters of the laser resonator. To mode match the laser TEM_{00} mode to that of the external cavity mode, lenses must usually be used to shape the incoming beam so that the parameters of the beam match those of the cavity. In other words, if the external cavity were actually a second laser, the beam exiting this second laser would overlap perfectly with the beam we're sending into that second cavity (see fig. 1.2 for a pictorial representation of this condition).

As usual, the experimental situation is more complex than the ideal case. If the laser beam is not perfectly aligned and mode-matched to the external cavity, the input beam will partially couple to many different transverse modes of the cavity (the mode of the input beam can be described as a superposition of external cavity eigenmodes- such as the Hermite-Gaussian modes), and the light that exits the cavity will resemble whatever cavity modes happen to be excited rather than maintaining the spatial profile of the input beam. Since we will not stabilize the length of the cavity, the coupling of the laser beam to the longitudinal and transverse cavity modes will change with time, and the light patterns that are observed exiting the cavity will fluctuate with time, revealing a time-dependent coupling of the laser beam to many different high-order Hermite-Gaussian modes.

1.3 Prelab

The external Fabry Perot cavity you will build for this lab consists of two mirrors shown in Fig. 1.1. M1 is planar mirror meaning the radius of curvature (ROC) is infinite and M2 is a curved mirror with $ROC = 30$ cm. As discussed in section 1.2, to efficiently couple the laser into the lowest order Gaussian mode of the cavity (the TEM_{00}), one must mode-match the laser beam to the cavity mode. In other words the Gaussian Beam mode of the laser must be matched (by choosing an appropriate lens) such when the beam is incident on the cavity, it "matches" the particular (and stable) Gaussian or eigen-mode of the cavity.

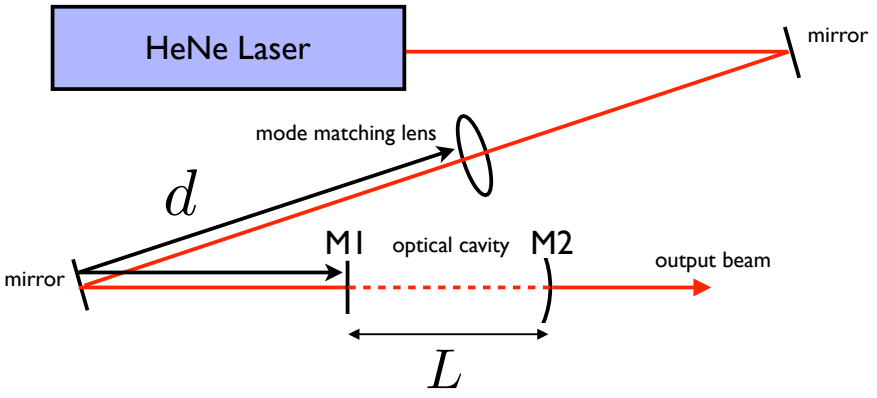


Figure 1.1: Cavity setup. The length of the cavity is the distance between M1 and M2 (L), and the distance from the mode matching lens to the input coupler of the cavity (M1) is denoted d. A photograph of the cavity can be seen in fig. 1.8.

Mode matching means both the beam diameter and the radial phase must be similar. For your pre-lab, you will determine the correct lens and its location to accomplish mode-matching for your particular laser to the external cavity. Toward this end for your pre-lab. you will need to perform three calculations prior to the lab:

- Determine the stability condition of the two mirror cavity shown Fig. 1.1. In other words, what mirror separations L produce a stable mode in the cavity?
- Produce a plot showing the size of beam waist of the cavity mode at **M1** as a function of L.
- Figure out the proper focal length of a lens and its position you need to use to transform the lasers beam to match that cavity mode at M1. See Fig. 1.2.

It will be easiest to use Matlab, Maple, or Mathematica for these calculations.

First, there is a range of mirror separation (L) when the cavity is stable. While a stability analysis is possible using ray optics, treatment of the problem in terms of gaussian beams is more appealing. For this calculation use the fact that in one round trip the $q(z)$ of the cavity mode (starting just after **M1**) must be self-consistent in one round trip and the $ABCD$ Law matrix formalism discussed in the text. In this calculation you will arrive at a quadratic expression $q(z)$. To determine what values of L are stable in the specific cavity at hand a physical solution means the q-parameter of the beam at **M1** must be purely imaginary (i.e. $q(z) = -iz_o$). Why? Use this fact to find a range of L.

Next, over this range of mirror separations (L) where the cavity is stable, at each L there will be a slightly different Gaussian mode in the cavity. Using your results from above, generate a plot of L vs beam diameter at **M1**. You

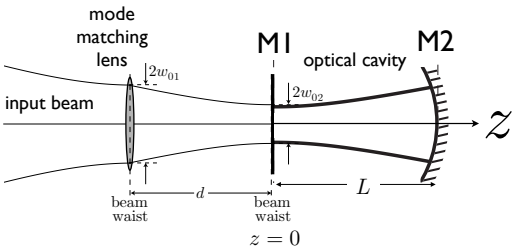


Figure 1.2: Coupling the input beam into the cavity. At $z = -d$ the input beam from the laser has a waist of w_{01}. The resonant mode of the cavity has a waist w_{02} at $z = 0$ (i.e. at **M1**). You need to determine the focal length of a lens and its position d such that the new beam waist of the input beam at z=0 is the same as w_{01}. As the input laser beam is somewhat collimated, you may use the following approximation: its depth of focus $z_{o1} >> d$.

have now found the the size of the Gaussian or eigen mode for the cavity at M1 as a function of L.

Finally, you need to figure out how to match the incoming laser beam to this eigen mode. Your hard-working professor has already measured the properties of the laser beam for you: you may assume it is a collimated (slowly diverging) beam with a diameter of 1.2 mm. Using the analysis from the text on Gaussian beam propagation through lens, determine the focal length of the lens and its distance d from **M1** that transforms the laser's Gaussian beam to the cavity's Gaussian Beam assuming the cavity length is 15 cm (this is in the middle of the cavity lengths you will explore in this lab). It will be easiest for you to mode-match by coupling into **M1** as opposed to **M2**. Think about why.

