Physics 458/408

Applied Optics Lab Manual

Department of Physics and Astronomy
UBC

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Chapter 1

Lab Rules

1. It is **essential and required** that you read the entire lab writeup for a particular experiment and do the corresponding prelab work **before** coming to lab. This prelab reading and work will probably take at least one hour and it is worth 20% of your lab mark. Your prelab work will be checked on the first day of each lab experiment.

2. Do not attempt to move optics mounts or other hardware fixed to the bench with screws painted in red. These are elements that are already in their proper place and moving them will make doing the lab impossible. If you don't have a tool to loosen a screw, you probably shouldn't be trying to move that component.

3. No food or drink is allowed in the lab rooms. If you must eat, do so in the hallway.

4. You should do your best to fully answer all questions posed in the lab manual. Particularly important questions are usually in **bold**.

Chapter 2

Fourier Optics

2.1 Introduction: image formation

In the figure below a mesh object is illuminated with collimated coherent radiation (produced by a laser) and a magnified image of the mesh is formed by the second lens, referred to as the transform lens. The magnified image is located in the plane a distance s_i away from the lens (at position $z = B$) which is itself a distance s_0 from the object. In the thin lens approximation, the magnification is equal to the ratio

$$m = \frac{s_i}{s_o}, \tag{2.1}$$

and the object and image distances (s_o, s_i) are related to the focal length f by the image equation

$$\frac{1}{s_i} + \frac{1}{s_0} = \frac{1}{f} \tag{2.2}$$

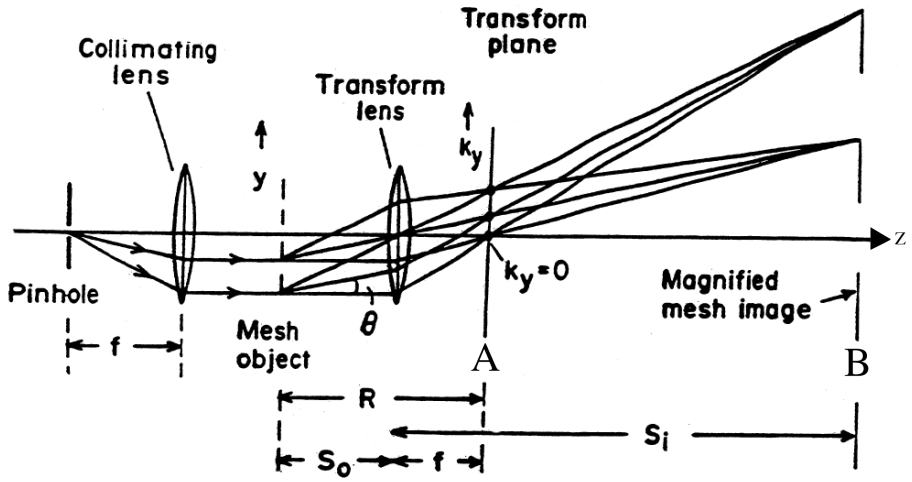


Figure 2.1: Magnified image formation with parallel coherent incident radiation.

In contrast, if the screen is placed at $z = A$, something else is produced. In particular, as will be shown below, the spatial 2-D Fourier transform [1] of the object image will appear in the plane at $z = A$

2.1.1 Optical Fourier Transform Produced by a Lens

In order to understand how a lens generates the Fourier Transform of the light wave emanating from the object, let's imagine that we break up this object wave into a superposition of plane waves (this can always be done for linear optical systems). Now, the lens simply focuses each of these plane waves to a different spot in the focal plane. So, the optical wave at the focal plane behind the lens is the 2-D Fourier transform of the optical wave leaving the object.

Let's see how this works. Consider a plane wave of complex amplitude $U(x, y, z) = A\exp[i(k_x x + k_y y + k_z z)]$ with wavevector $\mathbf{k}= (k_x, k_y, k_z)$, wavelength λ, wavenumber $k = (k_x^2 + k_y^2 + k_z^2)^{1/2} = 2\pi/\lambda$, and complex envelope A. The vector $\mathbf{k}$ makes angles $\theta_x = \sin^{-1}(k_x/k)$ and $\theta_y = \sin^{-1}(k_y/k)$ with the $y - z$ and $x - z$ planes, respectively, as illustrated in Fig. 4.4. The complex amplitude in the $z = 0$ plane, $U(x, y, 0)$, is a spatial harmonic (i.e. sinusoidal) function $f(x, y) = A\exp[2i\pi(v_x x + v_y y)]$ with spatial frequencies $v_x = k_x/2\pi$ and $v_y = k_y/2\pi$ (cycles/mm). The angles of the wavevector are therefore related to the spatial frequencies of the harmonic function by

$$\begin{aligned} \theta_x &= \sin^{-1} \lambda v_x \\ \theta_y &= \sin^{-1} \lambda v_y \end{aligned} \tag{2.3}$$

Recognizing $\Lambda_x = 1/v_x$ and $\Lambda_y = 1/v_y$ as the spatial periods of the harmonic function in the x and y directions, we see that the angles $\theta_x = \sin^{-1}(\lambda/\Lambda_x)$ and $\theta_y = \sin^{-1}(\lambda/\Lambda_y)$ are governed by the ratios of the wavelength of light to the period of the harmonic function in each direction. These geometrical relations follow from matching the wavefronts of the wave to the periodic pattern of the harmonic function in the $z = 0$ plane, as illustrated in Fig. 4.4.

If $k_x \ll k$ and $k_y \ll k$, so that the wavevector $\mathbf{k}$ is paraxial, the angles θ_x and θ_y are small (therefore $\sin\theta_x \approx \theta_x$ and $\sin\theta_y \approx \theta_y$) and

$$\begin{aligned} \theta_x &\approx \lambda v_x \\ \theta_y &\approx \lambda v_y \end{aligned} \tag{2.4}$$

Thus the angles of inclination of the wavevector are directly proportional to the spatial frequencies of the corresponding harmonic function.

Now, the different plane-wave components that constitute a wave may be spatially separated (i.e. isolated) by the use of a lens. Recall that a thin spherical lens transforms a plane wave into a paraboloidal wave focused to a point in the lens focal plane. If the plane wave arrives at small angles θ_x and θ_y, the paraboloidal wave is centered about the point$(\theta_x f, \theta_y f)$, where f is the focal length (see Fig. 4.5). The lens therefore focuses (maps) each plane wave propagating in the direction (θ_x, θ_y) onto a single point $(\theta_x f, \theta_y f)$ in the focal plane and thus spatially separates the contributions of the different harmonic functions.

[1] If you don't remember what a Fourier transform is, now is a good time to review it. Also see $http://www.falstad.com/fourier/$ for an instructional java applet.

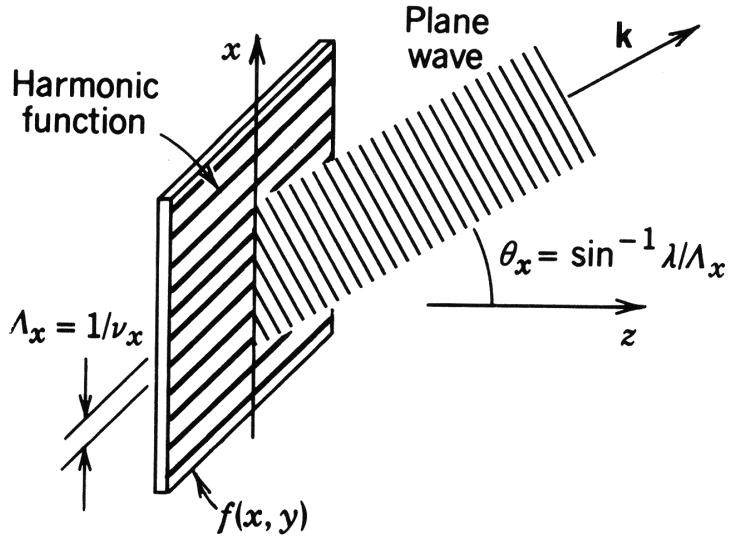


Figure 2.2: A harmonic function of spatial frequencies v_x and v_y at the plane $z = 0$ is consistent with a plane wave traveling at angles $\theta_x = \sin^{-1} \lambda v_x$ and $\theta_y = \sin^{-1} \lambda v_y$.

In reference to the optical system shown in Fig. 4.1, let $f(x, y, z)$ be the complex amplitude of the optical wave in the x–y plane at some position z. The optical wave emanating from the object in the $z = 0$ plane located to the left had side of the lens is then $f(x, y, 0)$. Think of the light coming from the object as a superposition of plane amplitude waves, with each wave component traveling at a small angle $\theta_x = \lambda v_x$ and $\theta_y = \lambda v_y$ having a complex amplitude proportional to the Fourier transform of $f(x, y, 0)$, namely $F(v_x, v_y)$. Each of these plane wave component is then focused by the lens to a unique point (x', y') in the focal plane where $x' = \theta_x f = \lambda f v_x$, $y' = \theta_y f = \lambda f v_y$. The complex amplitude of the wave at point (x', y') in the output or focal plane (located at $z = A$) is therefore proportional to the Fourier transform of $f(x, y, 0)$ evaluated at $v_x = x/\lambda f$ and $v_y = y/\lambda f$. Thus we have

$$f(x, y, z = A) \propto F\big(\frac{x}{\lambda f}, \frac{y}{\lambda f}\big) \tag{2.5}$$

Short range irregularities in $f(x, y, 0)$ (sharp edges) contribute primarily to the diffraction field away from the optic axis, *i.e.* for large (x', y') or (k_x, k_y). Long range irregularities (soft edges) contribute primarily to the small spatial frequencies (k_x, k_y) near the optic axis in the diffraction pattern. Obviously, spatial filtering of the beam is performed in the transform plane. This filtering can be of simple low-pass or high-pass type, or it can involve selective phase-change of any particular spatial frequency component. Later we will investigate the filtering properties.

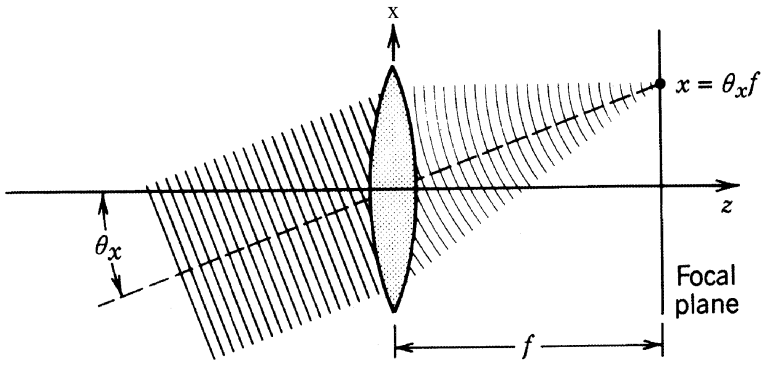


Figure 2.3: Focusing of a plane wave into a point. A direction (θ_x, θ_y) is mapped into a point $(x, y) = (\theta_x f, \theta_y f)$ at $z = f$.

2.1.2 Inverse Fourier Transform

So if the Fourier transform appears at $z = A$, how does an image appear at $z = B$? Very simply, propagation of the wave through free space from $z = A$ to $z = B$ generates the inverse Fourier transform! In this way, the object wave is modified by the lens so that propagation of the wave after the lens produces the Fourier transformed at the transform plane $z = A$, and then, by additional propagation through free space, the inverse Fourier transform is created at $z = B$. This process is what produces an image of the object at $z = B$.

To see how propagation generates the inverse transform, consider this: the electromagnetic wave which crosses the plane at $z = A$ can be though of as originating from a distribution of point emitters in that plane, and each of those point emitters illuminates the image plane (at $z = B$) after propagation by the distance d between A and B. If you write the image wave at $z = B$ as the sum of spherical waves generated by all these point emitters, you get something which, after some approximations, is exactly the inverse Fourier Transform!

Here's the same argument but with the math too. First consider just a single point source of light located on the z-axis in the Transform plane at $z = A$. This point source generates a spherical wave propagating outward (i.e. $E = \frac{1}{r}e^{i(kr-\omega t)}$ where r is the length of the vector pointing from the point source to the observation point). If we examine this spherical wave far from the source but close to the z axis, we can safely approximate it as a paraboloidial wave (i.e. $E \simeq \frac{1}{z}e^{ikz}e^{ik(x^2+y^2)/2z}$). If the point source emitter weren't located exactly *on* the z-axis (i.e. at $x = 0, y = 0$) but rather just off the z-axis, say at (x_i, y_i), then we simply replace $x \to x - x_i$ and $y \to y - y_i$ since the position variables come from the vector pointing from the source at $\vec{r_i}$ to the observation point at $\vec{r}$. In this case, the electric field produced in the plane at $z = B$, a distance d from the point emitter at $z = A$ is then

$$E(x, y, z)|_{z=B} \simeq \frac{1}{d}e^{ikd}e^{ik((x-x_i)^2+(y-y_i)^2)/2d} \tag{2.6}$$

Okay, now if we have a whole distribution of point sources of different amplitudes (brightnesses) in the plane $z = A$ described by the function $E(x, y, z = A)$, then

the electric field observed in the plane $z = B$ far from $z = A$ will just be the sum of all these point sources each located at some initial position (x_i, y_i) and with some amplitude $E(x_i, y_i, z = A)$. This sum is just the integral

$$E(x, y, z)|_{z=B} \simeq \frac{1}{d} e^{ikd} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(x_i, y_i, z)|_{z=A} \, e^{ik((x-x_i)^2+(y-y_i)^2)/2d} \, dx_i \, dy_i. \tag{2.7}$$

This equation expresses the form of the diffracted field after propagation by a distance d. In the far field limit this reduces to the Fraunhofer diffraction expression

$$E(x, y, z)|_{z=B} \simeq \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(x_i, y_i, z)|_{z=A} \, e^{-i2\pi(xx_i+yy_i)/\lambda d} \, dx_i \, dy_i. \tag{2.8}$$

where k has been replaced by it's value $2\pi/\lambda$, we have dropped the unimportant phase prefactor ($\frac{1}{d} e^{ik(d+[x^2+y^2]/2d)}$) which doesn't matter if we only look at the field amplitude. Also, we have dropped the quadratic terms inside the exponent ($kx_i^2/2d$). This last step is okay so long as we're only interested in the field generated by point emitters lying close to the z-axis such that $kx_i^2/2d \ll \pi$. After these simplifications, we are left with just the integral which is the inverse Fourier Transform of the wave at $z = A$. The result is that if we modify the wave at $z = A$ by placing there a mask, the image which results at $z = B$ will be the convolution of the original image and the Fourier transform of the mask.

The Abbé theory of image formation arose from Abbé's consideration of these effects when trying to view small objects through a microscope. It became clear to him that a lens (the objective lens of the microscope) always acts as a low-pass filter, since its finite size will admit only spatial frequencies up to an obvious geometrical limit. The bigger the lens and the larger the range of spatial frequencies admitted, the greater the image fidelity. In the limit of admitting only the undeflected central order ($\theta = 0$), no detail of the object at all will be present in the image. Only a uniform illumination (the so-called d.c. level) will be obtained in the image plane. This, therefore determines the size of the smallest objects which can be viewed by a particular microscope. This relationship is exploited to "clean" up the laser beam coming from the HeNe laser. By focussing the beam through a pinhole, we spatially filter out the high spatial frequencies (i.e. the transverse spatial structures) of the laser to give a clean uniform beam without structure.

2.2 Prelab

Consider the setup shown in figure 2.4. This is part of the basic optical setup for this lab. The beam from the laser is first expanded and collimated by the spatial filter and collimating lens. The beam then passes through the mesh object (one of the objects you will use in this lab). The optical wave transmitted by the mesh passes through the Fourier Transform lens and, due to the magic of wave propagation, at the focal distance (f) after that lens, the Fourier transform of the transmitted beam is generated.

The optical Fourier transform phenomenon arising from wave propagation through a lens plays an important role in pattern recognition, image restoration,

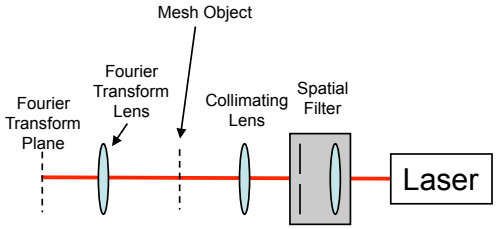


Figure 2.4: Part of the imaging setup for Fourier Optics lab. The spatial filter and collimating lens expand and collimate the laser beam which then illuminates the mesh object. The optical wave transmitted by the mesh passes through the Fourier Transform lens and at the focal distance (f) after that lens, the Fourier transform of the transmitted beam appears. A picture of the mesh Fourier transform intensity pattern is shown in Fig. 2.5

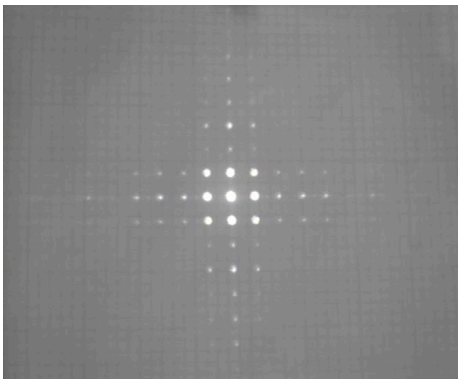


Figure 2.5: Picture of the intensity profile at the Fourier Transform plane produced by a wire mesh object. See Fig. 2.4 for the optical setup.

and optical image processing. A picture of this mesh Fourier transform intensity pattern is shown in Fig. 2.5.

Given the intensity pattern shown in Fig. 2.5, the Fourier Transform Lens focal length is $f = 40$ cm, and the graph paper squares are 0.5 mm on a side, **compute the distance between the wires in the mesh.**

To make imaging the intensity pattern at the Fourier transform plane, you will normally (in this lab) use a second lens to magnify the image before taking a picture of it.

If the wave is allowed to pass the Fourier transform plane, after some more propagation through free space, the optical wave undergoes an inverse Fourier transform, and the image of the transmitted optical wave appears some distance away from the lens. In general, wave propagation by a large distance through free space leads to a Fourier transform of the input wave. The lens simply shapes the input wave so that the Fourier transform of the input wave is complete at a distance f from the lens. Wave propagation after this point produces a second Fourier transform which is complete at a very specific location in z at the image

plane (s_i is the distance from the Image screen to the lens, s_o is the distance of the object to the lens, and $1/f = 1/s_i + 1/s_o$). This second transform is the same as an inverse transform but with the axes inverted, that's why the object image is inverted in this single lens setup. With an appropriate two lens imaging setup, there are 4 transforms and the inversion is made twice producing a non-inverted image.

2.3 The Spatial Filter

The purpose of the spatial filter is to remove any higher order spatial frequencies from the output of the laser to produce a "clean" Gaussian beam with which to illuminate different objects in this lab.

The spatial filter is comprised of a microscope objective and a pinhole mounted on translation stages and should be placed just after the laser. The microscope objective lens focusses the laser through the tiny ($\sim$ 25 to 50 μm) pinhole, and this produces a divergent, coherent beam with a clean spatial profile (i.e. no transverse spatial structure).

Remove everything from the rail except the laser. Align the laser so that it produces a beam which is centered over the optical rail and parallel to it. To check the alignment of the laser, place the pointed metal rod on the optical rail in a holder which allows no horizontal motion. Adjust the laser mount so that the beam remains on the point of the rod as it is moved along the entire rail, approximately 20 cm above the rail. If you find that there is a place where the rail rolls a bit and the beam misses the point (so to speak), try to avoid placing lenses or objects at this position.