Since the lab does not have lenses of arbitrary focal length, but rather only a 60, 50, and 30 cm lens, choose the lens that most closely matches your ideal lens and compute the cavity length and the corresponding mode size at M1 at which the coupling is optimal.

1.4 Mirror reflectivity

In order to predict the expected finesse of your optical cavity, you will need to know the reflectivity of both mirrors and any losses inside the optical cavity. Since there's nothing but air in your optical cavity (and the loss is quite low) you only need to characterize the mirror reflectivity. To do this, you will need to measure the laser power reflected and transmitted by the mirrors. Be sure to capture all of the light (either reflected or transmitted) on the power meter and compare that to the incident optical power. Check that your reflectivity and transmission coefficients add to unity! Finally, be aware that the mirror reflectivity is angle dependent. You should therefore measure the reflectivity for the smallest angle possible to obtain the reflectivity at normal incidence to the beam as the mirrors will be inside the optical cavity. Please be careful not to bump or damage any components on the table while doing this measurement.

1.5 Input beam

From your pre-lab, you computed which lens you should use to optimally couple the light into the TEM_{00} mode of the optical cavity. Unfortunately, the lab probably doesn't have the exact lens you require. Therefore, choose from those available one that is close to your optimal lens. In this part, you will verify your prediction. Start by aligning the beam with the two turning mirrors after the HeNe laser to propagate down the center of the cavity mirror mounts AND to be level with respect to the optical table. Use the alignment card that is mounted since its hole is already at the optimal height for the laser beam. Next, place your chosen lens into the beam path (according to the figure). Be sure that the beam is centered on the lens. Measure the beam waist at the focus (i.e. where M1 will be placed) using a knife-edge measurement (see the writeup provided on the lab webpage regarding knife-edge measurements). You can locate the beam focus by looking at the beam on an index card. It's probably easiest to look at the beam spot generated by the light that propagates through the card to the back side since it's not as intense as the scattered light from the front of the card. Be sure not to change the power meter's "max-power setting" during your knife-edge measurements since the gain on the power meter is different at the different scales. To move the translation stage using the computer, open up the **APTUser** program on the desktop. Using the *Settings* button on the GUI make sure the max velocity is 0.5 mm/s for both the move and jog. Also, the jog setting of the micrometer stage needs to be ≤ 10 μm. The smaller the value, the more accurate your measurement will be.

Is the beam waist what you predicted in your prelab? If not, why are they different? Since diffraction at the knife edge can be quite significant, be careful to capture all of the light after the knife edge (razor blade) by placing the detector very close to the knife edge.

1.6 Optical cavity setup

Setup the resonator (a planar mirror for M1 and a 30 cm mirror for M2) with a short cavity length and couple the light in with M1 placed at the focus of the input beam. *From now on, don't worry about changing your input coupling lens even if your cavity length isn't optimal for coupling since you should get some light coupled into your cavity.* See the section on entitled "Notes on alignment" for additional tips on how to align your cavity.

Start by insuring your input beam is at the correct height to pass through the center of M1 and M2. Next, position the planar mirror (M1) at the position of the beam focus produced by the lens. The coated mirror surface should face into the cavity that you are building. There will be a fraction of light that is reflected from M1 back towards the laser, possibly resulting in multiple spots due to the beam bouncing back and forth between M1 and other optics (such as the laser's output coupler). Adjust M1 so that these spots overlap the forward-propagating beam. An index card with a hole in it is provided to help with this alignment.

How does this single mirror affect the spatial characteristics of the low-power beam that is transmitted through the mirror? Does the transmitted intensity fluctuate in time or is it relatively stable? How does this compare to

the situation where a second mirror is placed behind that first mirror and the two are aligned so that an optical cavity is formed. **In that case, does the cavity affect the spatial characteristics of the low-power beam that is transmitted? Does the intensity transmitted by the two mirrors of a cavity fluctuate in time or is it relatively stable?** Write down your expectations for the answer (a guess will suffice) and return to answer these questions fully after your cavity is aligned and you can observe the behavior. **Did you guess correctly the behaviors?**

Next, install the 30 cm curved mirror (M2) - again so that the coated mirror surface faces the inside of the cavity. Place the index card inside the cavity and position it so that the beam transmitting through M1 passes through the hole and hits M2. Adjust M2 so that the reflection passes back through the hole and onto M1. Insure this rough alignment is good even with the card held right at the face of M1. If the alignment is very close, you should see flickering of light on the mirrors as the cavity length drifts into and out of resonance. Remember that unless the cavity is confocal (this cavity is not), different transverse modes will in general be resonant at different frequencies or equivalently different cavity lengths, thus as the mirrors vibrate the narrow-frequency laser light couples to different transverse modes of the cavity. There are actually a few frequency components to the laser light, so the picture is even more complicated!

It may help to ramp slowly the cavity length with the PZT while you are aligning the cavity. Since the length of the cavity is constantly changing due to mirror vibrations, scanning the length with the piezo ensures that the cavity will at some time during the sweep be just the right length to be resonant with the input beam. When aligning, it also helps to look at the light transmitted through the cavity (i.e. after M2) and to adjust the end mirror (M2) so that the transmitted light is a single symmetric spot (and not a line of light).

Suppose your input beam is a TEM_{00} mode and it is perfectly mode matched to the TEM_{00} mode inside the cavity. Because the Hermite-Gaussian modes are *orthogonal*, you will *only* excite that one cavity mode and the transmitted beam will be identical to the input beam. Since your mode matching will not be perfect (the input beam parameters and/or alignment will be off slightly) you will couple to other transverse modes of the cavity. By adjusting the cavity alignment, position of the incident beam waist (by translating the coupling lens L1 or by translating the input mirror M1), and external cavity length L, see how close you can come the ideal situation. The transmitted TEM_{00} mode should be very bright when resonant, and other modes greatly suppressed.

1.7 Piezoelectric actuator

Fine adjustments of the length of the cavity are produced by applying a voltage to the piezoelectric actuator behind the mirror. The piezoelectric actuator can be seen in fig. 1.9. Voltage is supplied to the piezoelectric actuator ("piezo" for short) by a high voltage supply (see fig. 1.13). This supply has a manual adjustment knob and an external input. The external input is connected to a ramp generator. **Be aware** that the voltage of the ramp as seen on the scope is **different** from the actual output of the high voltage supply applied to the piezo! You will need to find out the gain on the input voltage which can be found in the user's manual (it should be the full output divided by the modulation

range). The distance the actuator moves given a certain voltage change can be found on the actuator datasheet; however, each actuator is slightly different and you need to characterize the response of your piezo. To do this, you will need to align the cavity with a *very short cavity length*, couple light into it, and measure the voltage swing (either manually or electronically) required to change the length of the cavity by exactly $\lambda/2$ (where λ is the wavelength of the light you are coupling into the cavity). Since the transmission function is periodic in the cavity length L with period $\lambda/2$, you only need look at the output (either with a CCD camera or the photodiode - see figs. 1.10 and 1.11) and watch over what change in voltage does the transmission repeat itself. This voltage variation will be the same independent of the cavity length, but a short cavity typically has a simpler transmission pattern with strong transmission for only the lowest order modes. You can, for example, look for all the voltage values at which the TEM_{00} mode is transmitted. Because there will be random fluctuations of the cavity length (due to mirror vibrations) this measurement is best done by quickly scanning the cavity length electronically and identifying the periodicity of the transmitted light by looking at the transmitted intensity on the photodiode.

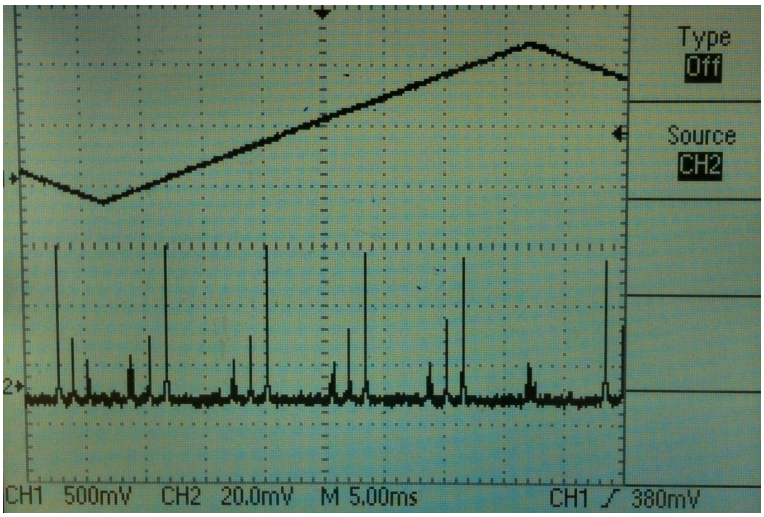


Figure 1.3: Oscilloscope traces showing the linear voltage ramp applied to the cavity piezo (thereby linearly ramping the cavity length) and the optical power transmitted as measured by the photodiode (PD). The PD signal is clearly periodic with the voltage applied to the piezo. Different transverse modes couple into the cavity at different voltages (lengths). By measuring the periodicity of the transmitted pattern, the length change induced by the piezo can be calibrated. By measuring the width of the transmitted peaks (in time or voltage) and comparing this with the width (in time or voltage) between the repeated patterns, the finesse of the cavity can be determined.

Align the output beam from a short length cavity onto the PD and identify the voltage variation that scans the cavity through one free spectral range (FSR). For your cavity, different transverse modes will in general be resonant at

different frequencies or equivalently different cavity lengths, thus as the cavity length is scanned the narrow-frequency laser light couples to different transverse modes of the cavity (see for example eqn. 10.2-31 in Fundamentals of Photonics, 2nd edition). Fig. 1.3 shows an example of oscilloscope traces showing the linear voltage ramp and the PD signal. Why does the pattern of peaks at the far left side of this image switch from large, medium, small to small, medium, large?

Be aware that your PD signal will probably look different depending on the alignment which will change which transverse modes are strongly coupled. Also, depending on your alignment, you might see that your pattern changes in time. In this case, you should take a single trace with the scope and make your measurements on a static trace. If your cavity is well aligned and you have coupled well into the TEM_{00} mode, the photodiode signal should look periodic in the voltage variation as in fig. 1.3. Double check the periodicity by varying the bias voltage manually on the high voltage driver (which adds to the input voltage ramp) to shift the whole pattern to the right or left by one or more periods. Check this also by computing the length change expected from the piezo given the information provided in the data sheet. **Do these methods all agree? What is the voltage variation that moves the piezo by $\lambda/2$?**

1.8 Observations

Once the cavity is aligned, observe the transmitted beam pattern using the CCD camera while slowly scanning the piezo voltage by hand. Use WinTV2000 to observe the output from the CCD camera. Try to stabilize the cavity length on a particular mode by adjusting the length of the cavity using the piezo driver manual adjust knob. When the coupling is optimal and the cavity is very well aligned, you will probably see that most of your higher order modes are circularly symmetric - i.e. Laguerre-Gaussian instead of Hermite-Gaussian modes. See how many different pure Laguerre and/or Hermite-Gaussian modes you can isolate and identify for a given length. Record and sketch or plot your results, clearly indicating the different transverse mode structures of output beams. **What is the largest transverse mode (TEM_{lm}) you can observe as you adjust the cavity length? What is the brightest mode? Which mode should be brightest if the cavity is properly aligned and mode matched?**

With M2 close to M1, to create a 1 cm long cavity, observe the mode patterns transmitted. How many transverse modes are visible? How much of the surface of M2 is filled with laser light?

Next, move mirror M2 so that the cavity is 25 cm long, and re-align the cavity. **What effect does this have on the mode patterns transmitted by the cavity? Are there generally more transverse modes than there were for the original short cavity? Now, how much of the surface of M2 is filled with laser light? Explain any differences you see between these results and the results for the 1 cm cavity.**

Move M2 so that the cavity length is just over 30 cm long, and try to re-align the cavity. Can you align the cavity so that mode patterns are visible in transmission? Consider the cavity stability criterion to see why this is so. **Describe what happens to the power transmitted through the cavity as the cavity length nears the edge of stability.**

1.9 Cavity finesse, linewidth, and Q-factor

In this section, your task is to measure the linewidth or quality of the cavity in units of Hz at several different cavity lengths. You will compare these measurements to the theoretically predicted resonator quality given the mirror reflectivities.