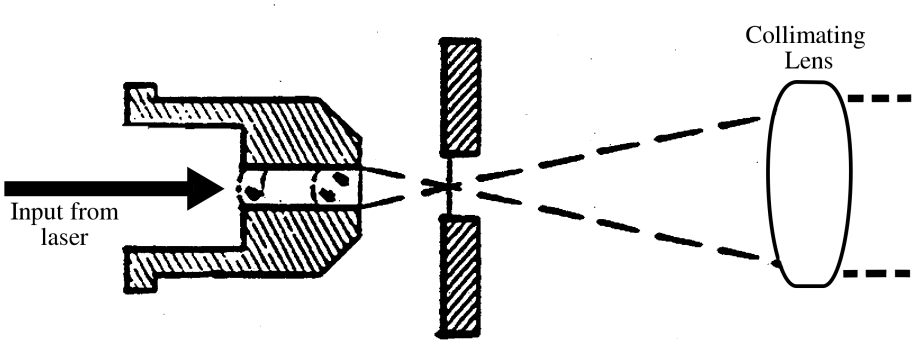


Figure 2.6: A spatial filter consists of a microscope objective, which can be adjusted to focus a laser beam through a small pinhole.

Next, insert and align the spatial filter. With the white screen on the rail, you want to see a large uniform beam without diffraction rings. There are six adjustments which you can make on the spatial filter assembly. It can be rotated about a vertical axis by turning the clamped rod in the holder. This is not a critical adjustment. The assembly as a whole can be raised or lowered, and moved side to side by turning the black knobs on the optical mount. The pinhole itself can be moved up or down, and side to side using the silver knobs attached to it. These are critical adjustments. Finally, the microscope objective lens can be focused on the pinhole using the knob near it parallel to the optic axis.

With this many knobs, alignment is clearly going to be an iterative process. First, align the system horizontally and vertically to get a clear beam along the optic axis. Then focus the system. Then fix the transverse position once again to return the beam to the axis, and so on.

It is best to make the two horizontal adjustments simultaneously. Move the assembly a little, then the pinhole, then the assembly, etc. Treat the vertical adjustments in the same manner. Watch the beam to see if you are making an improvement each time you refocus.

When adjusting the spatial filter, or any other device, assume that the previous user has left it close to the proper alignment. So remember how you found it, start with small adjustments in both directions, and do not make changes without observing their effect.

Check that the beam still runs parallel to the rail. Before you move on, ask a TA to check your spatial filter. If you can't get a uniform circle in ten minutes, ask a TA to help. It is often easier to absorb a demonstration than written instructions. Useful tip in case of difficulty: If you don't see any light through the pinhole, increase the distance between the lens and the pinhole until you can see where the beam hits the area around the pinhole. You can now centre the beam at the pinhole and see light transmitted. You can then readjust the distance until the diffraction pattern disappears.

2.4 The Lenses

Converging lenses, which are generally thicker at the center than at their margins, focus collimated light (a parallel beam) to a point in their rear, or secondary focal plane. Diverging lenses are wider at the perimeter and "focus" collimated light in their rear focal plane (see 2.7). This is the definition of a lens' focus.

Images that are produced on the side of the lens opposite to the object are termed real because they can be seen on a screen. Images on the same side of the lens as the object are called virtual, and are seen only by looking through the refractive surface (from the opposite side, of course). Diverging lenses always produce virtual images, but a converging lens will produce either depending on the position of the object. In this experiment, we concern ourselves only with real images. From a simple lens, real images are usually inverted, whereas virtual images are upright (see 2.7).

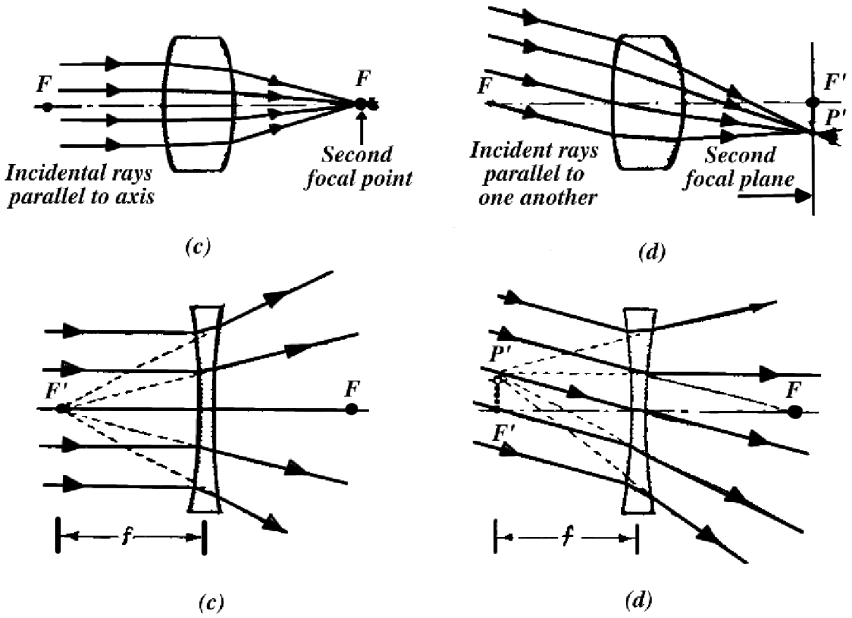


Figure 2.7: Focal planes of converging and diverging lenses.

2.5 Alignment

Figure 2.8 illustrates the *approximate* positions of the major optical components on the 4m optical bench. First check the alignment of the spatial filter pinhole and ensure that output beam is round and symmetrical. Then make fine adjustments to the transverse position of the spatial filter system with respect to the laser beam (i.e. the laser beam propagates along the z axis and you are to adjust the x and y position of the entire spatial filter system) to make the output beam propagate down and along the optical rail at a constant height.

Then place the collimating lens as shown in fig. 2.8 so that the laser light diverging from the spatial filter is collimated along the entire length of the rail. Make sure the collimating lens transverse (x and y) position is correct so that the beam propagates along the rail (along z) at a constant height. Position the end mirror so that the beam runs parallel to the secondary rail and check the laser beam spot size on the wall to see that it remains collimated up to that point.

Next, place the transform lens on the bench about 70cm from the collimating lens, and lock it. Avoid the junction of the two rail components. Center the transform lens, checking that the reflections return to the laser. With the screen or your hand, find the place along the rail where the light is brought back to a focus. Place the screen on the rail beyond the focus, so that the beam has re-expanded to a disc of 10 or 15 cm.

Finding the object plane. In the following sections, you will have to place various apertures and objects in the object plane. The best way to handle this is to first find the plane with an easy aperture/object and then change

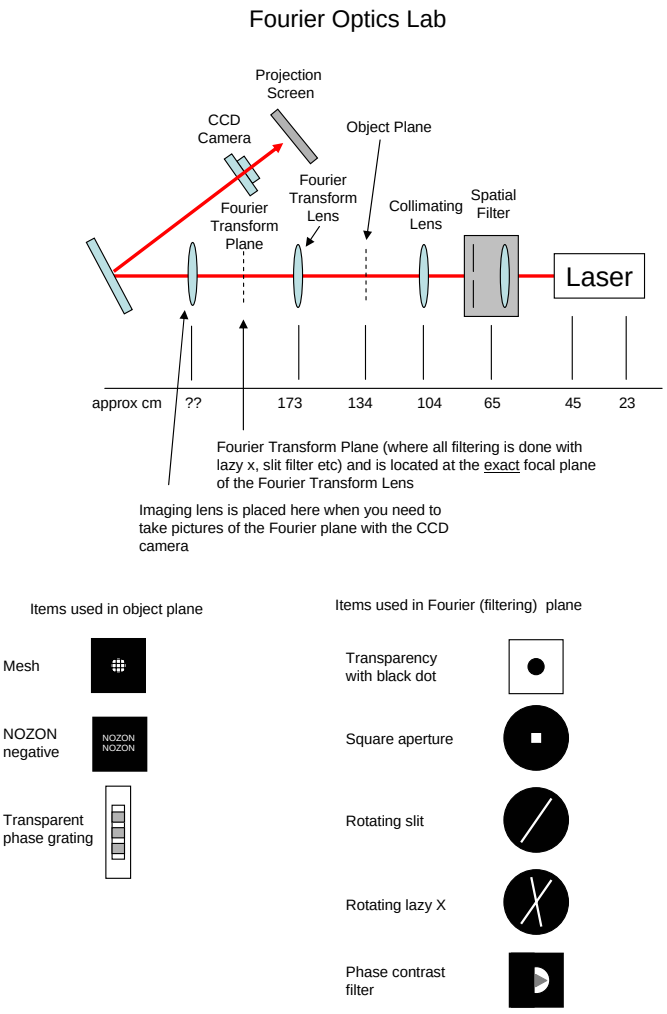


Figure 2.8: Experimental arrangement showing the *approximate* positions of the major optical components on the 4m optical bench as well as the various object and filter components used in this lab. The laser is mounted on one end, the plane mirror on the other. Position the mirror so that the beam illuminates a screen on the secondary rail.

apertures while leaving the mount locked down in that position. To find the object plane, use either the NOZON negative aperture or the aperture which consists of a razor blade with a wedge of clear glass glued to it. Place this in between the collimating lens and the transform lens and move it along the rail (i.e. along z) until a clear and sharp image appears on the screen on the secondary rail. You can have your lab partner (or CCD camera) watch carefully for the exact position at which the image of the razor blade becomes sharp and in focus without any additional structure arising from diffraction. Lock down the position of this mount and replace the razor blade aperture with your aperture of choice.

Finding the Fourier transform plane. In the following sections, you will have to find the exact location of the FT plane and to place objects there either to completely block or simply phase shift certain spatial frequencies. The easiest way to do this is to start with the mesh object in the object plane since it gives a nice FT pattern. Then put the aperture of choice onto a mount with an x and y translation stage. Move the translation stage in the horizontal direction x far enough that the beam emerging from the transform lens lands to the side of the aperture on the black metal surface. Push the entire mount in the z direction until the Fourier Transform pattern is sharp and crisp. Then lock down the mount and remove or change out the aperture in the FT plane leaving the mount in place.

2.6 Mesh Filtering Experiment

2.6.1 Experimental Arrangement

Put in the wire mesh aperture and check that its image is focussed on the screen. It should be perfectly in focus assuming you found the object plane correctly from before. Using the mesh to find the object plane is very tricky because the Fresnel diffraction pattern from the mesh can be mistaken for a focused image. Be careful you have the correct location for the mesh since the following measurements depend critically on this step. The easiest way to verify you have found the correct location is to translate the mesh off-axis (say in the $x-$direction perpendicular to the optical axis) so that you can see in your image of the mesh part of the circular aperture to which the mesh is fastened. Ensure the image of the edge of the aperture is well focused and centre the mesh afterwards. Note the positions of the two lenses and the object (mesh), so that they may be easily replaced when moved. Also note the screen position and the angle between the two rails so that you can compute the object and image distances from the lens.

From the number of wires per cm given on the mesh, and the spacing of the image wires on the screen, estimate the magnification of this system. **Does this magnification agree with the thin lens formula (Equation 2.1)?**

2.6.2 The Transform Plane

In this arrangement, the Fourier transform plane is the back focal plane (a distance f from the lens) of the transform lens (See figure 4.1). A white screen placed here allows the Fourier transform of the mesh to be seen easily and

clearly, even with the room lights on. It is an extensive and intricate pattern. Each point in the pattern represents a unique spatial frequency, and is therefore the focus of all parallel rays in the object space making a certain angle with the optic axis. **What is the effect of moving the mesh?** Move it in the x, y, and z directions and rotate it. **Comment. How would this be useful for optical scanning systems, such as UPC symbols at the grocery check-out?**

The transform may be enlarged and viewed on the secondary rail by inserting a converging lens between the rear focal plane and the screen. Adjust the position of the converging lens until the diffraction pattern is focused on the screen. In this case the screen on the secondary rail and the plane of the spatial filter pinhole are conjugate, as may be seen by removing the mesh. Off-axis points in the pattern correspond to differences in optical path length from repeating parts of the mesh to the screen by integral multiples of the wavelength.

Attach a sheet of black graph paper to the screen. Position the CCD camera such that the entire screen will be imaged. Turn off the lights, but leave the door open so that the white lines of the graph paper are still visible in the image. Take a picture of the magnified diffraction pattern (directions can be found in Appendix A). Now measure the spatial frequencies present in the transform.

Are these frequencies what you expect given the wire spacing of the object? You can compensate for any distortions in the image due to the camera lens by using the grid on the graph paper. The information given in Appendix B might be useful in completing the required analysis.

2.6.3 Spatial Filtering of Vertical Lines

Return the lenses and the mesh to the positions for which a magnified image of the mesh is seen on the screen. Insert the slit in a holder and place it in the transform plane. The diffraction spots in the vertical direction are produced by the vertical spacings between the horizontal wires of the mesh. By aligning the slit to allow only the central vertical lines of the Fourier transform to pass, nearly all of the information about the horizontal separation of the vertical wires is blocked out. **What does the resulting image look like?** Record your observations and **explain them in reference to the mathematical expression for the Fourier Transform.**

It is possible to confirm the optical Fourier transform through image processing of the mesh grid image. This can be done using Matlab or Octave. Octave can be found on any of the Optics Lab computers by typing 'octave' in a terminal window, while Matlab can be run from any physics.ubc.ca xterminal by typing 'matlab&'. This confirmation will be considered bonus, and extra marks will be given accordingly.

2.6.4 Spacial Filtering to Produce 45° Lines

Strange as it may seem, an image pattern containing lines at 45° to the vertical may be produced by rotating the slit-filter to the symmetric 45° position.

Verify this and take a picture. **Explain why this is the case.**

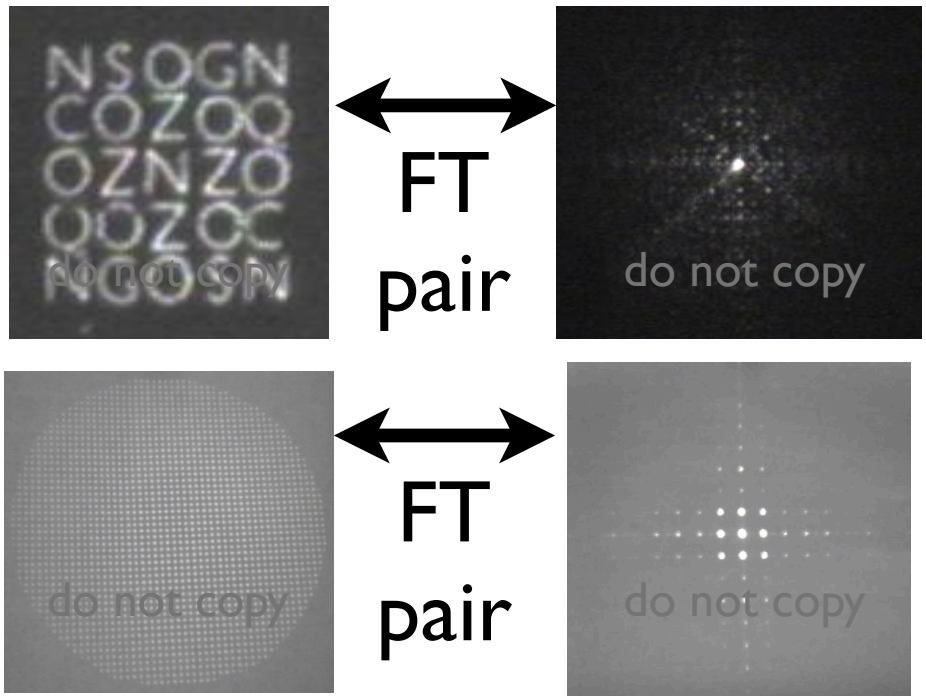


Figure 2.9: Images of the object-character array and mesh and their respective Fourier Transform images produced by a lens. These each form a "transform pair".

2.7 Dark-field Image

A "dark-field image" is obtained when the central (zeroth) order spot in the transform is blocked. The dark regions in the unfiltered image are a result of destructive interference of all the light coming from the object. If the DC (zero frequency) component is removed, complete destructive interference will not occur. Thus, previously dark regions of the image will appear bright. Since the high spatial frequency components required to define the edges of the apertures are still transmitted, the edges haven't lost any definition, so will appear more clearly. Here it is essential that the image be in focus. A difference in focussing of even a few millimetres can destroy the effect.

For this experiment, you should use a different aperture other than the wire mesh since the dark-field image of the mesh is somewhat difficult to interpret. Instead, you should use the aperture which consists of a razor blade with a wedge of clear glass glued to it. This aperture is used later in this lab to realize phase-contrast imaging. Place this aperture in the object plane (where the wire mesh was) and translate it until its image is clear and crisp. Now place the transparency with the black dot on it into the FT plane.

Describe your observations and take a picture for your lab book.

2.8 Character Recognition

2.8.1 Spatial Filtering to Transmit "N"s

Place the object-character array negative NOZON at the object position. Place a rotatable lazy-x spatial filter in the transform plane. This filter should transmit the N's if properly aligned. It should be in a holder with vertical and lateral adjustment. Orient the filter to transmit the N's but remove the O's and Z's. Most of the non-N letters will not be completely removed because of cross-correlation effects. If the filtering isn't perfect, your spatial filter might not be exactly at the Fourier Transform plane. The position of the filter can be optimized by observing the Fourier Transform pattern hitting the back side of the filter and adjusting the filter position to make the pattern as sharp as possible.

Take a picture. **How does this filtering work?**

2.8.2 Character Transform

The transform of these letters is a very detailed "snowflake" pattern and is shown in Fig. 2.9. As in Section 2.6.2, add a converging lens behind the transform lens and adjust its position until the transform is focused on the screen. Take a picture.

2.9 Phase-Contrast

The only spatial filtering discussed so far has involved altering the amplitude of the transmission function, by passing only selected spatial frequencies through the Fourier Transform plane. However, the phase of the light wave is also available at the Fourier Transform plane, and altering the phase of selected spatial frequencies using phase changing spatial filtering masks offers different possibilities for optical image manipulation and reconstruction.

Consider biological imaging applications where the objects you are trying to observe (say a protist swimming in water) are almost as transparent as the medium in which they are situated. In addition, different parts of the cell will typically have the same absorption coefficient and therefore result in the same output wave amplitude, but they have different indices of refraction. As a result, a substantial amount of information is encoded in the *phase* of the light. However, because photodetectors (like your eye) measure the intensity of the wave, this phase information is lost in a normal microscope. Phase contrast imaging is a technique by which spatial filtering is used to transform the phase variation across the wave into amplitude variations which can then be detected. The invention of this method was first applied in Phase-Contrast Microscopy, which earned Fritz Zernike a Nobel Prize.

Returning our attention to the protist, the index of refraction of this creature is close to that of water, so the transmission function will be approximately equal to that of water but with a small position dependent phase rotation (see figure 2.10A). A plane wave passing through the water will have the form e^{ikd} where d is the thickness of the water layer. This same plane wave will pick up an extra phase term from the modification of the transmission function due to the protist in the water. The complex amplitude of the plane wave passing through

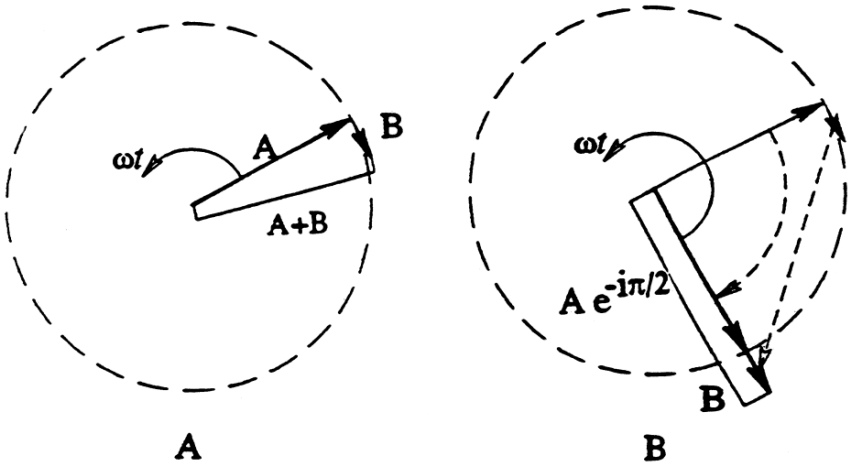


Figure 2.10: Phasor diagram illustrating the principle underlying phase contrast imaging. In the left image (A) we see the two parts of the field **A** and **B** adding. In the text, the field is $f(x,y) = 1 + i\theta(x,y)$, so here $\mathbf{A} = 1$ and $\mathbf{B} = i\theta$. Because one is real and the other imaginary, they are orthogonal and the length of the resulting vector is almost the same length as the original **A**. Therefore the contribution from **B** is hard to see. In the right image (B), The **A** component is first phase shifted by 90° and then added to **B**. In this case, they add in the same direction, and any changes in **B** produce a much larger change in the length of the resulting vector (the intensity of the field) than before when the two components were orthogonal. This increases the intensity contrast produced by the variation of the **B** component making it more visible. Phase contrast imaging is therefore especially useful for seeing clear objects, such as a fingerprint on glass, where the object encodes its presence into the *phase* of the light.

the protist will be $f(x,y) = e^{ikd}e^{i\theta(x,y)}$, where $\theta(x,y)$ is the position dependent additional phase shift induced by the protist body and its organelles. If we were to image this wave on a detector, the intensity profile, $I(x,y) \propto |f(x,y)|^2 = 1$, is completely uniform and we can't possibly see the creature like this. So, we need to implement phase contrast imaging to reveal the phase information. Let's see how this works.

Assuming the additional phase shift is small, $\theta(x,y) \ll 1$ (this assumption is not so important but makes the math less messy), the exponent can be approximated as $e^{i\theta} \simeq 1 + i\theta$. Dropping the overall phase of the plane wave in $f(x,y)$, we have

$$f(x,y) \simeq 1 + i\theta \tag{2.9}$$

$$|f(x,y)|^2 \simeq 1 + i\theta - i\theta - i^2\theta^2 = 1 + \theta^2 \tag{2.10}$$

Under this approximation, we can see that (again) there is no information showing up in the square of the field (intensity), since $\theta \ll 1$. So, what should we do? What would happen if we first changed the "1" in "$1 + i\theta$" to an "i"? This

would correspond to a 90° phase shift of the DC term (the "1") while leaving the spatially varying part (the θ) unchanged (See Figure 2.10B). In this case we have

$$|i + i\theta|^2 = -i^2 - i^2\theta - i^2\theta - i^2\theta^2 \quad (2.11)$$
$$= 1 + 2\theta + \theta^2. \quad (2.12)$$

Now the intensity has a 2θ term which is still smaller than the DC term but much larger than θ^2 and would give enough contrast against the flat illumination of the plane wave to see the protist. So, how do we affect this selective 90° phase shift of the DC term?

Since the DC term has no spatial variation, representing just a flat and featureless illumination from the plane wave, it will be focused to a point in the Fourier Transform plane and comprise the zeroth order of the diffraction pattern. This suggests a simple way to alter the phase of the uniform illumination, just insert a thin dielectric layer (film) at the central spot in the transform plane. A quarter wavelength phase change would produce the best results (turning the "1" into an "i"), which is what Zernike did, but clearly any phase change would help us distinguish between the object and the water. Commercial phase contrast microscopes often have an adjustable phase.

2.9.1 Experimental Arrangement

Place the transform lens back on the rail, and introduce, at the object position, a transparent phase grating consisting of parallel ridges and troughs ruled on a microscope slide. (See Figure 2.11.) The added path length in glass produces a small phase difference between the ridges and troughs. However, the image on the screen should be invisible, as this difference is very small. Ensure that the image is properly in focus at which point it should show no signs of the grating.

PLEASE DO NOT TOUCH THE GRATING SURFACE !

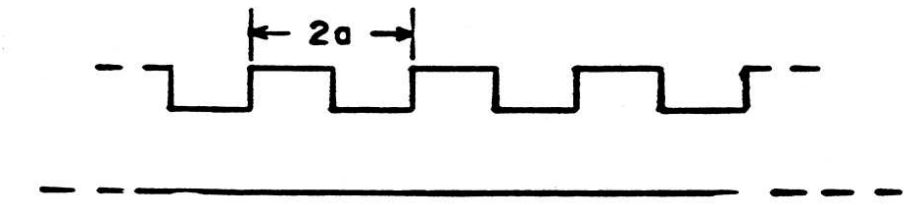


Figure 2.11: Cross-section of the transparent phase-grating.

Insert the vertical razor blade with a wedge of clear plastic taped to it. Adjust until the wedge covers just the central diffraction spot. The magnified image of the grating should be visible on the screen, with the ridges brighter than the troughs. Measure the grating spacing in the image. Compare this magnification with that found earlier in section 2.6.1.

Take a picture of the filtered image.

2.10 Diffraction

The term diffraction is generally understood to mean the propagation of light "around" the edges of obstacles or apertures. Diffraction phenomena are traditionally divided into two classes - Fresnel and Fraunhofer diffraction - although they are both manifestations of the same phenomenon. Fraunhofer diffraction refers to what happens to an optical pattern after propagation over a very large distance z in the paraxial approximation. Fraunhofer diffraction is the *far field* limiting case of Fresnel diffraction that arises when both source and observer are at very large distances from the diffracting screen such that the incident and diffracted waves are effectively plane. In this limit, the curvatures of the incident and diffracted waves can be safely neglected and the Fraunhofer diffraction pattern is just the Fourier transform of the initial pattern. If, however, either the source or receiving point is close to the diffracting aperture such that the wave front curvature is significant, then the result will be Fresnel diffraction [see Fig. 2.12 [2]].

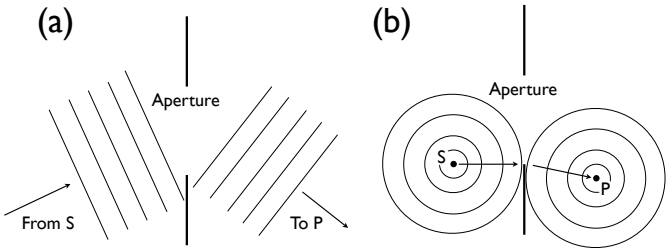


Figure 2.12: Diffraction by an aperture in the (a) Fraunhofer limit and (b) Fresnel limit

In order to be in the Fraunhofer limit, the distances between the source and diffracting aperture and between the diffracting aperture and receiving point should be large compared to a characteristic distance, R_0 given by;

$$R_0 = \frac{2\mathrm{w}^2}{\lambda} \tag{2.13}$$

where λ is the wavelength of the light, and w is a dimension of the aperture or obstacle. This relation for R_0 guarantees that the phase change across the width of the aperture is less than $\frac{\lambda}{4}$. Perfect Fraunhofer diffraction can only be achieved with collimated incident and diffracted beams.

2.10.1 Diffraction Pattern of a Slit

In this experiment, you will investigate the two (Fraunhofer and Fresnel) diffraction regimes by studying the diffraction pattern from a slit aperture of variable width. You should start by inserting the collimating lens after the spatial filter so that the beam wave fronts are parallel. Thus the source of illumination is effectively at infinity. Place the slit after the collimating lens and reflect the

[2] figure adapted from "Introduction to Modern Optics" by Grant R. Fowles

transmitted light with the secondary mirror onto the screen with the image visible by the CCD camera.

The diffraction pattern observed can be characterized by a dimensionless parameter, Δv, defined as;

$$\Delta v = \mathrm{w}\left(\frac{2}{R\lambda}\right)^{1/2} = \left(\frac{R_0}{R}\right)^{1/2} \tag{2.14}$$

where w is the width of the slit, and R is the slit-screen distance. Fresnel diffraction occurs for $\Delta v \geq 1$. Fraunhofer diffraction, the so called far-field approximation is characterized by $\Delta v \ll 1$. This condition can be established with either a large slit-screen distance as the name implies, or a narrow slit. In the lab, the slit width is the more convenient adjustment to vary the Δv value, since a large range of w is easily achieved with a turn of the micrometer. However, varying R is equally acceptable. For each measurement you make, you should compute your Δv value on the spot to insure that you are in the proper limit (recall that the wavelength of the He-Ne laser is 632.8 nm). Because the Fraunhofer diffraction pattern is essentially the Fourier Transform of the input wave, the Fraunhofer diffraction can also be observed at the focal plane of a converging lens (see Figure 2.13)

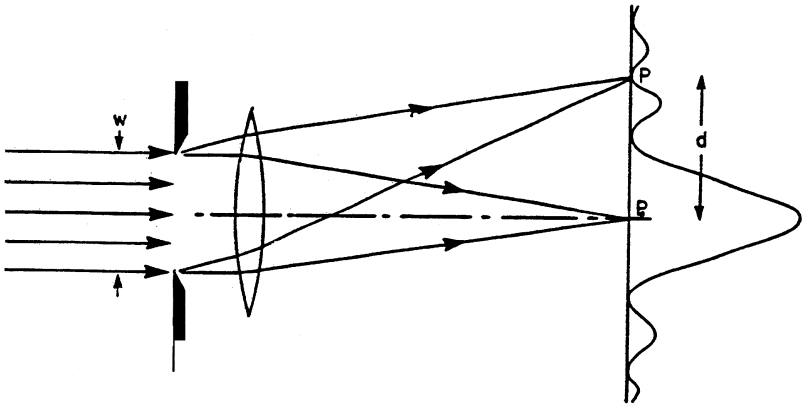


Figure 2.13: Fraunhofer diffraction pattern of a slit formed at the focal plane of a lens. In this part of the lab, you will generate your Fraunhofer patterns *without* the aid of a "Fourier Transform" lens placed after the aperture.

Start by adjusting the slit width to be very narrow until you see the characteristic Fraunhofer pattern. Record and reproduce in your lab write-up the image of the observed diffraction pattern. **Are the minima zero? Why or why not?**

Now slowly widen the slit width watching the diffraction pattern as you do. You should be able to see the transition from Fraunhofer to Fresnel diffraction, with the initial loss of the zeros, and subsequent increase of secondary maxima. Record the image of the Fresnel patterns at three interesting Δv's. For each measurement you make, you should compute your Δv value to insure that you are in the proper limit. Your images should look something like those in Fig. 2.14. Since the CCD has an auto gain circuit, it may help to turn on the

room lights so that the patterns are visible and not completely saturated. Your analysis will be done by taking a line out of the image, so as long as there is a part of the image that is not saturated, you can extract the data. Because the CCD has a limited dynamic range, the image may not show clearly the low intensity wings of the diffraction pattern that will be obvious to you by eye.

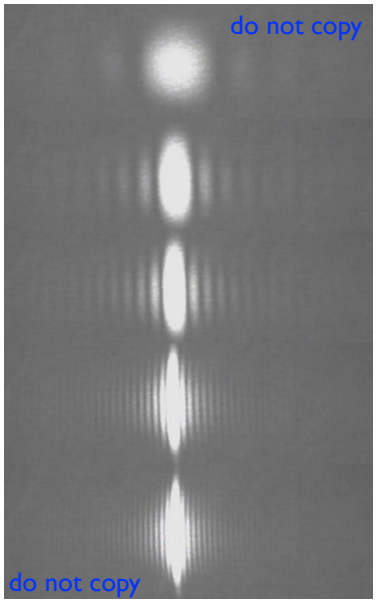


Figure 2.14: Diffraction patterns from a slit of various widths.

2.10.2 Analysis of the slit diffraction

Take your images and import them into an array in MATLAB, maple, mathematica, octave, or your favorite math tool. Plot the intensity as a function of position along an axis perpendicular to the slit axis (let's call that the x-axis, and this axis is horizontal in the images shown in Fig. 2.14). To improve the signal to noise of your data, you may need to integrate (add pixel intensities) the image along the y direction. Since there might be some distortion of the image along the vertical (y) direction (as seen in Fig. 2.14), you should probably only sum up the 10 adjacent pixels along a vertical line for each sample along x. Be sure that you choose an area of the image that is not saturated. In Fig. 2.14, this area would correspond to the top of the images above the point where the central spot is completely saturated. The problem of saturation is probably most apparent in the lowest image where the interference fringe spacing is the smallest. Calculate your values of Δv, and compare your diffraction patterns to theory by plotting your experimentally measured data against the predicted diffraction patterns at the corresponding Δv. For this you should numerically evaluate the form of the diffraction intensity using Matlab or Mathematica using the theory presented.

It is convenient to represent the intensity of the diffraction pattern at a

distance, d from the optic axis in terms of the dimensionless parameter,

$$z = \frac{d}{\mathrm{w}} \tag{2.15}$$

Thus the edges of the geometric shadow correspond to $z = \pm 0.5$.

In the Fraunhofer limit ($\Delta v \ll 1$), the intensity variation in the diffraction pattern is given by the Fourier transform of a square function,

$$I(z) \propto (\Delta v)^2 \sin^2 \beta/\beta^2 \tag{2.16}$$

where $\beta = (z\pi/2)(\Delta v)^2$. The pattern has a global maximum at $\beta = 0$, with subsidiary maxima on both sides. These maxima are separated by zeroes in the intensity for

$$\pm\beta = n\pi \text{ or } \pm z = \frac{2n}{(\Delta v)^2} \quad n = 1, 2, 3, \ldots$$

Notice that the intensity maximum is proportional to $(\Delta v)^2$ and therefore to w^2, while the width of the central maximum is proportional to $(\Delta v)^{-2}$ or to w^{-2}.

For Fresnel diffraction, ($\Delta v > 1$), the general expression for the intensity is given by;

$$I(z) \propto \Big[C^2(v) + S^2(v)\Big]_{v_1}^{v_2} \tag{2.17}$$

where $C(v)$ and $S(v)$ are the Fresnel cosine and sine integrals, respectively, given by

$$C(v) = \int_{v_1}^{v_2} \cos(\frac{\pi x^2}{2})\mathrm{d}x$$

and

$$S(v) = \int_{v_1}^{v_2} \sin(\frac{\pi x^2}{2})\mathrm{d}x$$

The limits on the variable of integration are

$$v_1 = -(z + 0.5)\Delta v \qquad \text{and} \qquad v_2 = -(z - 0.5)\Delta v$$

In the limit of small Δv, equation 2.17 reduces to equation 2.16.

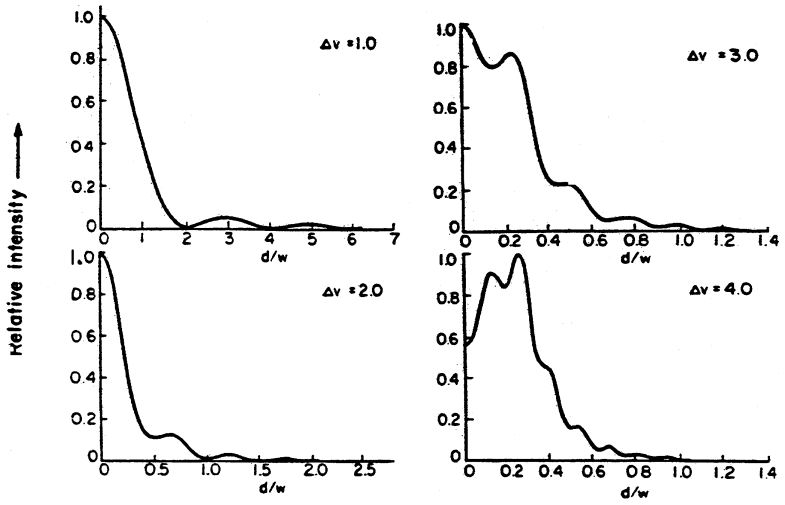


Figure 2.15: Line intensities of Fresnel diffraction patterns of a slit for various values of the parameter Δv. Only one half of the symmetrical pattern is shown.

Chapter 3

The Michelson Interferometer

3.1 Prelab

In this lab you have to find the position of mirror M1 (see Fig. 3.1) such that the optical path length from the beam splitter to M1 and back is identical to the optical path length from the beam splitter to M2 and back again. This special position insures the path length difference is zero and is referred to as the "zero path length" ZPL. As the position of M1 approaches the position of ZPL, you will observe that the periodicity of the circular interference fringes (which are circular for the lamps - sodium and the white light) becomes larger and they are the largest at the ZPL (see Fig. 3.2). Explain qualitatively why this is. Next, model this situation numerically. Derive an expression for the interference pattern as a function of z from two spherical waves where the sources are separated on the z-axis by a distance d apart (representing the two different beam paths in the interferometer). *Hint: write down the electric field amplitudes (U_1 and U_2) from two spherical wave sources and compute the total intensity* $I = |U_1 + U_2|^2$. Plot the total intensity (i.e. interference pattern) for two different separations d. Comment on the change of the ring periodicity as d gets smaller. Also, for a given d comment on the ring periodicity as you move radially away from the z-axis along x or y.

In this lab, you will also use the interferometer to measure the index of refraction of air by putting a vacuum cell in between the beamsplitter and mirror M2 and measuring the additional phase shift acquired when you slowly introduce air into the evacuated cell. How many times does the beam traverse the cell in the interferometer? Compute the total optical path length difference for light at 632.8 nm with and without air in a vacuum cell 10 cm in length. What is the expected number of fringes that will pass as air is leaked into a cell of this length initially under a perfect vacuum?

Also, before doing this lab, you should prepare a **MATLAB** `.m` file which performs the analysis outlined in section 3.10.2. The code for this **MATLAB** script is spelled out in this section, and you should test it with the sample Fourier spectroscopy data (of a white light spectrum) provided on the optics lab webpage next to the link to the `.pdf` of this document. The sample data is

given in both `.csv` format and as a saved **MATLAB** workspace.

3.2 Introduction

A well known interferometer, operating on the principle of division-of-amplitude, is the Michelson interferometer. The incident light beam is divided into two parts by means of a beam-splitter. The divided beams traverse different paths and then are recombined. Depending on the difference in optical path lengths of the two beams, the recombined beams may be 180° out of phase, producing destructive interference, or in phase, producing constructive interference. Observation of these interference fringes, with changing optical path difference allows one to determine wavelengths, wavelength differences, and/or small differences between the two optical paths.

Use of this interferometer as a spectrometer to measure the power spectrum of an emitting source became feasible with the advent of high speed, large storage capacity, computers which could calculate the Fourier transform of the output irradiance of the interferometer. You will use this instrument to realize such a Fourier transform spectrometer.