1.9.1 Resonator Theory

The transmission function of the optical resonator in this lab (which is a Fabry-Perot interferometer) depends on the *quality* (or Q-factor) of the resonator (equivalently the *finesse*) and the spectrum of the laser light. For a laser input with an infinitely narrow optical spectrum, the cavity transmission is

$$T = \frac{T_{\max}}{1 + \left(\frac{2F}{\pi}\right)^2 \sin^2(\Delta\phi_{\rm rt}/2)} \tag{1.1}$$

where $T_{\max}$ is the maximum transmission (depending on the mirror reflectivity), F is the cavity *finesse* and $\Delta\phi_{\rm rt}$ is the round trip optical phase. The finesse is defined by

$$F = \frac{\pi\sqrt{r}}{1-r} \tag{1.2}$$

where the amplitude of the wave is reduced by a factor r on each round trip. Given intensity reflection coefficients R_1 and R_2, we have that $r = \sqrt{R_1 R_2}$. For a plane wave inside a cavity of length L made of planar mirrors, $\Delta\phi_{\rm rt} = 2kL$, where $k = n2\pi/\lambda$ and n is the index of refraction of the material inside the cavity. The cavity transmission is maximum when $\Delta\phi_{\rm rt}/2 = q\pi$ where q is an integer or equivalently when $2kL = 2\pi q$.

Okay, here's the main point: the resonance condition is then $L/\lambda = q/2$ (where $q = 1, 2, 3\ldots$). That is, the cavity length must be an *integer multiple of half the wavelength of the input light*! That means that monitoring the resonance of optical cavities is great way to detect small changes (on the order of λ) in the cavity length. This is the principal of operation for LIGO, the cosmic gravitational wave detectors run by MIT and Caltech. **Notice**: since we can *either* vary the input **laser frequency** (i.e. wavelength) OR the **cavity length** to move the laser and the cavity into resonance, we get *two conditions on the positions of the resonances*. For a **fixed cavity length**, the resonance condition on the wavelength or frequency of the input beam is

$$\lambda_q = \frac{2L}{q}; \quad \nu_q = \frac{cq}{2L} = q\,\nu_{\rm FSR} \tag{1.3}$$

where the so-called "free-spectral range" is $\nu_{\rm FSR} = c/2L$ and the speed of light in the cavity is $c = c_0/n$ where c_0 is the speed of light in a vacuum. The time it takes a photon to travel from M1 to M2 and back to M1 (the round trip time) is simply $\tau_{\rm rt} = 1/\nu_{\rm FSR}$. From this, we see that the cavity transmission is periodic in the input laser frequency with period $\nu_{\rm FSR}$. On the other hand, for a **fixed laser frequency**, the resonance condition on the length of the cavity is that it must be an integer number of half wavelengths

$$L_q = q\frac{\lambda}{2}. \tag{1.4}$$

This implies that for a fixed input frequency, the cavity transmission is periodic in the length L of the cavity with period $\lambda/2$. The periodic cavity transmission is shown in Fig. 1.4.

The minimum cavity transmission is achieved when $\sin^2 \Delta\phi_{\rm rt}/2 = 1$ and is

$$T_{\rm min} = \frac{T_{\rm max}}{1 + \left(\frac{2F}{\pi}\right)^2}. \tag{1.5}$$

The minimum intensity only goes to zero in the limit of large finesse - that is when the *mirror reflectivity becomes nearly perfect* ($r \to 1$). As a way to characterize the width of the resonances, we can find the full width at half maximum of the transmission peaks. The points at which the transmission falls to $T_{\rm max}/2$ (i.e. when $\sin^2(\Delta\phi_{\rm rt}/2) = \left(\frac{\pi}{2F}\right)^2$) are given by $\Delta\phi_{\rm rt} = 2\sin^{-1}(\pi/2F)$. The width of a resonance is really only a sensible concept in the limit of large finesse, when the resonances are well resolved. In this limit, we can write the half-maximum intensity phases as $\delta_{\rm HM} \simeq \pm(\pi/F)$. And thus the full width of the resonances at half maximum is $\delta_{\rm FWHM} = 2\pi/F$ or equivalently $L/\lambda = 1/(2F)$. Again, since we can either vary the input **laser frequency** OR the **cavity length** to move the laser and cavity through resonance, we get the following conditions on the width of the resonances:

$$\nu_{\rm FWHM} = \frac{\nu_{\rm FSR}}{F} \tag{1.6}$$

$$L_{\rm FWHM} = \frac{\lambda}{2F} \tag{1.7}$$

$$\lambda_{\rm FWHM} = \frac{1}{2LF} \tag{1.8}$$

Fig. 1.4 shows the transmission (or intensity inside the cavity) as a function of the cavity length given a *fixed laser frequency* and as a function of the input frequency ν given a *fixed cavity length* L. From this, it is clear that the cavity finesse can be obtained experimentally by taking the ratio of the cavity periodicity and dividing this by the width of the transmission peaks.

$$F = \frac{\nu_{\rm FSR}}{\nu_{\rm FWHM}} = \frac{\frac{\lambda}{2}}{L_{\rm FWHM}} \tag{1.9}$$

Alternatively, if the mirror reflectivity (thus finesse) and cavity length L are known, the frequency or length resolving power of the cavity can be computed. Fig. 1.5 shows the transmission of the cavity at different values of the finesse. As the finesse is increased, the resonances become more and more sharp and the transmitted light off of resonance becomes smaller and smaller.