3.3 The Michelson Interferometer

Fig. 3.1 illustrates the physical arrangement of the Michelson interferometer. The beam-splitter, oriented at 45° to the incoming light, is partially metallized on one surface reflecting half of the beam to mirror M1, and transmitting the other half of the beam to mirror M2. After reflection from these plane mirrors the two beams are recombined at the beam-splitter, and part of this light passes out of the interferometer towards the CCD camera. Mirror M1 is mounted on a computer controlled stage which can be moved via software or the jog/toggle buttons on the DC servo controller. **You will notice there is a second piece of glass in one arm of the interferometer. What is the purpose of this object?**

3.4 Finding the Fringes

Be sure that the vacuum cell is **not** in the interferometer for this part. Place the sodium lamp at the input of the interferometer and make sure the diffusing screen is in front of it (see Fig. 3.1). Note that the lamp will take a few minutes to warm up. The output of the interferometer will be examined using the CCD camera. In this case use the CCD camera with its lens assembly installed. Also, you should reset the translation stage zero so that your measurements are properly referenced. Open up the **APTUser** program on the desktop. Using the *Settings* button on the GUI make sure the max velocity is 0.5 mm/s for both the move and jog. Next, push the *Home/Zero* button to zero the stage. The stage will move to the end of its travel and the computer will note this as the home or zero position. The interferometer has been set so that the position of zero path difference falls near the centre of the range of motion of the computer-controlled stage that M1 sits on (you should ask the TA about where it might be since the exact position depends on the setup and changes from week to week

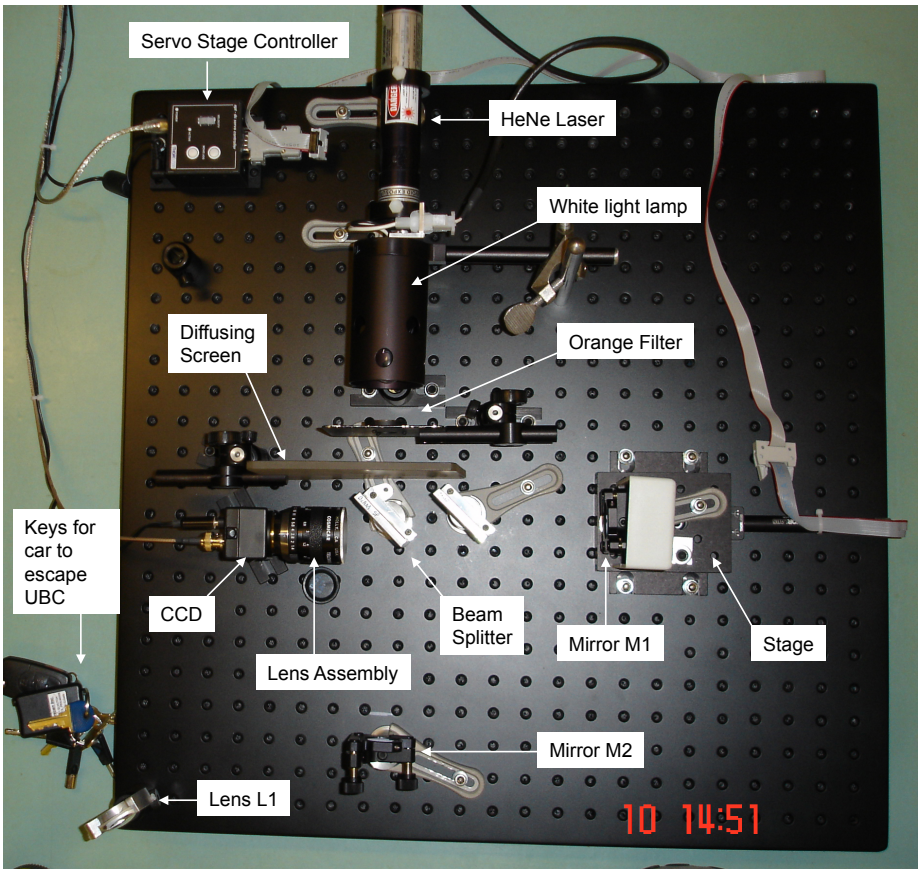


Figure 3.1: The Michelson Interferometer. Note, the sodium lamp is not pictured

due to misalignments). Position the stage near the center of the range (around 10 to 12 mm). Then align the mirrors by adjusting the two knobs of M2 and making the two images of the focussing pin coincide. Fringes should appear on the monitor. You will need to make sure the CCD camera is not saturated by adjusting the aperture on the lens assembly. Fringes of different shapes may be produced by appropriate adjustment of the mirror M2.

3.4.1 Circular Fringes

When the mirrors are effectively parallel to each other (which means they are actually perpendicular since the arms of the interferometer are perpendicular - see Fig. 3.2) circular fringes are produced (see Fig. 3.3). These fringes, normally formed at infinity, may be focussed on a screen (or onto the CCD camera chip) with the use of a lens, as shown in Fig. 3.2.

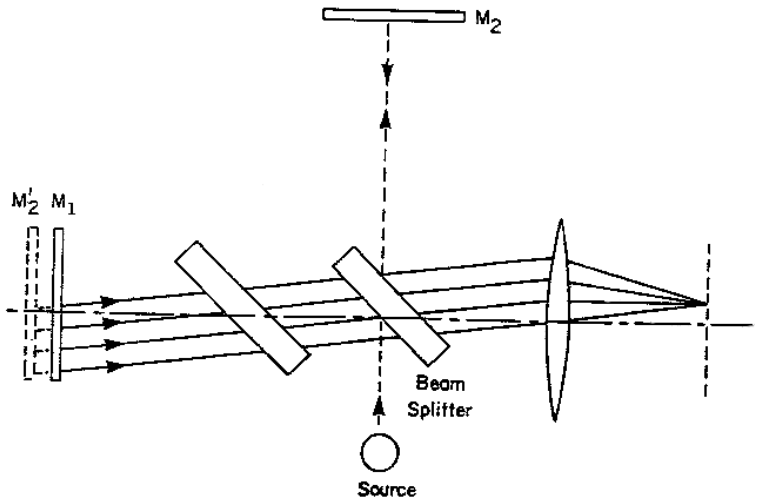


Figure 3.2: Formation of circular fringes. M_2' indicates the effective position of the mirror M_2. When both mirrors are effectively parallel to each other but at slightly different positions, the optical path lengths are different and circular fringes occur. These fringes are focused with a lens on to the CCD camera chip.

3.4.2 Straight Fringes

If the mirrors are slowly angled so that they are not effectively parallel to each other, the centre of the circular fringes moves away and the fringes will appear like a series of curved lines (the edges of a set of concentric circles). When the path length is set to be very near the ZPL, straight line fringes, or "fringes of equal thickness" occur (see Fig. 3.3). They are identical to the fringes formed by an air wedge; the smaller the wedge angle, the broader the fringes.

3.4.3 Changing from One Type of Fringe to Another

When the interferometer is set up, it is unlikely that the mirrors will be aligned or that stage will be already close to the zero path length. Therefore the fringes may be either horizontal or vertical, or anything in between. By adjusting the mirrors the fringes may be changed from one form to another. As the wedge angle between the mirrors is decreased, the fringes become more curved until circular fringes (theoretically) appear.

Find the fringes described above, and practice forming straight and circular fringes until fringes of a specific kind can be produced at will. Note that perfectly straight fringes will only occur right at the ZPL. In practice, of course, the fringes may also be distorted due to imperfections in the flatness of the mirrors. In fact, Michelson and other amplitude division interferometers are used in industrial fabrication plants exactly for this purpose - to verify the flatness or precise curvature of mirrors. Using the **WinTV2000** program on the desktop, record some images of various fringe patterns for your lab book. The GUI button *SNAP* will grab a frame for you. Right click on the captured image to save it

to a file.

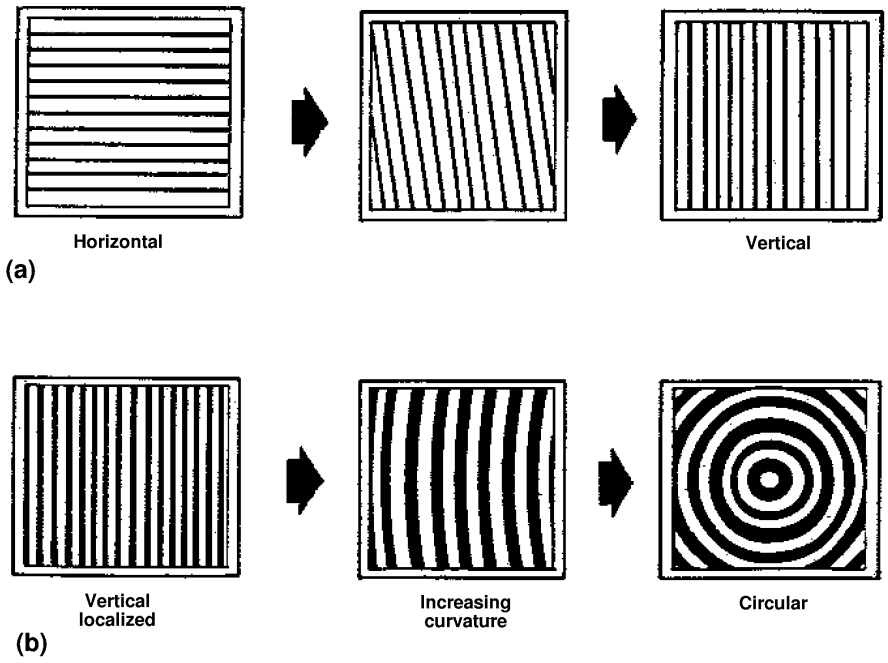


Figure 3.3: (a) Straight fringes and (b) circular fringes which occur when the misalignment of the mirrors is corrected so that they are effectively parallel (see Fig. 3.2).

3.5 The Position Of Zero Path Difference

For some applications in this lab, it will be necessary to locate the position of zero path difference. The following section describes how this is done.

3.5.1 Procedure

Be sure that the vacuum cell is **not** in the interferometer for this part.

If you haven't already done so, you should reset the stage zero so that your measurements are properly referenced. It will take some careful adjustment to find the exact position for the translation stage to achieve a zero path length difference. Therefore, start by setting up the sodium lamp and diffuser and search for the location with the strongest (highest contrast) fringes possible. Do your best to insure the fringes are circular (nicely concentric), well spaced, and clearly defined during your search.

Actually, you will find that there is a series of stage positions where the contrast is high then low and then high again (see Fig. 3.6). Take note of the exact stage positions where the contrast is minimum (or disappears) near the

stage position where the contrast appears best. You will next look systematically and precisely between each of these minima for the ZPL using the white light source.

Another sign that you are near the zero path length difference is that the radius of curvature of the circular fringes is the largest. **Why is that?** At this location, you should be within a fraction of one millimeter from the zero path length difference. To look for changes of the radius of curvature as you move the stage position, you could also look at the curvature of the straight fringes with the mirrors slightly angled.

Be very careful not to bump the equipment at this stage since you can easily knock the system out of alignment and ruin your pre-alignment work!

Now, replace the sodium lamp with the white light source. Note that the Sodium lamp will be hot, so only grab it by the post. You will notice that fringes do not appear. When using the white light source, they will only appear when the arms of the interferometer have equal path length. **Why? What does this tell you about the "coherence length" of the white light source? What is the definition of the "coherence length?"** You must adjust the stage until the path lengths are equal and fringes are observed.

Since finding the white light fringes is like looking for a needle in a haystack, you should cheat by using a narrow band filter with the white light source to find fringes with filtered light and then remove the filter to find the zero path length difference with the white light. **Why would this filter help?** With the narrow band filter in place, slowly scan the stage with the jog buttons to the positions precisely between the minima you found before. When you finally try the correct maxima, the stage will be near the ZPL and fringes will be observed. If you are spending more than 15 minutes without finding the white light fringes, ask a TA to help check you are doing everything correctly. At the point where the fringes appear with the unfiltered white light source, the stage is set for zero path length difference. Write down the stage position for future reference and note the direction you approached the zero-path difference. The latter is important for mechanical backlash in the micrometer screw. Since you will be using a **MATLAB** script to take data, and this script moves the stage forward (i.e. to higher values on the indexed stage), then backwards, then forward again back to the starting point, you should always approach from below your position of zero-path length difference. The fringes will appear and disappear within 2 μm, so you will probably have to adjust the max velocity and step distance of the stage so that you don't go too fast and miss the appearance of the fringes. Also, use the actuator toggle with care since rapid changes in motion of the stage might also misalign the interferometer.

3.6 Calibration Of The Interferometer

Before the interferometer can be used to measure wavelengths it must first be calibrated. By "calibrated" we mean that the distance which the stage actually moves needs to be correlated to the approximate reading provided by the computer.

3.6.1 Procedure

The distance by which the stage travels is found by counting the number of fringes which pass given a certain mirror movement and given a light source whose wavelength is well known. The Helium-Neon Laser (λ =632.8nm) is ideal for this purpose. Slide the lamp up and rotate the diffusing screen and orange filter out of beam path of the laser. When using the laser, place an imaging lens (L1) in the post just after the output of the laser. You should remove the lens assembly on the CCD camera. By adjusting the height and tilt of the imaging lens in the post holder, locate the the main spot of the HeNe laser on the backplane of the CCD camera. Temporarily put a piece of paper just in front of the CCD and look at the pattern cast on this paper. Align the mirror (M2) to see fringes. Now take out the paper and use the monitor to view the CCD output, and further adjust mirror M2 so that about five vertical straight fringes fill the image on the CCD camera (see Fig. 3.3).

Using the monitor to view the CCD output, adjust mirror M2 so that about five vertical straight fringes fill the image on the CCD camera (see Fig. 3.3). You should adjust the camera aperture and attach some neutral density filters to the CCD camera so that it isn't saturated.

The overall plan is to move the stage a certain distance and count the number of fringes that pass. You should therefore take the data, plot it, and count the fringes on the plot. The mirror moves a distance of $\lambda/2$ as one fringe passes a reference point on the CCD Camera. Thus, if N fringes pass through as the mirror moves a distance d, we have,

$$2d = N\lambda \tag{3.1}$$

If the servo controller readings change by ΔD from D_1 to D_2, then the mirror travel is linearly related to the stage displacement through the calibration constant K:

$$d = K\Delta D = K \mid D_2 - D_1 \mid \tag{3.2}$$

To analyze the fringe movement (i.e. to count the fringes), you will use **MATLAB** to control the translation stage (i.e. mirror) position AND to detect the variation of the intensity on a given pixel in the fringe pattern as the fringe pattern moves. Several helpful **MATLAB** files are located in the desktop folder "Michelson files". At this point you should NOT be running the **WinTV2000** program as it will not allow **MATLAB** to access the video card. By reading the comments of the M-files you will be able to tell what each of their functions are.

In order to take data for this section, you will be using the **MATLAB** script called "michelson_timestream.m". You will need to quit the APT user program when you use the "michelson_timestream" file since **MATLAB** will open APT user again with the correct parameters. When this script is executed, it will prompt you to click on the image the location where the fringe pattern is the most visible. The script will then move the stage back by a distance (`span`) specified in mm, then it will move the stage forward by twice that distance (`2 x span`) at a slow velocity (`speed`) specified in mm/s, and then it will move the stage back to the starting point. To use this script, your code will look like this

```
>> data = michelson_timestream(span, speed);
```

where `span` and `speed` are to be chosen by you, and the data will be returned into the array `data`. As an example, consider this code

```
>> data = michelson_timestream(0.01, 0.001);
```

This will move the stage back by 10 μm and then forward by 20 μm at a speed of 1 μm/s (which will take 20 seconds), and then back 20 μm to the starting point. The data will be returned into the array `data` which will have the intensity on the pixel you selected for each frame of the video. Since the video frame rate is 30 frames per second, this means the stage will have moved $\frac{1\ \mu m/s}{30\ \text{frames}/s} = \frac{33\ \text{nm}}{\text{frame}}$.

For a given amount of distance moved by the stage, determine the number of fringes that passed through the reference point (region of interest,ROI) and then calculate the calibration constant K. Repeat this measurement as many times as necessary to obtain consistent results for K and estimate its accuracy.

3.7 Measuring the index of refraction of air

Now that the constant K has been determined, you can use this interferometer to characterize extremely small changes in the optical path length differences between the two arms. You can use this sensitivity to even measure the phase shift induced by the propagation of light through air. You will measure the phase change by counting fringes as air is slowly introduced into one of the interferometer arms. Using the vacuum pump and glass vacuum cell provided, insert the glass vacuum cell between the beam-splitter and M2 and measure the index of refraction of air at 632.8 nm (i.e. with the HeNe laser).

For your measurements, you can use the `vid_capture` **MATLAB** script to capture the intensity on a small area in the CCD image as a function of time. Your code will look like this "`>> d = vid_capture(time);`" where you choose `time` appropriately (try 10 seconds).

To do your measurement, you should first prepare a vacuum in the cell: close the "leak valve" and open the "pump valve" and turn on the vacuum pump. After the pressure in the cell (as read by the meter) has dropped to the minimum on the scale, close the "pump valve" and turn off the pump. Start the data acquisition and slowly open the "leak valve" to let in air while you count fringes. You may need some practice introducing the air fast enough that the pressure rises to atmospheric pressure during the `time` you have set while not rising too quickly so that you miss the passage of fringes due to the finite sampling time of the video capture. Be careful to insert the glass vacuum cell without hitting or misaligning the mirrors (see Fig. 3.4).

Would you have counted the same number of fringes if you had used the Na lamp instead of the HeNe laser for this measurement? Why or why not? What is your value for the index of refraction of air for 632.8 nm light? Does this agree with the "accepted" value? What are your sources of error? Could you use this apparatus as a barometer or thermometer?

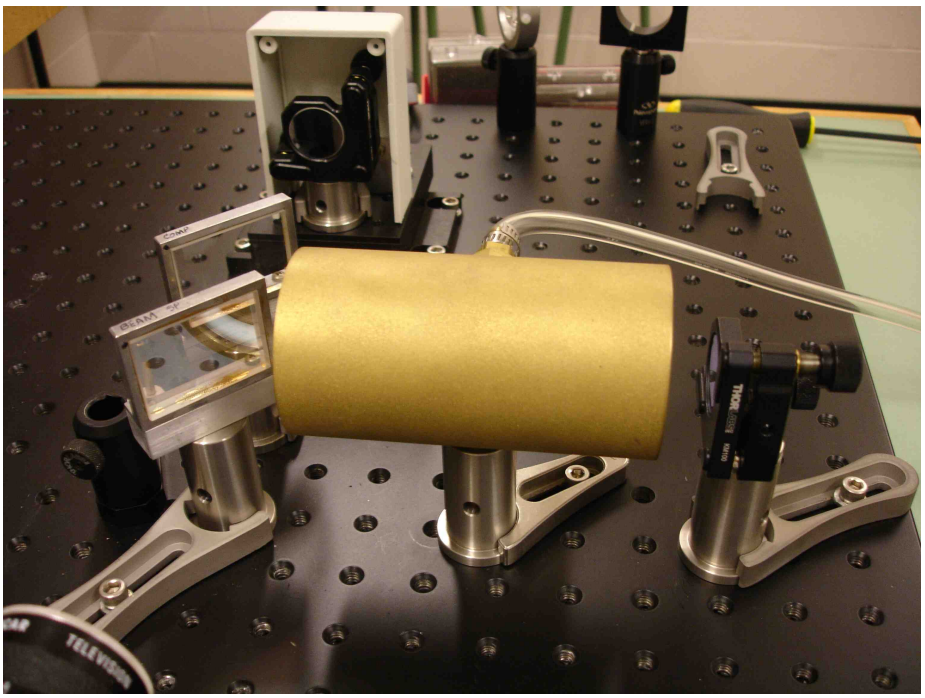


Figure 3.4: Vacuum vessel inserted into M2 arm of interferometer.

3.8 Determination Of Wavelengths

Be sure that the vacuum cell is **not** in the interferometer for this part.

Also, knowing the calibration constant K, an unknown wavelength may be measured by the same procedure; i.e., by noting the stage readings for the passage of a certain number of fringes. Carry out this measurement for the sodium source to get the average wavelength for the two Na D1 and D2 lines. You should use the same span and speed as the previous section. The fringes for these two lines coincide at the position of zero path difference and the measurement should be made near this position. On either side, the fringes become indistinct as they start to overlap. This feature will be studied in the next section.

3.9 Wavelength Differences

In 1887 Michelson and Morley, while studying the "visibility of fringes" in Michelson's interferometer, observed that the H2 line ($n = 3 \rightarrow n = 2$) is not one line but actually a doublet with about $\tilde{\nu} = 0.33$ cm^{-1} separation. The wavenumber, $\tilde{\nu}$, is given in units of inverse centimeters and is the spatial analog of frequency. One can convert to frequency by multiplication with the speed of light, $f = c\tilde{\nu}$. The wavenumber of electromagnetic radiation is defined as $\tilde{\nu} = 1/\lambda$, where λ is the wavelength in vacuum.

In this part of the lab, you will determine the separation between the D1 and D2 lines in sodium using the Na lamp. The fringes from any two lines

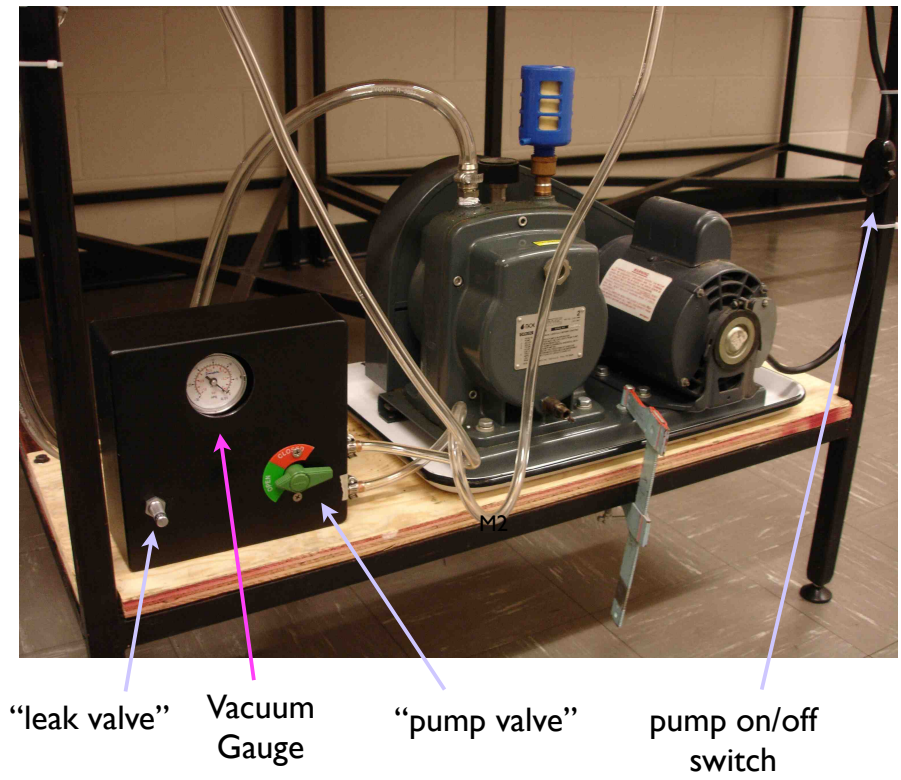


Figure 3.5: Vacuum pump, valves, and vacuum gauge for vacuum cell.

(wavelengths λ_1 and λ_2), such as the Na D1 and D2 lines, always overlap at the position of zero path difference. If the stage is moved away from this position, either forwards or backwards, then a position will be reached (at a distance d) where the bright fringes of one line fall on the dark fringes of the other. The resulting fringe pattern is then very indistinct − the fringes are said to have low visibility. This particular distance is

$$2d = (N+1)\lambda_1 = N\lambda_2 \qquad (\lambda_1 < \lambda_2) \tag{3.3}$$

where N is an integer. The wavelength difference is, therefore;

$$\Delta\lambda = \lambda_2 - \lambda_1 = 2d\left(\frac{1}{N} - \frac{1}{N+1}\right) = \frac{\lambda_1\lambda_2}{2d} \tag{3.4}$$

If λ_1 and λ_2 differ by very little, it is sufficient to replace their product by λ^2, where λ is the average of the two. The expected value of $\Delta\lambda$ ∼0.6nm.

3.9.1 Procedure

Be sure that the vacuum cell is **not** in the interferometer for this part.

Set up a white light source and locate the white light fringes as you did previously. This will locate the stage at zero path difference. Recall that due to backlash the stage position will differ if you approach this position from above

versus from below. Since you will eventually be using a **MATLAB** script to take data, and this script moves the stage forward (i.e. to higher values on the indexed stage), then backwards, then forward again back to the starting point, you should always approach from below your position of zero-path length difference.

Replace the white lamp with the sodium lamp. Now move the stage using the toggle lever on the controller, move the stage over a distance of at least 2 mm noting the stage readings at several positions of minimum visibility. When recording the positions, you should always move the stage in same direction to avoid backlash errors. Repeat this process several times and average the stage readings. You can do this manually or you can use the script from before. Your code will again look like this

```
>> d = michelson_timestream(span, speed);
```

where you choose `span` and `speed` appropriately. A span = 1.0 and speed = 0.01 are probably good starting values for this part.

From the average value of ΔD obtained, and the measured calibration constant, K, calculate d from equation 3.2, and hence obtain a value for $\Delta\lambda$ from equation 3.4. Use the average value of the two Na D1 and D2 lines that you determined above and find the difference between the two lines.

3.10 Fourier Transform Spectroscopy

So far you have investigated the behavior of the interference fringe pattern as a function of the optical path difference for different optical sources. In general the fringe pattern intensity versus the optical path difference (or equivalently versus the time delay between the two interfering beams) is related to the *power spectrum* of the light entering the amplitude division interferometer by a Fourier transform. Therefore one can easily measure the optical power spectrum using the measurement of the intensity variation as a function of stage position.

Let's see how this works [1]. The electric field incident on the camera is the sum of the electric fields $\mathbf{E}_1$ and $\mathbf{E}_2$ arriving from M1 and M2 respectively after having passed through the beam splitter. Assuming for the moment that the input source is a purely *monochromatic* wave, we can characterize these electric fields with a vector amplitude (encoding the strength and polarization of the wave) and a spatial and time dependent phase:

$$\mathbf{E}_1 = \mathbf{A}_1 e^{i(\mathbf{k}_1\cdot\mathbf{r}-\omega t+\phi_1)} \tag{3.5}$$

$$\mathbf{E}_2 = \mathbf{A}_2 e^{i(\mathbf{k}_2\cdot\mathbf{r}-\omega t+\phi_2)}. \tag{3.6}$$

The phase includes terms ($\phi_1 = |\mathbf{k}_1|\, l_1$ and $\phi_2 = |\mathbf{k}_2|\, l_2$) which depend on the optical path lengths from the beamsplitter along path l_1 (encountering M1) and path l_2 (encountering M2). The intensity on the camera is simply the square of the total electric field:

$$I = |\mathbf{E}|^2 = \mathbf{E}\cdot\mathbf{E}^* = (\mathbf{E}_1+\mathbf{E}_2)\cdot(\mathbf{E}_1^*+\mathbf{E}_2^*) \tag{3.7}$$

$$= |\mathbf{A}_1|^2 + |\mathbf{A}_2|^2 + 2\mathbf{A}_1\cdot\mathbf{A}_2\cos\theta \tag{3.8}$$

$$= I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\theta \tag{3.9}$$

[1] This derivation follows that in "Introduction to Modern Optics" by Grant R. Fowles

where we have assumed in the last step that the waves have the same polarization and that $\theta = (\mathbf{k}_1 - \mathbf{k}_2) \cdot \mathbf{r} + \phi_1 - \phi_2$. You might be wondering about a white light source which is probably completely unpolarized. Is this assumption still valid in that case? This assumption is justified since the two fields $\mathbf{E}_1$ and $\mathbf{E}_2$ are copies of the incident field generated by the beam splitter. As long as the beam splitter and the optics which follow preserve the polarization of the light, these two fields will arrive at the detector with exactly the same polarization (whatever it was at the input of the interferometer). **What could the beam splitter and following optics do to change the polarization of the input light?**

If the interferometer is well aligned so that the waves are co-linear ($\mathbf{k}_1 = \mathbf{k}_2$) and the 50/50 beamsplitter generates two waves of the same intensity ($I_1 = I_2 = I/2$), then the interference pattern is simply

$$I(x) = I(1 + \cos kx) \tag{3.10}$$

where $x = l_1 - l_2$ is the path length difference and $k = |\mathbf{k}_1| = 2\pi/\lambda$ is the magnitude of the wavevector.

Okay, now in general, the light illuminating the interferometer will be composed of *many* different frequencies. So, let's model the power spectrum of the incident light as a continuous distribution, $G(\omega)$. Then the total optical power emitted into the interferometer in the frequency range from ω_0 to $\omega_0 + d\omega$ (where $d\omega$ is an infinitesimally small frequency band) is simply $G(\omega_0)d\omega$. For convenience, we can instead use the distribution over wavelength $G(k)$ (where $\omega = ck$). Then the optical power, P, hitting the detector from illuminating the interferometer with this polychromatic source will be the sum of the interference patterns from each monochromatic wave composing $G(k)$. Assuming the interferometer is ideal and has no losses, we have then:

$$\begin{aligned} P(x) &= \int_0^\infty G(k)\,(1 + \cos kx)\,dk & (3.11)\\ &= \int_0^\infty G(k)\,dk + \int_0^\infty G(k)\frac{e^{ikx} + e^{-ikx}}{2}\,dk & (3.12)\\ &= \frac{1}{2}P(0) + \frac{1}{2}\int_{-\infty}^\infty G(k)e^{ikx}\,dk & (3.13) \end{aligned}$$

where $P(0)$ is simply the power measured at zero path length difference. We can rearrange this expression and define the power function $W(x)$

$$W(x) = 2P(x) - P(0) = \int_{-\infty}^\infty G(k)e^{ikx}\,dk. \tag{3.14}$$

We see then that the power function $W(x)$ is the *inverse* Fourier transform of the power spectral density $G(k)$. Inverting this expression we have that the power spectrum is the Fourier transform of the power function

$$G(k) = \frac{1}{2\pi}\int_{-\infty}^\infty W(x)e^{-ikx}\,dx, \tag{3.15}$$

or equivalently

$$G(\nu) = \frac{1}{2\pi}\int_{-\infty}^\infty W(x)e^{-i2\pi\nu x/c}\,dx. \tag{3.16}$$

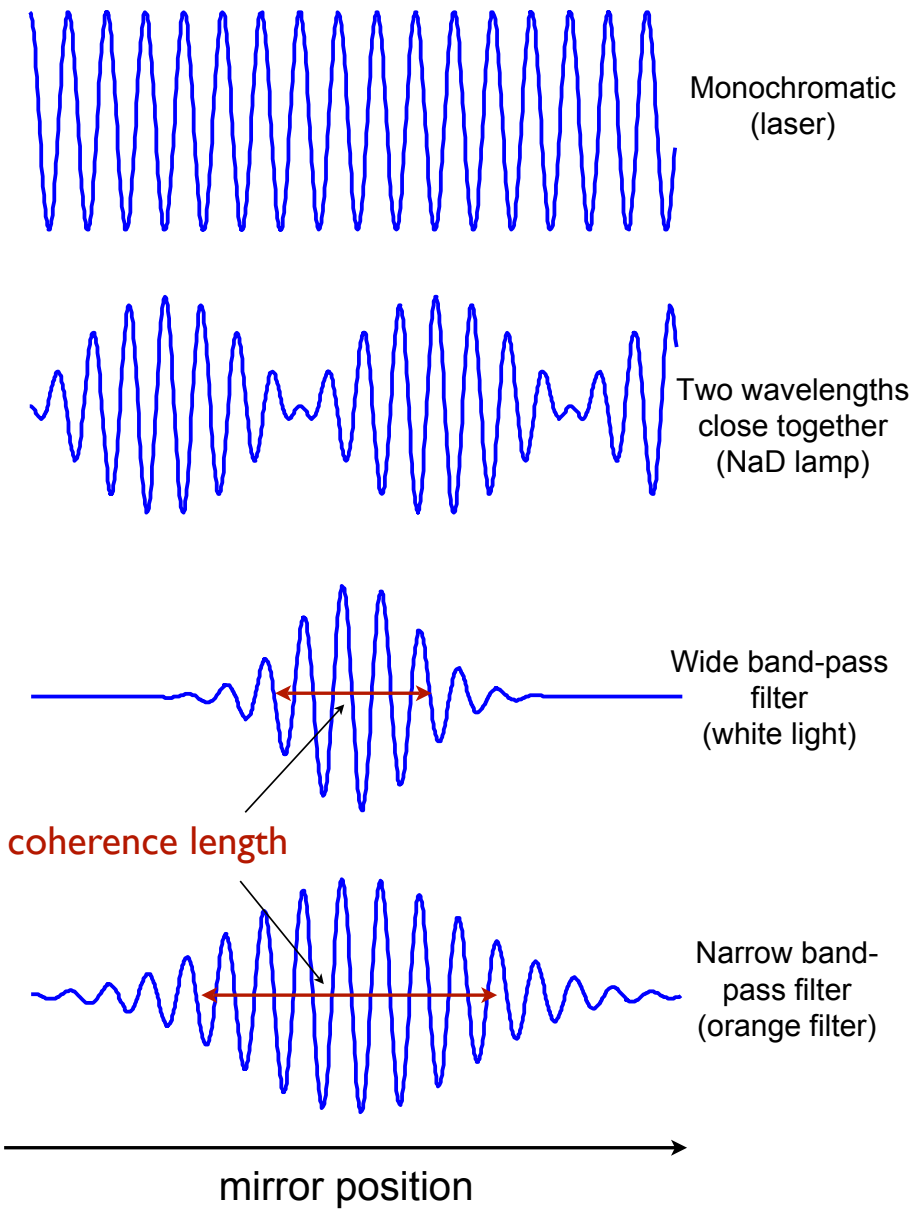


Figure 3.6: Fringe patterns plotted as a function of the mirror position. Notice how the fringe visibility collapses and then revives as a function of the mirror position when the spectrum is discrete. Also, note that if the mirror is moved a distance d, the optical path length between the two arms in the interferometer change by $2d$.

3.10.1 Procedure

Be sure that the vacuum cell is **not** in the interferometer for this part.

In this stage of the experiment, you will use the interferometer to realize

a Fourier transform spectrometer. The idea is to translate the stage around the zero path length position (ZPL), record the fringe pattern, and use it to determine the power function $W(x)$ and then compute numerically the corresponding spectral density $G(k)$ of various optical sources. To do this, you will (as before) use the **MATLAB** script "michelson_timestream.m" which records an array of power values for each video frame (i.e. each stage position). You will use your calibration constant for the motion, K, to deduce the stage position at each frame and therefore generate the power function $W(x)$ (be sure the stage velocities are set the same or else recalibrate K for this part). From this you will numerically compute the Fourier transform to determine the power spectrum of the light.

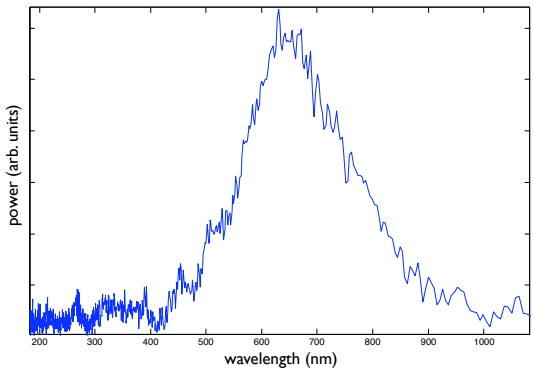


Figure 3.7: Example spectrum from the incandescent bulb measured by the Michelson Fourier transform spectrometer. Your reference white light spectrum should look something like this one. Keep in mind that this measurement of the spectrum is ultimately limited by the wavelength dependent transmission of the optical elements in the Michelson interferometer and by the sensitivity of the CCD detector. See Fig. 3.8 for a typical response curve of a silicon CCD image sensor.

As before, set up a white-light source, find the fringes at zero path length difference, make them vertical, and note the stage setting. Before taking each data run, you should carefully adjust the iris on the camera lens to maximize the fringe brightness without saturating the camera. This will optimize the signal to noise and produce much better data. Start by recording the fringe pattern and computing the spectrum of the white light source. You should get a spectrum similar to that shown in Fig. 3.7. This is your reference spectrum which you will be modifying with a filter. Keep in mind that this measurement of the white light source spectrum is ultimately limited by the transmission of the optical elements in the Michelson interferometer and by the sensitivity of the CCD detector. See Fig. 3.8 for a typical response curve of a silicon CCD image sensor.

To take the data for this section, you will again be using the **MATLAB** script "michelson_timestream.m", only this time, you should use a *very* slow `speed` and a very narrow `span` since the filtered white light has a short coherence length and you will need good spatial resolution (which will determine the

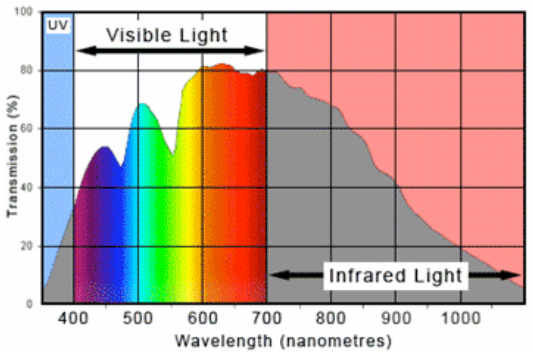


Figure 3.8: The spectral response of a typical silicon CCD image sensor ranges from 400 to 1000 nm.

highest frequency in your spectrum) so you can see the full spectrum without any truncation. Recommended values are `speed = 0.001` and `span = 0.05`. Note that this span might not be wide enough for the narrow band filter but that it already will take around 100 seconds given the speed! So, first check with a higher speed that this is the correct range for your filter and ONLY THEN take your data at the appropriate slow speed. Be aware: if you choose a span smaller than the backlash in the micrometer screw, the stage will not return to where you started. You should therefore insure the span is wide enough that the stage returns to the starting point properly.

Carefully (without disturbing the interferometer alignment) insert the 589 nm interference filter and record the fringe pattern generated. Take your spectrum and normalize it so that the peak of the spectrum has an amplitude of 1. Do the same with the reference spectrum and divide the filtered spectrum by the reference to deduce the transmission function of the filter. **Does the 589-nm interference filter have a Gaussian transmission function? Why or why not?**

You should now take a spectrum of the HeNe laser to check that your calibration of the stage velocity is correct. Is the peak of the spectrum where you would expect it? If not, why not? You should use this information to refine and/or correct the spectra you took of the filtered white light. What is the measured width of the spectral peak of the HeNe output in nm? The linewidth of the HeNe laser is well below 100 MHz. Is the measured width larger than this? If so, why?