Photon survival time and Q-factor

A photon in the cavity completes one round trip every $\tau_{\rm rt} = 1/\nu_{\rm FSR}$ seconds. Over this round trip, it has a probability $P_{\rm s} = R_1 R_2$ of surviving the trip (i.e. not being lost from the cavity). Here R_1 and R_2 are the intensity reflection coefficients. Therefore the lifetime of a photon inside the cavity is

$$\tau_{\rm p} = \frac{\tau_{\rm rt}}{1 - P_{\rm s}} = \frac{1}{\nu_{\rm FSR}(1 - P_{\rm s})} \tag{1.10}$$

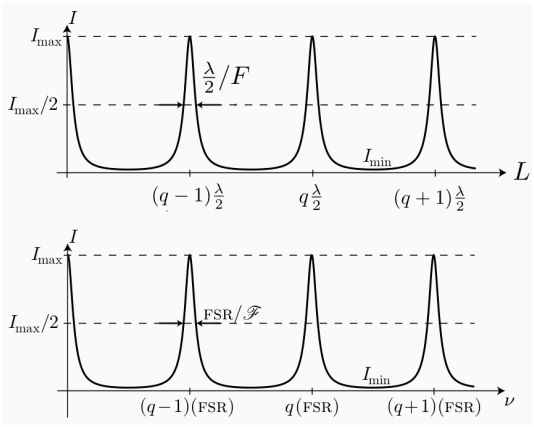


Figure 1.4: Transmission of cavity.

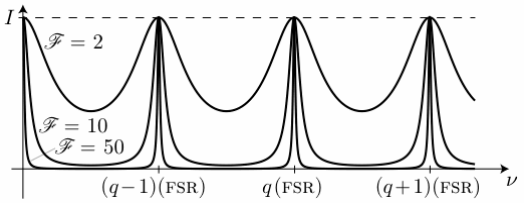


Figure 1.5: Transmission of cavity for various values of the finesse.

The finesse also depends on the mirror reflectivity and can be written as

$$F = \frac{\pi P_{\rm s}^{1/4}}{1-\sqrt{P_{\rm s}}}. \tag{1.11}$$

For large finesse (large survival probabilities), we can approximate $P_{\rm s}^{1/4} \simeq 1$ and $(1-P_{\rm s}) \simeq 2(1-\sqrt{P_{\rm s}})$ which allows us to rewrite the photon lifetime as

$$\tau_{\rm p} = \frac{1}{2\nu_{\rm FSR}(1-\sqrt{P_{\rm s}})} = \frac{F}{2\pi\nu_{\rm FSR}} = \frac{1}{2\pi\nu_{\rm FWHM}} \tag{1.12}$$

and we get an "uncertainty relation" (analogous to the time/energy uncertainty principle in quantum mechanics) of

$$\tau_{\rm p}\,\nu_{\rm FWHM} = \frac{1}{2\pi} \tag{1.13}$$

The resonator *quality* or Q-factor is 2π times the ratio of the total energy stored in the cavity divided by the energy lost in a single cycle. We can write this as

$$Q = 2\pi\nu_q\tau_{\rm p} = \frac{\nu_q}{\nu_{\rm FWHM}} = qF. \tag{1.14}$$

1.9.2 Experimental study of the resonator finesse

Measure the full width at half maximum of the transmitted cavity resonances (L_{FWHM}) as you scan the cavity length by several factors of $\lambda/2$ (corresponding to the free spectral range) for a few (3 or more) different cavity lengths L. Include data from a very short cavity length $L \leq 1$ cm and a very long cavity near the edge of stability. From these measurements, compute the finesse (F) and the corresponding cavity linewidth in units of Hz for each cavity length. **Is your measurement of the finesse different for different cavity lengths? Should it be different? Explain why or why not?**. From your measurements of the mirror reflectivity, compute the finesse of the cavity and compute the expected cavity linewidth in units of Hz for your different cavity lengths. Plot your cavity linewidth as determined from your measurement of the finesse and as determined from your expectation of the finesse from the mirror reflectivity. **From these results, what can you say about the spectral linewidth of the HeNe laser? Are your measurement results consistent with your theory? If not, explain why not?**

Hint: when measuring the length (or voltage) change of the cavity corresponding to a free spectral range (i.e. a distance of $\lambda/2$), it helps to trigger the scope on the voltage ramp you are sending to the piezo. See Section 1.11 for helpful hints on scope usage for this part. It also helps to carefully measure the voltage ramp time and amplitude so that you can easily convert time differences of peaks to voltage differences of the ramp. When measuring the cavity length change (and voltage change) required to scan the cavity through one of the transmission resonances (i.e. the distance L_{FWHM}), it helps to trigger the scope off the rising edge of the photodiode signal peak so that the transmission peak doesn't move around and you can zoom the time-base in to more easily measure the full width at half maximum. Be sure that you "zoom in" enough so that your measurement of the peak full width half maximum is not significantly limited by the finite sampling time of the scope. To capture a single trace, the trigger setting on the oscilloscope needs to be set to "single" and then you press the "run" button to capture a single trace (again, see Section 1.11 for helpful hints on scope usage). The peak separation and FWHM of the peaks is most easily measured using the cursors on the oscilloscope. Press the cursor button, select time type (not voltage type), and adjust their position so that they line up with the features you are measuring. Be aware that their time difference depends on the time scale of the scope, so if you change scales the cursors separation in time changes while their position on the screen remains fixed. Double check that their separation, given the time per division, agrees with the number displayed on the scope!

For these same experimental measurements, plot the experimentally determined finesse versus the cavity length and compare with the expected finesse. **Is the finesse changing with cavity length? Should it? What is the Q-factor of your cavity? Consider a tuning fork oscillating at 440 Hz (this is what musicians often use to tune their instruments). How long would the tuning fork ring given it had the same Q-factor as your optical cavity? Is this reasonable?**

Is there a cavity distance L where the geometric factors drop out (i.e. $g_1 g_2 = 0$) and you can see the modes becoming degenerate? If so, take a spectrum away from this point and at this point and put

them in your lab report and discuss your observations.

1.10 Important Rules: for safety and efficient use of time

Do not be apprehensive of this lab. If you are careful and heed the following warnings, the danger involved in this lab is extremely minimal.