3.10.2 Analysis with MATLAB

In order to compute the power spectra from your fringe pattern data, you will need to do some data processing. First, you will need to properly truncate your data because the vector of data, `data`, may include points when the stage was moving backwards and then sitting stationary before and after the slow sweep through the ZPL position. Therefore, find the points `AA` and `BB` which are at the beginning and end of the data and truncate the data. Your code will look

something like the following:

```
>> rawdata = michelson_timestream(span, speed);
>> data = rawdata(AA:BB);
```

Inspect a plot of your data to check that it has been properly truncated. Now, you should transform your vector of data into something equivalent to $W(x)$ (well equivalent up to a constant and a scale factor) by subtracting off the mean and then normalizing the data vector with

```
>> ndata = data-mean(data);
>> ndata = ndata/(max(ndata));
```

Now, to take the Fourier transform, you will use the "Fast Fourier Transform" function (`fft`) which, because of the structure of this super-efficient numerical algorithm, requires that you feed it a vector whose length is equal to a power of 2. Fortunately you can feed the function your vector and tell it the length is longer by rounding up to the next higher power of 2 (and the function will automatically pad your data with zeros at the end to fill in the gap). for this, use some code like the following:

```
>> L = length(ndata);
>> NFFT = 2^nextpow2(L);
>> fftdata = abs(fftshift(fft(ndata,NFFT)));
>> fftdata_oneside = fftdata(NFFT/2+2:NFFT);
>> nfftdata_oneside = fftdata_oneside/max(fftdata_oneside);
```

This will give you the absolute value of the Fourier transform since we aren't interested in the phase factors, only the amplitude versus wavelength. Also, we are only interested in the positive frequency components of the FT, so we will plot `fftdata_oneside`. The last line just normalizes the result by the max value. Why are the array limits `NFFT/2+2` and `NFFT` for the command "`>> fftdata_oneside = fftdata(NFFT/2+2:NFFT);`"? The reason is that the (N/2+1) element is the zero frequency bin and the (N/2) element is actually one of the negative frequency bins. The positive frequencies start at (N/2 + 2).

Now, the data contained in `fftdata` are the values of $|G(k)|$ (technically $|G_n|$ since we just have a list of values indexed by n) where the sample size is $\Delta_k = \frac{2\pi}{N_s \Delta_x}$. N_s = `NFFT` is the number of samples in the data which was "Fast Fourier Transformed" and Δ_x is the step size in, for example, nanometers between images. Remember since Δ_x is defined as the amount the optical path length varied between frames, it is twice the amount the carriage moved between images.

So, you know that each value of $|G_n|$ is paired with a wavevector of $k = n \times \Delta_k$. But you should plot your spectral data in terms of wavelength $|G(\lambda)|$ and so you need to create a vector of λ values corresponding to each point in $|G_n|$. Using the fact that $k = \frac{2\pi}{\lambda}$, we can write that $\lambda = \frac{2\pi}{n\Delta_k}$

```
>> n=1:1:NFFT/2-1;
>> lambda=zeros(size(fftdata_oneside));
>> lambda=NFFT*delta_x./n;
```

```
>> plot(lambda,fftdata_oneside);
```

The little dot (`.`) at the end of `delta_x.` insures that the division to create the vector `lambda` from the vector `n` happens element-wise.

An alternative and more transparent example is the following (given your data file has an EVEN number of elements):

```
>> data = rawdata(AA:BB);
>> data = data - mean(data);
>> N = length(data);
>> spec = abs(fft(data));
```

Now we remove (or just leave out) the zero frequency at index 1, and 'negative' frequencies (N/2+2:N)...

```
>> spec = spec(2:N/2+1);
>> spec = spec / max(spec);
```

And, since we've removed zero frequency, the lambda index starts at 1.

```
>> n = 1:N/2;
>> lambda = N*delta_x./n;
>> plot(lambda, spec);
```

Chapter 4

The Optical Cavity

4.1 Objectives

In this experiment the goal is to explore the spatial and temporal characteristics of the transverse and longitudinal modes a two-mirror optical resonator. This resonator is similar to the HeNe laser resonator (that you have or will investigate in another lab) except that there is no gain element in the cavity and it is therefore a "passive" resonator. The sections at the end of this chapter contain several useful notes and equations for this lab. Specific objectives are:

1. Explore the concepts of resonance, spatial mode matching, and resonator stability using the TEM_{00} transverse mode of a Helium-Neon laser and an external two-mirror optical cavity.

2. Couple higher-order Hermite-Gaussian modes into the cavity and determine the optical frequency spectrum associated with these different transverse modes of a spherical mirror cavity.

3. Compute from first principles the expected transmission function as a function of the length of a Fabry-Perot optical resonator and compare your predictions with the experimentally measured transmission function. Explore the frequency resolution of the Fabry-Perot optical resonator.

4.2 Introduction

One of the most common methods used in the characterization of laser light involves sending the light into an external optical cavity. Because laser light properties such as frequency components and transverse modes are dependent on the optical resonator that is part of the laser, we can think of the laser light as carrying information about the laser resonator. By sending the laser light into a separate optical cavity, the laser light can be analyzed as long as we know the characteristics of the external cavity. In a sense, by sending laser light into an external resonator, we are using one optical cavity to characterize another.

To understand better how this process works, in the first part of this lab you will send a beam of laser light into an external cavity that you will construct. Because external cavity length changes randomly due to mirror vibrations and

temperature fluctuations, the injected laser light will be resonantly coupled to different cavity modes (longitudinal and transverse) at different points in time. Remember that resonant coupling will only occur when at least one of the frequency components of the laser light matches at least one of the mode frequencies of the external cavity (which are changing in time due to mirror vibrations, etc.). Since resonant coupling of light into an external cavity also leads to enhanced transmission of the light through the external cavity, you will be able to see the effects of the time-dependent coupling by looking at the changes in the intensity profile of the beam as it exits the external cavity.

Suppose the laser light is purely monochromatic, and has a TEM_{00} beam profile. The ideal situation for transmitting the laser light through an external cavity would be to have the cavity length stabilized, with the frequency of the laser light resonant with the frequency of one of the modes of the cavity. Furthermore, the transverse mode of the light would ideally match a transverse mode of the cavity: if the laser light propagates in a TEM_{00} mode (by which we mean the fundamental transverse mode of the laser resonator), we would ideally want it to match the fundamental transverse mode (TEM_{00}) of the external cavity. Such mode matching does not occur automatically, since the fundamental external cavity mode will have its own set of Gaussian parameters of spatially varying beam width and radius of curvature, entirely independent of the parameters of the laser resonator. To mode match the laser TEM_{00} mode to that of the external cavity mode, lenses must usually be used to shape the incoming beam so that the parameters of the beam match those of the cavity. In other words, if the external cavity were actually a second laser, the beam exiting this second laser would overlap perfectly with the beam we're sending into that second cavity (see fig. 4.2 for a pictorial representation of this condition).

As usual, the experimental situation is more complex than the ideal case. If the laser beam is not perfectly aligned and mode-matched to the external cavity, the input beam will partially couple to many different transverse modes of the cavity (the mode of the input beam can be described as a superposition of external cavity eigenmodes- such as the Hermite-Gaussian modes), and the light that exits the cavity will resemble whatever cavity modes happen to be excited rather than maintaining the spatial profile of the input beam. Since we will not stabilize the length of the cavity, the coupling of the laser beam to the longitudinal and transverse cavity modes will change with time, and the light patterns that are observed exiting the cavity will fluctuate with time, revealing a time-dependent coupling of the laser beam to many different high-order Hermite-Gaussian modes.

4.3 Prelab

The external Fabry Perot cavity you will build for this lab consists of two mirrors shown in Fig. 4.1. M1 is planar mirror meaning the radius of curvature (ROC) is infinite and M2 is a curved mirror with $ROC = 30$ cm. As discussed in section 4.2, to efficiently couple the laser into the lowest order Gaussian mode of the cavity (the TEM_{00}), one must mode-match the laser beam to the cavity mode. In other words the Gaussian Beam mode of the laser must be matched (by choosing an appropriate lens) such when the beam is incident on the cavity, it "matches" the particular (and stable) Gaussian or eigen-mode of the cavity.

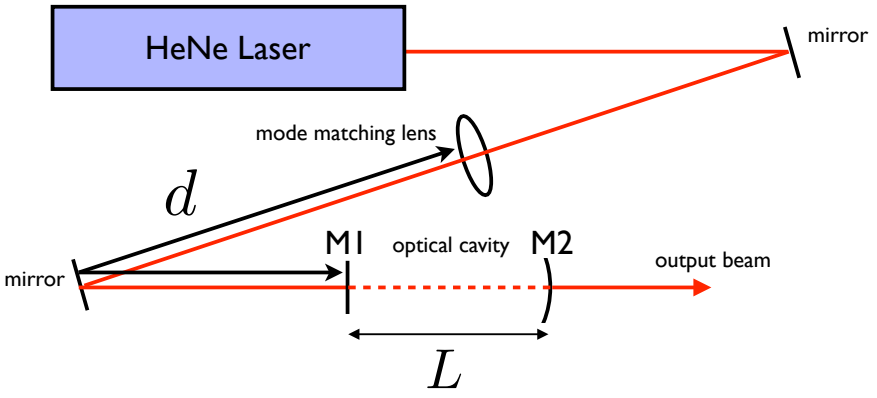


Figure 4.1: Cavity setup. The length of the cavity is the distance between M1 and M2 (L), and the distance from the mode matching lens to the input coupler of the cavity (M1) is denoted d. A photograph of the cavity can be seen in fig. 4.8.

Mode matching means both the beam diameter and the radial phase must be similar. For your pre-lab, you will determine the correct lens and its location to accomplish mode-matching for your particular laser to the external cavity. Toward this end for your pre-lab. you will need to perform three calculations prior to the lab:

- Determine the stability condition of the two mirror cavity shown Fig. 4.1. In other words, what mirror separations L produce a stable mode in the cavity?
- Produce a plot showing the size of beam waist of the cavity mode at **M1** as a function of L.
- Figure out the proper focal length of a lens and its position you need to use to transform the lasers beam to match that cavity mode at M1. See Fig. 4.2.

It will be easiest to use Matlab, Maple, or Mathematica for these calculations.

First, there is a range of mirror separation (L) when the cavity is stable. While a stability analysis is possible using ray optics, treatment of the problem in terms of gaussian beams is more appealing. For this calculation use the fact that in one round trip the $q(z)$ of the cavity mode (starting just after **M1**) must be self-consistent in one round trip and the $ABCD$ Law matrix formalism discussed in the text. In this calculation you will arrive at a quadratic expression $q(z)$. To determine what values of L are stable in the specific cavity at hand a physical solution means the q-parameter of the beam at **M1** must be purely imaginary (i.e. $q(z) = -iz_o$). Why? Use this fact to find a range of L.

Next, over this range of mirror separations (L) where the cavity is stable, at each L there will be a slightly different Gaussian mode in the cavity. Using your results from above, generate a plot of L vs beam diameter at **M1**. You

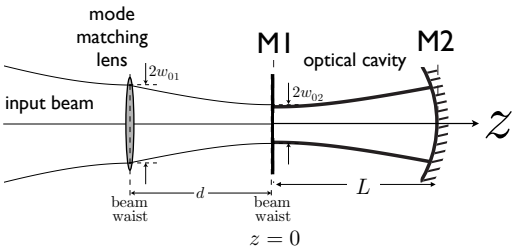


Figure 4.2: Coupling the input beam into the cavity. At $z = -d$ the input beam from the laser has a waist of w_{01}. The resonant mode of the cavity has a waist w_{02} at $z = 0$ (i.e. at **M1**). You need to determine the focal length of a lens and its position d such that the new beam waist of the input beam at z=0 is the same as w_{01}. As the input laser beam is somewhat collimated, you may use the following approximation: its depth of focus $z_{o1} >> d$.

have now found the the size of the Gaussian or eigen mode for the cavity at M1 as a function of L.

Finally, you need to figure out how to match the incoming laser beam to this eigen mode. Your hard-working professor has already measured the properties of the laser beam for you: you may assume it is a collimated (slowly diverging) beam with a diameter of 1.2 mm. Using the analysis from the text on Gaussian beam propagation through lens, determine the focal length of the lens and its distance d from **M1** that transforms the laser's Gaussian beam to the cavity's Gaussian Beam assuming the cavity length is 15 cm (this is in the middle of the cavity lengths you will explore in this lab). It will be easiest for you to mode-match by coupling into **M1** as opposed to **M2**. Think about why.

Since the lab does not have lenses of arbitrary focal length, but rather only a 60, 50, and 30 cm lens, choose the lens that most closely matches your ideal lens and compute the cavity length and the corresponding mode size at M1 at which the coupling is optimal.

4.4 Mirror reflectivity

In order to predict the expected finesse of your optical cavity, you will need to know the reflectivity of both mirrors and any losses inside the optical cavity. Since there's nothing but air in your optical cavity (and the loss is quite low) you only need to characterize the mirror reflectivity. To do this, you will need to measure the laser power reflected and transmitted by the mirrors. Be sure to capture all of the light (either reflected or transmitted) on the power meter and compare that to the incident optical power. Check that your reflectivity and transmission coefficients add to unity! Finally, be aware that the mirror reflectivity is angle dependent. You should therefore measure the reflectivity for the smallest angle possible to obtain the reflectivity at normal incidence to the beam as the mirrors will be inside the optical cavity. Please be careful not to bump or damage any components on the table while doing this measurement.

4.5 Input beam

From your pre-lab, you computed which lens you should use to optimally couple the light into the TEM_{00} mode of the optical cavity. Unfortunately, the lab probably doesn't have the exact lens you require. Therefore, choose from those available one that is close to your optimal lens. In this part, you will verify your prediction. Start by aligning the beam with the two turning mirrors after the HeNe laser to propagate down the center of the cavity mirror mounts AND to be level with respect to the optical table. Use the alignment card that is mounted since its hole is already at the optimal height for the laser beam. Next, place your chosen lens into the beam path (according to the figure). Be sure that the beam is centered on the lens. Measure the beam waist at the focus (i.e. where M1 will be placed) using a knife-edge measurement (see the writeup provided on the lab webpage regarding knife-edge measurements). You can locate the beam focus by looking at the beam on an index card. It's probably easiest to look at the beam spot generated by the light that propagates through the card to the back side since it's not as intense as the scattered light from the front of the card. Be sure not to change the power meter's "max-power setting" during your knife-edge measurements since the gain on the power meter is different at the different scales. To move the translation stage using the computer, open up the **APTUser** program on the desktop. Using the *Settings* button on the GUI make sure the max velocity is 0.5 mm/s for both the move and jog. Also, the jog setting of the micrometer stage needs to be ≤ 10 μm. The smaller the value, the more accurate your measurement will be.

Is the beam waist what you predicted in your prelab? If not, why are they different? Since diffraction at the knife edge can be quite significant, be careful to capture all of the light after the knife edge (razor blade) by placing the detector very close to the knife edge.

4.6 Optical cavity setup

Setup the resonator (a planar mirror for M1 and a 30 cm mirror for M2) with a short cavity length and couple the light in with M1 placed at the focus of the input beam. *From now on, don't worry about changing your input coupling lens even if your cavity length isn't optimal for coupling since you should get some light coupled into your cavity.* See the section on entitled "Notes on alignment" for additional tips on how to align your cavity.

Start by insuring your input beam is at the correct height to pass through the center of M1 and M2. Next, position the planar mirror (M1) at the position of the beam focus produced by the lens. The coated mirror surface should face into the cavity that you are building. There will be a fraction of light that is reflected from M1 back towards the laser, possibly resulting in multiple spots due to the beam bouncing back and forth between M1 and other optics (such as the laser's output coupler). Adjust M1 so that these spots overlap the forward-propagating beam. An index card with a hole in it is provided to help with this alignment.

How does this single mirror affect the spatial characteristics of the low-power beam that is transmitted through the mirror? Does the transmitted intensity fluctuate in time or is it relatively stable? How does this compare to

the situation where a second mirror is placed behind that first mirror and the two are aligned so that an optical cavity is formed. **In that case, does the cavity affect the spatial characteristics of the low-power beam that is transmitted? Does the intensity transmitted by the two mirrors of a cavity fluctuate in time or is it relatively stable?** Write down your expectations for the answer (a guess will suffice) and return to answer these questions fully after your cavity is aligned and you can observe the behavior. **Did you guess correctly the behaviors?**

Next, install the 30 cm curved mirror (M2) - again so that the coated mirror surface faces the inside of the cavity. Place the index card inside the cavity and position it so that the beam transmitting through M1 passes through the hole and hits M2. Adjust M2 so that the reflection passes back through the hole and onto M1. Insure this rough alignment is good even with the card held right at the face of M1. If the alignment is very close, you should see flickering of light on the mirrors as the cavity length drifts into and out of resonance. Remember that unless the cavity is confocal (this cavity is not), different transverse modes will in general be resonant at different frequencies or equivalently different cavity lengths, thus as the mirrors vibrate the narrow-frequency laser light couples to different transverse modes of the cavity. There are actually a few frequency components to the laser light, so the picture is even more complicated!

It may help to ramp slowly the cavity length with the PZT while you are aligning the cavity. Since the length of the cavity is constantly changing due to mirror vibrations, scanning the length with the piezo ensures that the cavity will at some time during the sweep be just the right length to be resonant with the input beam. When aligning, it also helps to look at the light transmitted through the cavity (i.e. after M2) and to adjust the end mirror (M2) so that the transmitted light is a single symmetric spot (and not a line of light).

Suppose your input beam is a TEM_{00} mode and it is perfectly mode matched to the TEM_{00} mode inside the cavity. Because the Hermite-Gaussian modes are *orthogonal*, you will *only* excite that one cavity mode and the transmitted beam will be identical to the input beam. Since your mode matching will not be perfect (the input beam parameters and/or alignment will be off slightly) you will couple to other transverse modes of the cavity. By adjusting the cavity alignment, position of the incident beam waist (by translating the coupling lens L1 or by translating the input mirror M1), and external cavity length L, see how close you can come the ideal situation. The transmitted TEM_{00} mode should be very bright when resonant, and other modes greatly suppressed.

4.7 Piezoelectric actuator

Fine adjustments of the length of the cavity are produced by applying a voltage to the piezoelectric actuator behind the mirror. The piezoelectric actuator can be seen in fig. 4.9. Voltage is supplied to the piezoelectric actuator ("piezo" for short) by a high voltage supply (see fig. 4.13). This supply has a manual adjustment knob and an external input. The external input is connected to a ramp generator. **Be aware** that the voltage of the ramp as seen on the scope is **different** from the actual output of the high voltage supply applied to the piezo! You will need to find out the gain on the input voltage which can be found in the user's manual (it should be the full output divided by the modulation

range). The distance the actuator moves given a certain voltage change can be found on the actuator datasheet; however, each actuator is slightly different and you need to characterize the response of your piezo. To do this, you will need to align the cavity with a *very short cavity length*, couple light into it, and measure the voltage swing (either manually or electronically) required to change the length of the cavity by exactly $\lambda/2$ (where λ is the wavelength of the light you are coupling into the cavity). Since the transmission function is periodic in the cavity length L with period $\lambda/2$, you only need look at the output (either with a CCD camera or the photodiode - see figs. 4.10 and 4.11) and watch over what change in voltage does the transmission repeat itself. This voltage variation will be the same independent of the cavity length, but a short cavity typically has a simpler transmission pattern with strong transmission for only the lowest order modes. You can, for example, look for all the voltage values at which the TEM_{00} mode is transmitted. Because there will be random fluctuations of the cavity length (due to mirror vibrations) this measurement is best done by quickly scanning the cavity length electronically and identifying the periodicity of the transmitted light by looking at the transmitted intensity on the photodiode.