1.10.1 Do Not Stare Into The Beam

The output of the HeNe laser is a medium power laser beam. You should therefore treat the beam with respect. Never place your head/eye in the path of the beam to see where it is going. A small index card (provided) is your tool to observe the path of the beam. It is also important to watch for stray reflections off of mirrors or other reflective surfaces. This means that any **watches, rings, or bracelets should be removed** before beginning this experiment. Laser goggles are provided and should be worn at all times.

1.10.2 Do Not Touch Any Optical Surface

It does not take many impurities on the optical surfaces to ruin the finesse of an optical cavity. Scratches, fingerprints, and even dust on the cavity elements can prevent the cavity from working well. If you suspect that an element is dirty, do not attempt to clean the optics yourself. Ask a TA and they will do it for you.

1.10.3 Do not spend more than 15 minutes trying and failing to get the cavity to resonate

If you spend more than 15 or 20 minutes trying to get the cavity aligned, there may be something more fundamentally wrong with the setup than alignment - like an optic is dirty or the external mirror mount height has been changed. You should not continue in vain but rather find a TA to help you diagnose and fix the problem.

1.11 Additional notes on using the scope

This section gives a few examples of scope usage.

1.11.1 Setting the scope to trigger on the PZT voltage ramp

1. Verify that the output of the function generator is going to channel 1 and the photodiode (hitherto referred to as “PD”) output is going to channel 2
2. press the ”TRIGGER MENU” button
3. toggle ”source” button to channel 1 (the voltage ramp channel)

4. toggle mode to "Normal" (there is "auto", "normal", and "single"

5. move the TRIGGER LEVEL knob up until the scope is properly triggered on the voltage ramp. Adjust the "HORIZONTAL POSITION" knob so that you can see at least one half period of the voltage triangle wave

6. The trace from channel 1 and channel 2 should look like those in Fig. 1.3

1.11.2 Setting the scope to trigger on the photodiode signal - the transmission peaks

1. press the "TRIGGER MENU" button

2. toggle "source" to channel 2 (the PD channel)

3. toggle mode to "Normal" (there is "auto", "normal", and "single"

4. move the TRIGGER LEVEL knob up until the main peak appears consistently appears centered on the screen. You may need to move the "HORIZONTAL POSITION" knob to get the peak to be centered when you zoom in on the peak by dialing the "HORIZONTAL SEC/DIV" knob

Sometimes it will be useful to average a few traces. This can be done with the following proceedure.

1. press the "ACQUIRE" button at the top in the "MENUS" section

2. select "Average" (there is "Sample", "Peak detect", and "Average")

3. toggle the "Averages" button to something not too large - like 8 or 16

1.12 Additional notes on alignment

1. Make sure the back reflection from the planar mirror (M1) goes back through the lens and back to the HeNe laser along the same path as the input beam. This ensures that M1 is properly aligned.

2. With M2 out, make sure that the transmitted light through M1 goes onto your detector

3. When you put in the second mirror (M2), watch the transmitted light (through M2) while adjusting the end mirror (M2) so that the transmitted light is a symmetric spot (and not a line of light).

4. When M2 is aligned you should see a flash of light on the surface of the mirrors facing the cavity. "Flash" occurs when the length of the cavity is just right and the power inside the cavity suddenly builds up to a large value and the scattered light at the mirrors becomes visible. Usually the cavity is only resonant for a brief moment in time causing the scattered light from the mirrors to "flash" or flicker on then off. You can roughly align the cavity while watching for flash.

5. You can do fine alignment by looking at the transmitted optical power on the photodiode (PD). Make adjustments by turning the screw and releasing the knob since the pressure from touching the mirror mount can produce a slight misalignment. It is often useful to find the limits of operation for each adjustment screw and set the screw position to the middle of this operating range.

6. Check PD alignment each time cavity length is adjusted.

7. Dust accumulation on the mirrors can be a problem since it can ruin the finesse of your cavity. If you suspect dusty or dirty mirrors, let a TA know and he/she can clean them.

1.13 Theory

1.13.1 Hermite-Gaussian Modes

This discussion of Gaussian modes assumes that you are familiar with the paraxial wave equation, its Hermite-Gaussian solutions, and the eigenmodes of spherical mirror cavities.

One often-encountered class of beam-like solutions to the wave equation may be written

$$E(\mathbf{r},t) = E_0(\mathbf{r})e^{ikz}e^{-i\omega t} \tag{1.15}$$

where the envelope $E_0(\mathbf{r})$ is a slowly varying function of z. Putting this solution into the wave equation yields the *paraxial wave equation*

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + 2ik\frac{\partial}{\partial z}\right] E_0(\mathbf{r}) = 0, \tag{1.16}$$

which is an equation for the envelope $E_0(\mathbf{r})$. One of several possible complete sets of solutions are the Hermite-Gaussians

$$E^{\mathrm{H}}_{nm} = \frac{Aw_0}{w(z)} H_n\left(\frac{\sqrt{2}x}{w(z)}\right) H_m\left(\frac{\sqrt{2}y}{w(z)}\right) \tag{1.17}$$

$$\times e^{-i(n+m+1)\arctan(z/z_0)} e^{ik(x^2+y^2)/2R(z)} e^{-(x^2+y^2)/w^2(z)} \tag{1.18}$$

where H_n and H_m are Hermite-polynomials of order n and m. These solutions are (approximate) eigenmodes of spherical mirror cavities and of Maxwell's equations in free space. That's why the output of a laser based on a spherical mirrors propagates over long distances as a well defined beam.

The phase of propagation accumulates like a plane wave phase (as kz) but has an additional Gouy phase that depends on the mode number ($(n+m+1)\tan^{-1}(z/z_0)$) and a radial curvature term. These solutions are appropriate in systems that lack full cylindrical symmetry, but still have a pair of symmetry axes x and y.

The beam size $w(z)$ and wavefront curvature $R(z)$ are closely related and characterized by the Rayleigh length z_0 according to

$$w(z) = w_0\sqrt{1+(z/z_0)^2} \tag{1.19}$$
$$R(z) = z + z_0^2/z \tag{1.20}$$
$$z_0 = \frac{\pi w_0^2}{\lambda} \tag{1.21}$$

Round trip phase for a Gaussian beam inside a cavity

Above, we have considered the resonant modes of a planar cavity with a plane wave inside. Here we

Fig. 1.6 shows a Gaussian beam inside a cavity. In order for the beam to be resonant with the cavity, the radius of curvature of the spherical wave fronts must match the mirror curvatures. These boundary conditions *fix* the position of the focus relative to the mirror positions. The focus is defined to be at $z = 0$ and the two mirrors are located at z_1 and z_2. Recall that the phase of a Gaussian

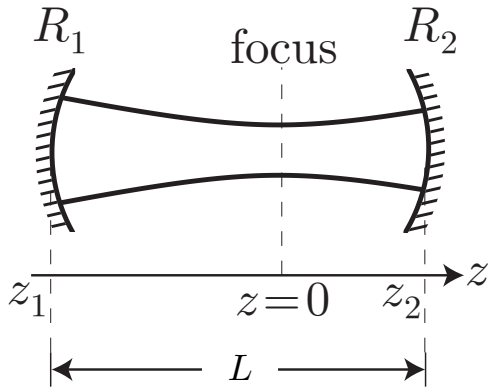


Figure 1.6: Schematic of a Gaussian beam inside a cavity.

beam *differs* from that of a plane wave. For a plane wave, the optical phase is simply $\phi(z) = kz$, but for a TEM_{lm} Gaussian beam it has an additional Gouy phase and a radially varying part:

$$\phi(r,z) = kz - (1+l+m)\tan^{-1}\left(\frac{z}{z_0}\right) + k\frac{r^2}{2R(z)}. \tag{1.22}$$

Since we have assumed the shape of the phase fronts is matched to the mirrors, we can consider just the on-axis phase $\phi(r=0,z)$. The round trip phase change of an optical wave inside the cavity is simply twice the phase change accumulated from z_1 to z_2

$$\Delta\phi_{\mathrm{rt}} = 2[\phi(0,z_2) - \phi(0,z_1)] = 2kL - (1+l+m)\left[\tan^{-1}\left(\frac{z_2}{z_0}\right) - \tan^{-1}\left(\frac{z_1}{z_0}\right)\right]. \tag{1.23}$$

On resonance the round trip phase change must be $\Delta\phi_{\mathrm{rt}} = 2\pi q$. It can be shown that the Gouy phase contribution can be written in terms of the geometric

factors $g_i = 1 + L/\rho_i$, where ρ_i is the radius of curvature of mirror i. Writing the resonance condition in terms of frequency, we have

$$\nu_q = \nu_{\mathrm{FSR}} \left(q + \frac{1}{\pi}(1 + l + m) \cos^{-1} \sqrt{g_1 g_2} \right). \tag{1.24}$$

The first term ($q\,\nu_{\mathrm{FSR}}$) is the planar resonance condition. The second term is an overall shift of the spectrum that depends on the cavity geometry (characterized by g_1 and g_2) and mode numbers l and m. In practice (except for confocal or concentric cavities), this means that different transverse modes (that are all at the same frequency) will be resonant with a cavity at different lengths. This is why when you couple the laser light into your spherical mirror cavity, you will see different transverse modes resonant (and transmitted) at slightly different cavity lengths. Alternatively, for a cavity of fixed length, different transverse modes are resonant at different optical frequencies.

Spherical-Mirror Resonators: stability

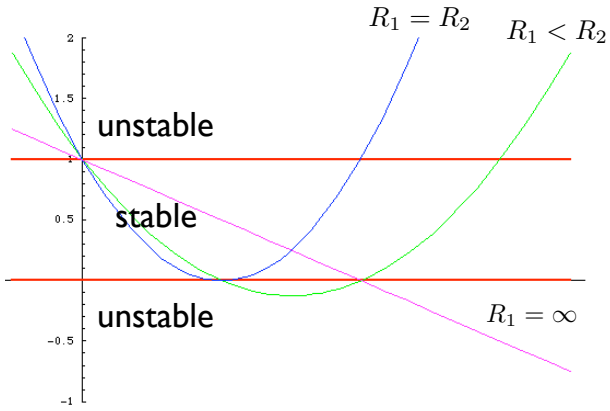


Figure 1.7: Plot of the middle term in expression (1.25) as a function of the mirror separation, d, for various mirror combiations (i.e. values of R_1 and R_2). The cavity is stable at all locations, d, for which the value of the term is between 0 and 1 (denoted by red lines).

An optical resonator composed of two planar mirrors ($R_1 = R_2 = \infty$) is stable for any mirror separation so long as they have been perfectly aligned. The difficulty with this arrangement is that in practice planar mirrors are extremely sensitive to misalignment; they must be perfectly parallel to each other and perfectly normal to the incident light rays. This sensitivity can be reduced by replacing the planar mirrors with spherical ones. The trade off, however, is that spherical-mirror resonators are only stable for specific geometric configurations. These mirrors can be either concave ($R < 0$) or convex ($R > 0$).

Limiting yourself to ray optics, and specifically to the methods of paraxial matrix-optics, it is possible to determine that the region of stability for any spherical-mirror resonator is given by;

$$0 \leq \left(1 + \frac{d}{R_1}\right)\left(1 + \frac{d}{R_2}\right) \leq 1 \tag{1.25}$$

where d is the optical cavity length, and R_1 and R_2 are the radii of curvature for the two mirrors. Typically, the two middle terms are written in terms of the ***g* parameters**

$$g_1 = 1 + \tfrac{d}{R_1} \quad \text{and} \quad g_2 = 1 + \tfrac{d}{R_2}$$

It is left as an exercise to demonstrate that this result is valid. You should record this derivation in your lab book. A good starting point for this analysis in located in your text book (Saleh and Teich, *Fundamentals of Photonics*).