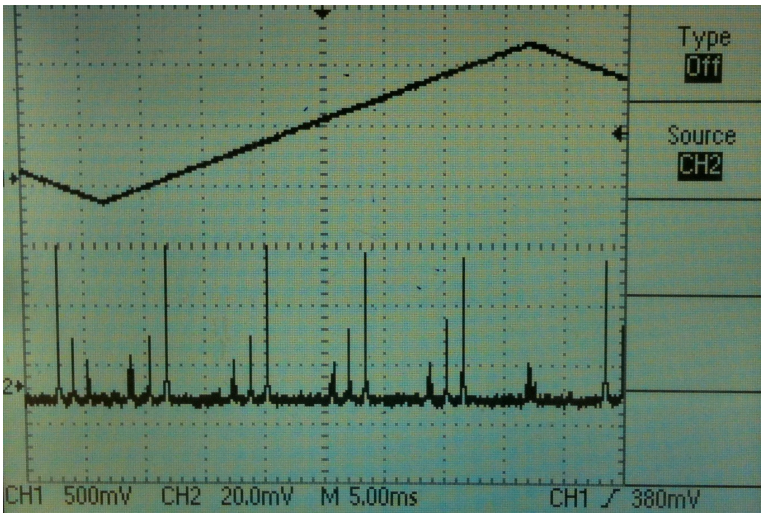


Figure 4.3: Oscilloscope traces showing the linear voltage ramp applied to the cavity piezo (thereby linearly ramping the cavity length) and the optical power transmitted as measured by the photodiode (PD). The PD signal is clearly periodic with the voltage applied to the piezo. Different transverse modes couple into the cavity at different voltages (lengths). By measuring the periodicity of the transmitted pattern, the length change induced by the piezo can be calibrated. By measuring the width of the transmitted peaks (in time or voltage) and comparing this with the width (in time or voltage) between the repeated patterns, the finesse of the cavity can be determined.

Align the output beam from a short length cavity onto the PD and identify the voltage variation that scans the cavity through one free spectral range (FSR). For your cavity, different transverse modes will in general be resonant at

different frequencies or equivalently different cavity lengths, thus as the cavity length is scanned the narrow-frequency laser light couples to different transverse modes of the cavity (see for example eqn. 10.2-31 in Fundamentals of Photonics, 2nd edition). Fig. 4.3 shows an example of oscilloscope traces showing the linear voltage ramp and the PD signal. Why does the pattern of peaks at the far left side of this image switch from large, medium, small to small, medium, large?

Be aware that your PD signal will probably look different depending on the alignment which will change which transverse modes are strongly coupled. Also, depending on your alignment, you might see that your pattern changes in time. In this case, you should take a single trace with the scope and make your measurements on a static trace. If your cavity is well aligned and you have coupled well into the TEM_{00} mode, the photodiode signal should look periodic in the voltage variation as in fig. 4.3. Double check the periodicity by varying the bias voltage manually on the high voltage driver (which adds to the input voltage ramp) to shift the whole pattern to the right or left by one or more periods. Check this also by computing the length change expected from the piezo given the information provided in the data sheet. **Do these methods all agree? What is the voltage variation that moves the piezo by $\lambda/2$?**

4.8 Observations

Once the cavity is aligned, observe the transmitted beam pattern using the CCD camera while slowly scanning the piezo voltage by hand. Use WinTV2000 to observe the output from the CCD camera. Try to stabilize the cavity length on a particular mode by adjusting the length of the cavity using the piezo driver manual adjust knob. When the coupling is optimal and the cavity is very well aligned, you will probably see that most of your higher order modes are circularly symmetric - i.e. Laguerre-Gaussian instead of Hermite-Gaussian modes. See how many different pure Laguerre and/or Hermite-Gaussian modes you can isolate and identify for a given length. Record and sketch or plot your results, clearly indicating the different transverse mode structures of output beams. **What is the largest transverse mode (TEM_{lm}) you can observe as you adjust the cavity length? What is the brightest mode? Which mode should be brightest if the cavity is properly aligned and mode matched?**

With M2 close to M1, to create a 1 cm long cavity, observe the mode patterns transmitted. How many transverse modes are visible? How much of the surface of M2 is filled with laser light?

Next, move mirror M2 so that the cavity is 25 cm long, and re-align the cavity. **What effect does this have on the mode patterns transmitted by the cavity? Are there generally more transverse modes than there were for the original short cavity? Now, how much of the surface of M2 is filled with laser light? Explain any differences you see between these results and the results for the 1 cm cavity.**

Move M2 so that the cavity length is just over 30 cm long, and try to re-align the cavity. Can you align the cavity so that mode patterns are visible in transmission? Consider the cavity stability criterion to see why this is so. **Describe what happens to the power transmitted through the cavity as the cavity length nears the edge of stability.**

4.9 Cavity finesse, linewidth, and Q-factor

In this section, your task is to measure the linewidth or quality of the cavity in units of Hz at several different cavity lengths. You will compare these measurements to the theoretically predicted resonator quality given the mirror reflectivities.

4.9.1 Resonator Theory

The transmission function of the optical resonator in this lab (which is a Fabry-Perot interferometer) depends on the *quality* (or Q-factor) of the resonator (equivalently the *finesse*) and the spectrum of the laser light. For a laser input with an infinitely narrow optical spectrum, the cavity transmission is

$$T = \frac{T_{\max}}{1 + \left(\frac{2F}{\pi}\right)^2 \sin^2(\Delta\phi_{\rm rt}/2)} \tag{4.1}$$

where $T_{\max}$ is the maximum transmission (depending on the mirror reflectivity), F is the cavity *finesse* and $\Delta\phi_{\rm rt}$ is the round trip optical phase. The finesse is defined by

$$F = \frac{\pi\sqrt{r}}{1-r} \tag{4.2}$$

where the amplitude of the wave is reduced by a factor r on each round trip. Given intensity reflection coefficients R_1 and R_2, we have that $r = \sqrt{R_1 R_2}$. For a plane wave inside a cavity of length L made of planar mirrors, $\Delta\phi_{\rm rt} = 2kL$, where $k = n2\pi/\lambda$ and n is the index of refraction of the material inside the cavity. The cavity transmission is maximum when $\Delta\phi_{\rm rt}/2 = q\pi$ where q is an integer or equivalently when $2kL = 2\pi q$.

Okay, here's the main point: the resonance condition is then $L/\lambda = q/2$ (where $q = 1, 2, 3\ldots$). That is, the cavity length must be an *integer multiple of half the wavelength of the input light*! That means that monitoring the resonance of optical cavities is great way to detect small changes (on the order of λ) in the cavity length. This is the principal of operation for LIGO, the cosmic gravitational wave detectors run by MIT and Caltech. **Notice**: since we can *either* vary the input **laser frequency** (i.e. wavelength) OR the **cavity length** to move the laser and the cavity into resonance, we get *two conditions on the positions of the resonances*. For a **fixed cavity length**, the resonance condition on the wavelength or frequency of the input beam is

$$\lambda_q = \frac{2L}{q}; \quad \nu_q = \frac{cq}{2L} = q\,\nu_{\rm FSR} \tag{4.3}$$

where the so-called "free-spectral range" is $\nu_{\rm FSR} = c/2L$ and the speed of light in the cavity is $c = c_0/n$ where c_0 is the speed of light in a vacuum. The time it takes a photon to travel from M1 to M2 and back to M1 (the round trip time) is simply $\tau_{\rm rt} = 1/\nu_{\rm FSR}$. From this, we see that the cavity transmission is periodic in the input laser frequency with period $\nu_{\rm FSR}$. On the other hand, for a **fixed laser frequency**, the resonance condition on the length of the cavity is that it must be an integer number of half wavelengths

$$L_q = q\frac{\lambda}{2}. \tag{4.4}$$

This implies that for a fixed input frequency, the cavity transmission is periodic in the length L of the cavity with period $\lambda/2$. The periodic cavity transmission is shown in Fig. 4.4.

The minimum cavity transmission is achieved when $\sin^2 \Delta\phi_{\rm rt}/2 = 1$ and is

$$T_{\rm min} = \frac{T_{\rm max}}{1 + \left(\frac{2F}{\pi}\right)^2}. \tag{4.5}$$

The minimum intensity only goes to zero in the limit of large finesse - that is when the *mirror reflectivity becomes nearly perfect* ($r \to 1$). As a way to characterize the width of the resonances, we can find the full width at half maximum of the transmission peaks. The points at which the transmission falls to $T_{\rm max}/2$ (i.e. when $\sin^2(\Delta\phi_{\rm rt}/2) = \left(\frac{\pi}{2F}\right)^2$) are given by $\Delta\phi_{\rm rt} = 2\sin^{-1}(\pi/2F)$. The width of a resonance is really only a sensible concept in the limit of large finesse, when the resonances are well resolved. In this limit, we can write the half-maximum intensity phases as $\delta_{\rm HM} \simeq \pm(\pi/F)$. And thus the full width of the resonances at half maximum is $\delta_{\rm FWHM} = 2\pi/F$ or equivalently $L/\lambda = 1/(2F)$. Again, since we can either vary the input **laser frequency** OR the **cavity length** to move the laser and cavity through resonance, we get the following conditions on the width of the resonances:

$$\nu_{\rm FWHM} = \frac{\nu_{\rm FSR}}{F} \tag{4.6}$$

$$L_{\rm FWHM} = \frac{\lambda}{2F} \tag{4.7}$$

$$\lambda_{\rm FWHM} = \frac{1}{2LF} \tag{4.8}$$

Fig. 4.4 shows the transmission (or intensity inside the cavity) as a function of the cavity length given a *fixed laser frequency* and as a function of the input frequency ν given a *fixed cavity length* L. From this, it is clear that the cavity finesse can be obtained experimentally by taking the ratio of the cavity periodicity and dividing this by the width of the transmission peaks.

$$F = \frac{\nu_{\rm FSR}}{\nu_{\rm FWHM}} = \frac{\frac{\lambda}{2}}{L_{\rm FWHM}} \tag{4.9}$$

Alternatively, if the mirror reflectivity (thus finesse) and cavity length L are known, the frequency or length resolving power of the cavity can be computed. Fig. 4.5 shows the transmission of the cavity at different values of the finesse. As the finesse is increased, the resonances become more and more sharp and the transmitted light off of resonance becomes smaller and smaller.

Photon survival time and Q-factor

A photon in the cavity completes one round trip every $\tau_{\rm rt} = 1/\nu_{\rm FSR}$ seconds. Over this round trip, it has a probability $P_{\rm s} = R_1 R_2$ of surviving the trip (i.e. not being lost from the cavity). Here R_1 and R_2 are the intensity reflection coefficients. Therefore the lifetime of a photon inside the cavity is

$$\tau_{\rm p} = \frac{\tau_{\rm rt}}{1 - P_{\rm s}} = \frac{1}{\nu_{\rm FSR}(1 - P_{\rm s})} \tag{4.10}$$

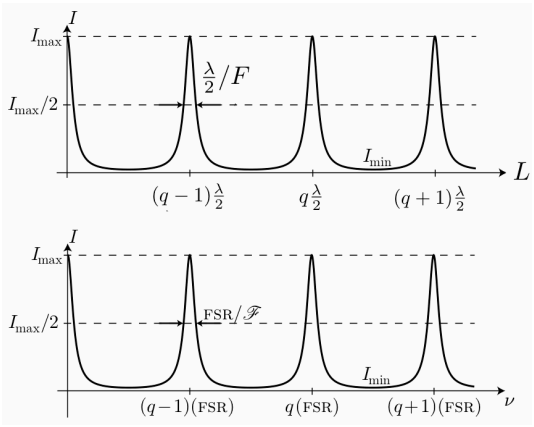


Figure 4.4: Transmission of cavity.

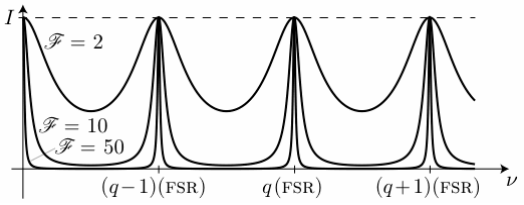


Figure 4.5: Transmission of cavity for various values of the finesse.

The finesse also depends on the mirror reflectivity and can be written as

$$F = \frac{\pi P_{\rm s}^{1/4}}{1-\sqrt{P_{\rm s}}}. \tag{4.11}$$

For large finesse (large survival probabilities), we can approximate $P_{\rm s}^{1/4} \simeq 1$ and $(1-P_{\rm s}) \simeq 2(1-\sqrt{P_{\rm s}})$ which allows us to rewrite the photon lifetime as

$$\tau_{\rm p} = \frac{1}{2\nu_{\rm FSR}(1-\sqrt{P_{\rm s}})} = \frac{F}{2\pi\nu_{\rm FSR}} = \frac{1}{2\pi\nu_{\rm FWHM}} \tag{4.12}$$

and we get an "uncertainty relation" (analogous to the time/energy uncertainty principle in quantum mechanics) of

$$\tau_{\rm p}\,\nu_{\rm FWHM} = \frac{1}{2\pi} \tag{4.13}$$

The resonator *quality* or Q-factor is 2π times the ratio of the total energy stored in the cavity divided by the energy lost in a single cycle. We can write this as

$$Q = 2\pi\nu_q\tau_{\rm p} = \frac{\nu_q}{\nu_{\rm FWHM}} = qF. \tag{4.14}$$

4.9.2 Experimental study of the resonator finesse

Measure the full width at half maximum of the transmitted cavity resonances (L_{FWHM}) as you scan the cavity length by several factors of $\lambda/2$ (corresponding to the free spectral range) for a few (3 or more) different cavity lengths L. Include data from a very short cavity length $L \leq 1$ cm and a very long cavity near the edge of stability. From these measurements, compute the finesse (F) and the corresponding cavity linewidth in units of Hz for each cavity length. **Is your measurement of the finesse different for different cavity lengths? Should it be different? Explain why or why not?**. From your measurements of the mirror reflectivity, compute the finesse of the cavity and compute the expected cavity linewidth in units of Hz for your different cavity lengths. Plot your cavity linewidth as determined from your measurement of the finesse and as determined from your expectation of the finesse from the mirror reflectivity. **From these results, what can you say about the spectral linewidth of the HeNe laser? Are your measurement results consistent with your theory? If not, explain why not?**

Hint: when measuring the length (or voltage) change of the cavity corresponding to a free spectral range (i.e. a distance of $\lambda/2$), it helps to trigger the scope on the voltage ramp you are sending to the piezo. See Section 4.11 for helpful hints on scope usage for this part. It also helps to carefully measure the voltage ramp time and amplitude so that you can easily convert time differences of peaks to voltage differences of the ramp. When measuring the cavity length change (and voltage change) required to scan the cavity through one of the transmission resonances (i.e. the distance L_{FWHM}), it helps to trigger the scope off the rising edge of the photodiode signal peak so that the transmission peak doesn't move around and you can zoom the time-base in to more easily measure the full width at half maximum. Be sure that you "zoom in" enough so that your measurement of the peak full width half maximum is not significantly limited by the finite sampling time of the scope. To capture a single trace, the trigger setting on the oscilloscope needs to be set to "single" and then you press the "run" button to capture a single trace (again, see Section 4.11 for helpful hints on scope usage). The peak separation and FWHM of the peaks is most easily measured using the cursors on the oscilloscope. Press the cursor button, select time type (not voltage type), and adjust their position so that they line up with the features you are measuring. Be aware that their time difference depends on the time scale of the scope, so if you change scales the cursors separation in time changes while their position on the screen remains fixed. Double check that their separation, given the time per division, agrees with the number displayed on the scope!

For these same experimental measurements, plot the experimentally determined finesse versus the cavity length and compare with the expected finesse. **Is the finesse changing with cavity length? Should it? What is the Q-factor of your cavity? Consider a tuning fork oscillating at 440 Hz (this is what musicians often use to tune their instruments). How long would the tuning fork ring given it had the same Q-factor as your optical cavity? Is this reasonable?**

Is there a cavity distance L where the geometric factors drop out (i.e. $g_1 g_2 = 0$) and you can see the modes becoming degenerate? If so, take a spectrum away from this point and at this point and put

them in your lab report and discuss your observations.

4.10 Important Rules: for safety and efficient use of time

Do not be apprehensive of this lab. If you are careful and heed the following warnings, the danger involved in this lab is extremely minimal.

4.10.1 Do Not Stare Into The Beam

The output of the HeNe laser is a medium power laser beam. You should therefore treat the beam with respect. Never place your head/eye in the path of the beam to see where it is going. A small index card (provided) is your tool to observe the path of the beam. It is also important to watch for stray reflections off of mirrors or other reflective surfaces. This means that any **watches, rings, or bracelets should be removed** before beginning this experiment. Laser goggles are provided and should be worn at all times.

4.10.2 Do Not Touch Any Optical Surface

It does not take many impurities on the optical surfaces to ruin the finesse of an optical cavity. Scratches, fingerprints, and even dust on the cavity elements can prevent the cavity from working well. If you suspect that an element is dirty, do not attempt to clean the optics yourself. Ask a TA and they will do it for you.

4.10.3 Do not spend more than 15 minutes trying and failing to get the cavity to resonate

If you spend more than 15 or 20 minutes trying to get the cavity aligned, there may be something more fundamentally wrong with the setup than alignment - like an optic is dirty or the external mirror mount height has been changed. You should not continue in vain but rather find a TA to help you diagnose and fix the problem.

4.11 Additional notes on using the scope

This section gives a few examples of scope usage.

4.11.1 Setting the scope to trigger on the PZT voltage ramp

1. Verify that the output of the function generator is going to channel 1 and the photodiode (hitherto referred to as “PD”) output is going to channel 2
2. press the ”TRIGGER MENU” button
3. toggle ”source” button to channel 1 (the voltage ramp channel)

4. toggle mode to "Normal" (there is "auto", "normal", and "single"

5. move the TRIGGER LEVEL knob up until the scope is properly triggered on the voltage ramp. Adjust the "HORIZONTAL POSITION" knob so that you can see at least one half period of the voltage triangle wave

6. The trace from channel 1 and channel 2 should look like those in Fig. 4.3

4.11.2 Setting the scope to trigger on the photodiode signal - the transmission peaks

1. press the "TRIGGER MENU" button

2. toggle "source" to channel 2 (the PD channel)

3. toggle mode to "Normal" (there is "auto", "normal", and "single"

4. move the TRIGGER LEVEL knob up until the main peak appears consistently appears centered on the screen. You may need to move the "HORIZONTAL POSITION" knob to get the peak to be centered when you zoom in on the peak by dialing the "HORIZONTAL SEC/DIV" knob

Sometimes it will be useful to average a few traces. This can be done with the following proceedure.

1. press the "ACQUIRE" button at the top in the "MENUS" section

2. select "Average" (there is "Sample", "Peak detect", and "Average")

3. toggle the "Averages" button to something not too large - like 8 or 16

4.12 Additional notes on alignment

1. Make sure the back reflection from the planar mirror (M1) goes back through the lens and back to the HeNe laser along the same path as the input beam. This ensures that M1 is properly aligned.