1.13.2 Eigenmodes of a spherical mirror resonator

The Hermite-Gaussian modes form a complete set of eigenmodes for a stable spherical mirror resonator. If the mirrors have radius of curvature R_1, R_2 and separation L, then the condition for stability is $0 \leq g_1 g_2 \leq 1$ where $g_i = 1 - L/R_i$. The mirror locations and waist size can be found from the following expressions:

$$z_1 = -L\frac{g_2(1-g_1)}{g_1(1-g_2)+g_2(1-g_1)} \tag{1.26}$$

$$z_2 = +L\frac{g_1(1-g_2)}{g_1(1-g_2)+g_2(1-g_1)} \tag{1.27}$$

$$w_0 = \sqrt{\frac{\lambda L}{\pi}}\left(\frac{g_1 g_2(1-g_1 g_2)}{(g_1+g_2-2g_1g_2)^2}\right)^{1/4} \tag{1.28}$$

The frequency of these eigenmodes are:

$$\nu_{nmq} = \frac{c}{2L}\left[q + \frac{1+n+m}{\pi}\arccos\sqrt{g_1 g_2}\right] \tag{1.29}$$

Here is a list of the first few Hermite polynomials

$$\begin{aligned} H_0(\xi) &= 1 \\ H_1(\xi) &= 2\xi \\ H_2(\xi) &= 4\xi^2 - 2 \\ H_3(\xi) &= 8\xi^3 \\ H_4(\xi) &= 1 \end{aligned}$$

1.14 Images of components for this lab

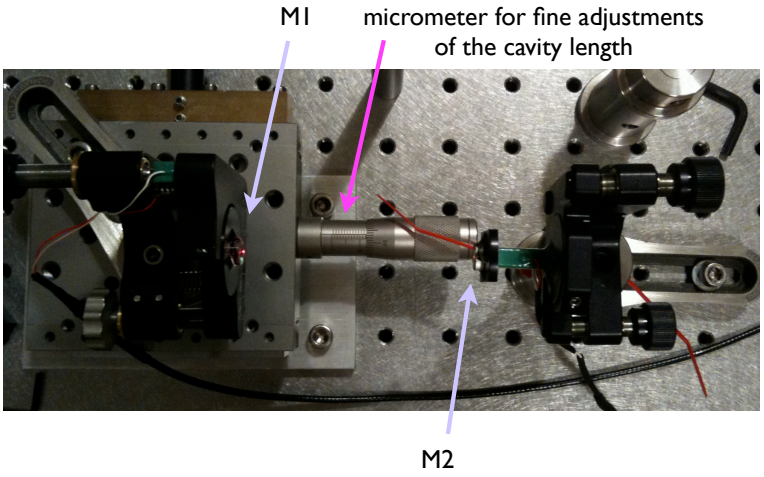


Figure 1.8: Image of the cavity showing M1 and M2.

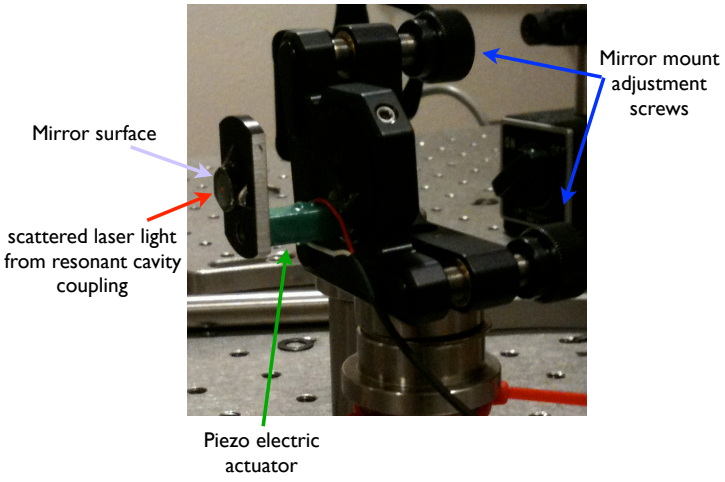


Figure 1.9: Close-up image of the mirror M2 showing the piezo actuator. When the cavity is resonant with the input laser light, the optical power inside the cavity is very large and scattered light from the mirrors becomes clearly visible.

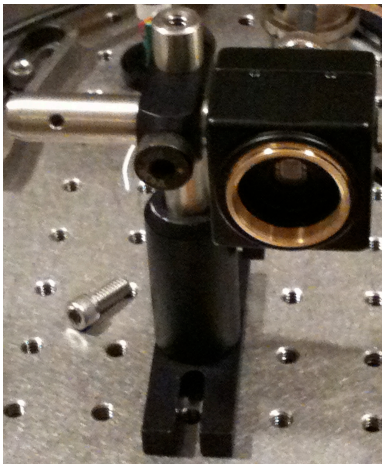


Figure 1.10: Image of the CCD camera. The active area of the sensor is the small rectangle (approximately 2x3 mm) centered in the region bordered by the threaded brass ring.

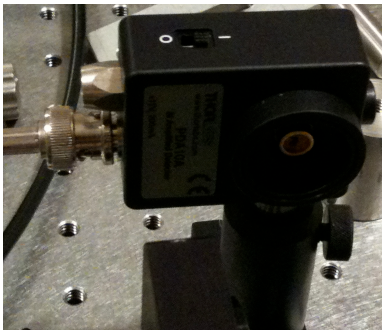


Figure 1.11: Close-up image of the photodiode. The active area of the sensor is a tiny (¡ 1 mm^2) square centered in the area bordered by the gold ring.

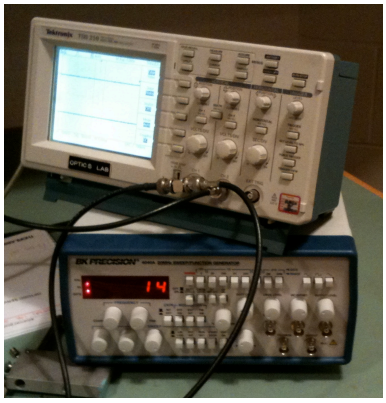


Figure 1.12: Oscilloscope and function generator. The output of the function generator should be sent to both the scope and the high voltage amplifier. The photodiode signal should also be displayed by the scope. An example trace is shown in fig. 1.3



Figure 1.13: High voltage (HV) amplifier (THORLABS model MDT694A). The display shows the output voltage. A voltage applied to the "EXT INPUT" on the front is amplified and that voltage is added to that set by the manual "OUTPUT ADJ" knob. The input multiplier changes depending on the voltage limit indicated by the green LED.