2. With M2 out, make sure that the transmitted light through M1 goes onto your detector

3. When you put in the second mirror (M2), watch the transmitted light (through M2) while adjusting the end mirror (M2) so that the transmitted light is a symmetric spot (and not a line of light).

4. When M2 is aligned you should see a flash of light on the surface of the mirrors facing the cavity. "Flash" occurs when the length of the cavity is just right and the power inside the cavity suddenly builds up to a large value and the scattered light at the mirrors becomes visible. Usually the cavity is only resonant for a brief moment in time causing the scattered light from the mirrors to "flash" or flicker on then off. You can roughly align the cavity while watching for flash.

5. You can do fine alignment by looking at the transmitted optical power on the photodiode (PD). Make adjustments by turning the screw and releasing the knob since the pressure from touching the mirror mount can produce a slight misalignment. It is often useful to find the limits of operation for each adjustment screw and set the screw position to the middle of this operating range.

6. Check PD alignment each time cavity length is adjusted.

7. Dust accumulation on the mirrors can be a problem since it can ruin the finesse of your cavity. If you suspect dusty or dirty mirrors, let a TA know and he/she can clean them.

4.13 Theory

4.13.1 Hermite-Gaussian Modes

This discussion of Gaussian modes assumes that you are familiar with the paraxial wave equation, its Hermite-Gaussian solutions, and the eigenmodes of spherical mirror cavities.

One often-encountered class of beam-like solutions to the wave equation may be written

$$E(\mathbf{r},t) = E_0(\mathbf{r})e^{ikz}e^{-i\omega t} \tag{4.15}$$

where the envelope $E_0(\mathbf{r})$ is a slowly varying function of z. Putting this solution into the wave equation yields the *paraxial wave equation*

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + 2ik\frac{\partial}{\partial z}\right] E_0(\mathbf{r}) = 0, \tag{4.16}$$

which is an equation for the envelope $E_0(\mathbf{r})$. One of several possible complete sets of solutions are the Hermite-Gaussians

$$E^{\mathrm{H}}_{nm} = \frac{Aw_0}{w(z)} H_n\left(\frac{\sqrt{2}x}{w(z)}\right) H_m\left(\frac{\sqrt{2}y}{w(z)}\right) \tag{4.17}$$

$$\times e^{-i(n+m+1)\arctan(z/z_0)} e^{ik(x^2+y^2)/2R(z)} e^{-(x^2+y^2)/w^2(z)} \tag{4.18}$$

where H_n and H_m are Hermite-polynomials of order n and m. These solutions are (approximate) eigenmodes of spherical mirror cavities and of Maxwell's equations in free space. That's why the output of a laser based on a spherical mirrors propagates over long distances as a well defined beam.

The phase of propagation accumulates like a plane wave phase (as kz) but has an additional Gouy phase that depends on the mode number ($(n+m+1)\tan^{-1}(z/z_0)$) and a radial curvature term. These solutions are appropriate in systems that lack full cylindrical symmetry, but still have a pair of symmetry axes x and y.

The beam size $w(z)$ and wavefront curvature $R(z)$ are closely related and characterized by the Rayleigh length z_0 according to

$$w(z) = w_0\sqrt{1+(z/z_0)^2} \quad (4.19)$$

$$R(z) = z + z_0^2/z \quad (4.20)$$

$$z_0 = \frac{\pi w_0^2}{\lambda} \quad (4.21)$$

Round trip phase for a Gaussian beam inside a cavity

Above, we have considered the resonant modes of a planar cavity with a plane wave inside. Here we

Fig. 4.6 shows a Gaussian beam inside a cavity. In order for the beam to be resonant with the cavity, the radius of curvature of the spherical wave fronts must match the mirror curvatures. These boundary conditions *fix* the position of the focus relative to the mirror positions. The focus is defined to be at $z = 0$ and the two mirrors are located at z_1 and z_2. Recall that the phase of a Gaussian

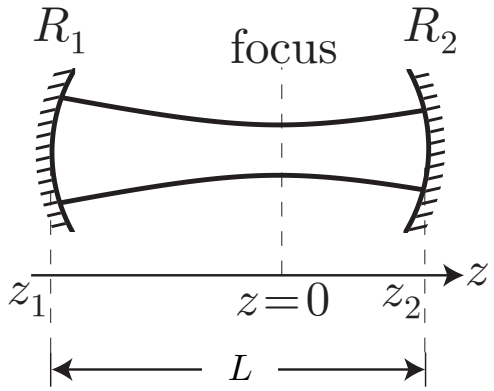


Figure 4.6: Schematic of a Gaussian beam inside a cavity.

beam *differs* from that of a plane wave. For a plane wave, the optical phase is simply $\phi(z) = kz$, but for a TEM_{lm} Gaussian beam it has an additional Gouy phase and a radially varying part:

$$\phi(r,z) = kz - (1+l+m)\tan^{-1}\left(\frac{z}{z_0}\right) + k\frac{r^2}{2R(z)}. \quad (4.22)$$

Since we have assumed the shape of the phase fronts is matched to the mirrors, we can consider just the on-axis phase $\phi(r=0,z)$. The round trip phase change of an optical wave inside the cavity is simply twice the phase change accumulated from z_1 to z_2

$$\Delta\phi_{\mathrm{rt}} = 2[\phi(0,z_2) - \phi(0,z_1)] = 2kL - (1+l+m)\left[\tan^{-1}\left(\frac{z_2}{z_0}\right) - \tan^{-1}\left(\frac{z_1}{z_0}\right)\right]. \quad (4.23)$$

On resonance the round trip phase change must be $\Delta\phi_{\mathrm{rt}} = 2\pi q$. It can be shown that the Gouy phase contribution can be written in terms of the geometric

factors $g_i = 1 + L/\rho_i$, where ρ_i is the radius of curvature of mirror i. Writing the resonance condition in terms of frequency, we have

$$\nu_q = \nu_{\mathrm{FSR}} \left(q + \frac{1}{\pi}(1 + l + m) \cos^{-1} \sqrt{g_1 g_2} \right). \tag{4.24}$$

The first term ($q\,\nu_{\mathrm{FSR}}$) is the planar resonance condition. The second term is an overall shift of the spectrum that depends on the cavity geometry (characterized by g_1 and g_2) and mode numbers l and m. In practice (except for confocal or concentric cavities), this means that different transverse modes (that are all at the same frequency) will be resonant with a cavity at different lengths. This is why when you couple the laser light into your spherical mirror cavity, you will see different transverse modes resonant (and transmitted) at slightly different cavity lengths. Alternatively, for a cavity of fixed length, different transverse modes are resonant at different optical frequencies.

Spherical-Mirror Resonators: stability

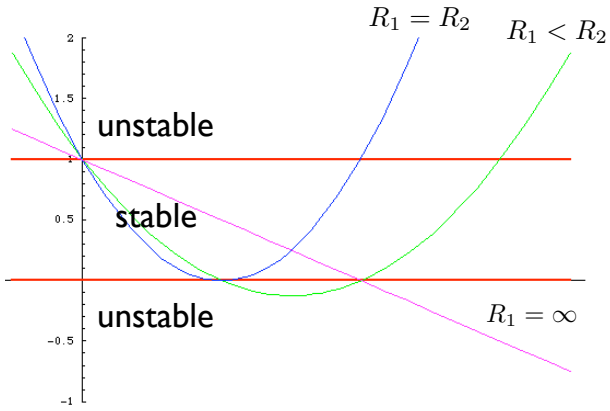


Figure 4.7: Plot of the middle term in expression (5.2) as a function of the mirror separation, d, for various mirror combiations (i.e. values of R_1 and R_2). The cavity is stable at all locations, d, for which the value of the term is between 0 and 1 (denoted by red lines).

An optical resonator composed of two planar mirrors ($R_1 = R_2 = \infty$) is stable for any mirror separation so long as they have been perfectly aligned. The difficulty with this arrangement is that in practice planar mirrors are extremely sensitive to misalignment; they must be perfectly parallel to each other and perfectly normal to the incident light rays. This sensitivity can be reduced by replacing the planar mirrors with spherical ones. The trade off, however, is that spherical-mirror resonators are only stable for specific geometric configurations. These mirrors can be either concave ($R < 0$) or convex ($R > 0$).

Limiting yourself to ray optics, and specifically to the methods of paraxial matrix-optics, it is possible to determine that the region of stability for any spherical-mirror resonator is given by;

$$0 \leq \big(1 + \frac{d}{R_1}\big)\big(1 + \frac{d}{R_2}\big) \leq 1 \tag{4.25}$$

where d is the optical cavity length, and R_1 and R_2 are the radii of curvature for the two mirrors. Typically, the two middle terms are written in terms of the ***g* parameters**

$$g_1 = 1 + \tfrac{d}{R_1} \quad \text{and} \quad g_2 = 1 + \tfrac{d}{R_2}$$

It is left as an exercise to demonstrate that this result is valid. You should record this derivation in your lab book. A good starting point for this analysis in located in your text book (Saleh and Teich, *Fundamentals of Photonics*).

4.13.2 Eigenmodes of a spherical mirror resonator

The Hermite-Gaussian modes form a complete set of eigenmodes for a stable spherical mirror resonator. If the mirrors have radius of curvature R_1, R_2 and separation L, then the condition for stability is $0 \leq g_1 g_2 \leq 1$ where $g_i = 1 - L/R_i$. The mirror locations and waist size can be found from the following expressions:

$$z_1 = -L\frac{g_2(1-g_1)}{g_1(1-g_2)+g_2(1-g_1)} \tag{4.26}$$

$$z_2 = +L\frac{g_1(1-g_2)}{g_1(1-g_2)+g_2(1-g_1)} \tag{4.27}$$

$$w_0 = \sqrt{\frac{\lambda L}{\pi}}\left(\frac{g_1 g_2(1-g_1 g_2)}{(g_1+g_2-2g_1g_2)^2}\right)^{1/4} \tag{4.28}$$

The frequency of these eigenmodes are:

$$\nu_{nmq} = \frac{c}{2L}\left[q + \frac{1+n+m}{\pi}\arccos\sqrt{g_1 g_2}\right] \tag{4.29}$$

Here is a list of the first few Hermite polynomials

$$\begin{aligned} H_0(\xi) &= 1 \\ H_1(\xi) &= 2\xi \\ H_2(\xi) &= 4\xi^2 - 2 \\ H_3(\xi) &= 8\xi^3 \\ H_4(\xi) &= 1 \end{aligned}$$

4.14 Images of components for this lab

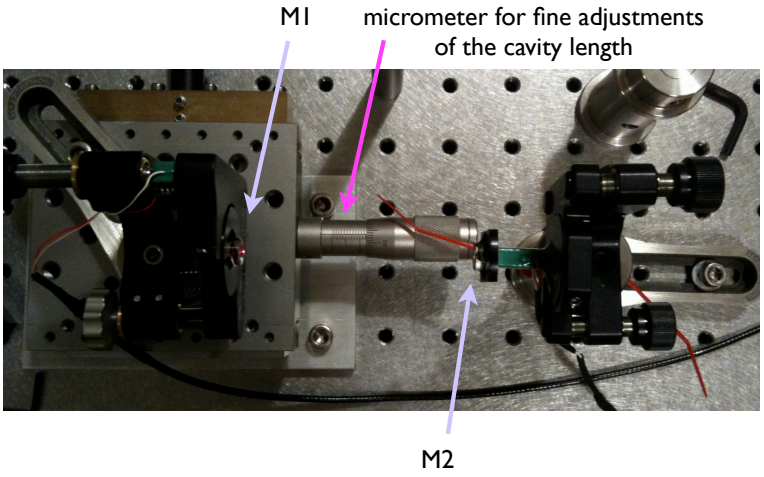


Figure 4.8: Image of the cavity showing M1 and M2.

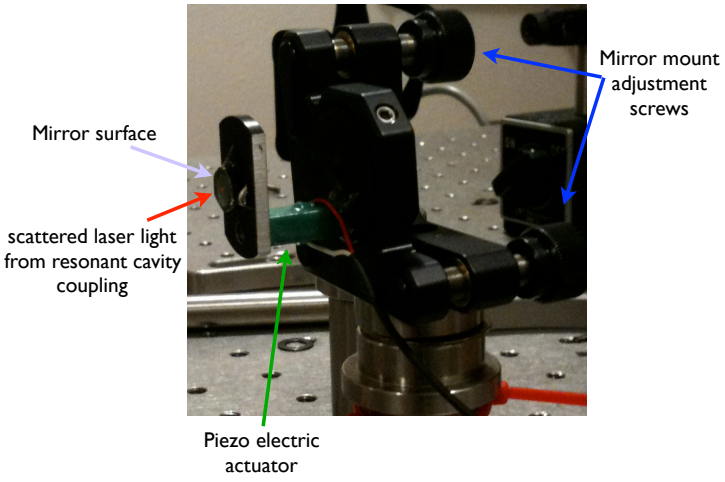


Figure 4.9: Close-up image of the mirror M2 showing the piezo actuator. When the cavity is resonant with the input laser light, the optical power inside the cavity is very large and scattered light from the mirrors becomes clearly visible.

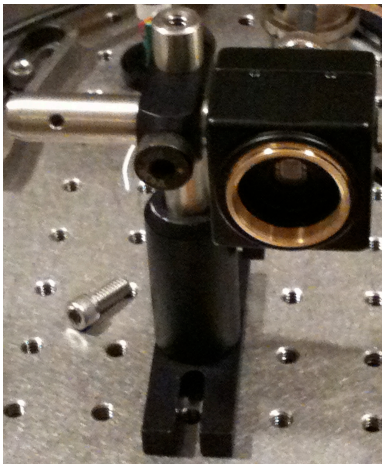


Figure 4.10: Image of the CCD camera. The active area of the sensor is the small rectangle (approximately 2x3 mm) centered in the region bordered by the threaded brass ring.

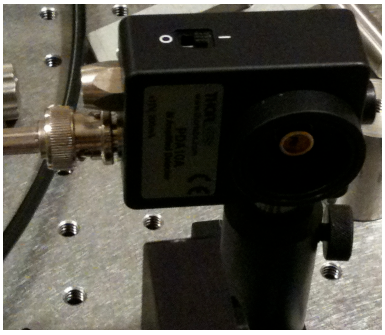


Figure 4.11: Close-up image of the photodiode. The active area of the sensor is a tiny (¡ 1 mm^2) square centered in the area bordered by the gold ring.

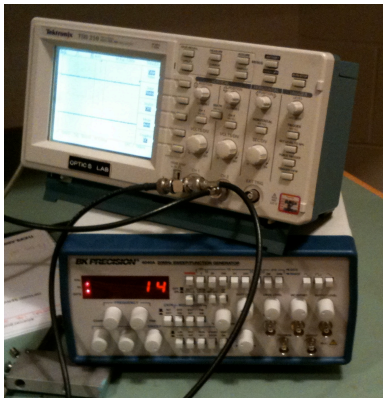


Figure 4.12: Oscilloscope and function generator. The output of the function generator should be sent to both the scope and the high voltage amplifier. The photodiode signal should also be displayed by the scope. An example trace is shown in fig. 4.3



Figure 4.13: High voltage (HV) amplifier (THORLABS model MDT694A). The display shows the output voltage. A voltage applied to the "EXT INPUT" on the front is amplified and that voltage is added to that set by the manual "OUTPUT ADJ" knob. The input multiplier changes depending on the voltage limit indicated by the green LED.

Chapter 5

The HeNe Laser

5.1 Prelab

In this lab, you will be building an optical resonator (a laser), and you will find that the laser cavity is only stable for certain mirror separations. The stability condition is derived based on ray optics in your text book (Saleh and Teich, *Fundamentals of Photonics*). Alternatively, the stability condition can be derived using Gaussian optics and ABDC matrices. This is your task for this prelab (do not copy the treatment given in Wikipedia).

The key to the derivation is the assumption (like for the ray optics derivation) that the Gaussian beam parameter, q, must be periodic - that is q must be the same after a cavity round trip. Using the ABCD matrix and the transformation law for q, derive the stability condition for the resonator. Recall that q_1 transforms to q_2 after passing through an optical system with an ABCD matrix as

$$q_2 = \frac{Aq_1 + B}{Cq_1 + D}. \tag{5.1}$$

Your result will be a quadratic equation in q. Using the physics of the problem, put limits on solution of q to find the regions of stability for the resonator. You should either (1) show that this condition is mathematically identical to the condition derived from ray optics or (2) describe qualitatively the implications of the condition you find by considering limiting cases and argue that it is equivalent to the one derived from ray optics.

5.2 Objective

In this lab you will construct a HeNe laser. By observing and investigating its fundamental properties and operating conditions, you will learn how a laser works and what determines its output characteristics.

5.3 Introduction to the Helium-Neon laser

The helium-neon (HeNe) laser was among the first lasers ever constructed, and has since proved to be among the most popular. Used throughout the scientific

laboratory and industrial workplace, these devices are easy to construct and relatively inexpensive. In general, a laser consists of three basics items: (1) an energy pump to provide energy/power to laser, (2) a gain medium to provide optical gain and (3) an optical resonator to provide positive optical feedback.

5.3.1 Energy Pump

A 1700 V high voltage, DC power supply maintains a glow discharge or plasma in a glass tube containing a mixture of helium and neon gas, as shown in Fig. 5.1. The discharge current is limited to about 5 mA by a 91 kW ballast resistor. Energetic electrons accelerating from the cathode to the anode collide with He and Ne atoms in the laser tube, producing a large number of neutral He and Ne atoms in electronically excited states. He and Ne atoms in excited states can de-excite and return to their ground states by spontaneously emitting light. This light makes up the bright pink-red glow of the plasma that is seen even in the absence of laser action.

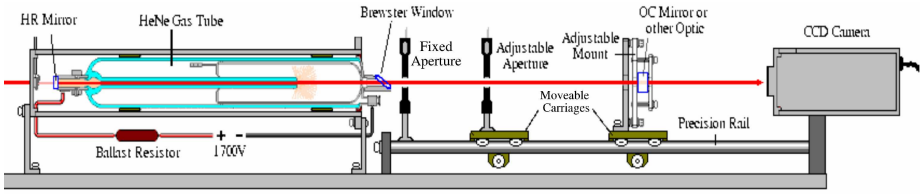


Figure 5.1: Open Frame HeNe Laser cavity. This is a schematic of the HeNe laser cavity that you will align in this lab.

The process of producing He and Ne in specific excited states is known as pumping and in the HeNe laser this pumping process occurs through electron-atom collisions in a discharge. In other types of lasers, pumping is achieved by light from a bright flashlamp or by chemical reactions. Common to all lasers is the need for some process to prepare an ensemble of atoms, ions or molecules in appropriate excited states so that a desired type of light emission can occur.

5.3.2 Gain Medium

As shown in Fig. 5.1, the laser you will build uses a glass tube filled with a helium-neon mixture (typically 5:1 or 7:1) at a low pressure (2-5 Torr) as the gain medium. To achieve laser action it is necessary to have a large number of atoms in excited states and to establish what is termed a population inversion. To understand the significance of a population inversion to HeNe laser action, it is useful to consider the processes leading to excitation of He and Ne atoms in the discharge. These processes can be identified using the simplified diagram of atomic He and Ne energy levels given in Fig. 5.2. A description of the rather complex HeNe excitation process can be given in terms of the following four steps.

(a) An energetic electron collisionally excites a He atom to the state labeled (in the Russell-Saunders notation) 2^1S_0. A He atom in this excited state is often written $He^*(2^1S_0)$, where the asterisk means that the He atom is in an excited state.

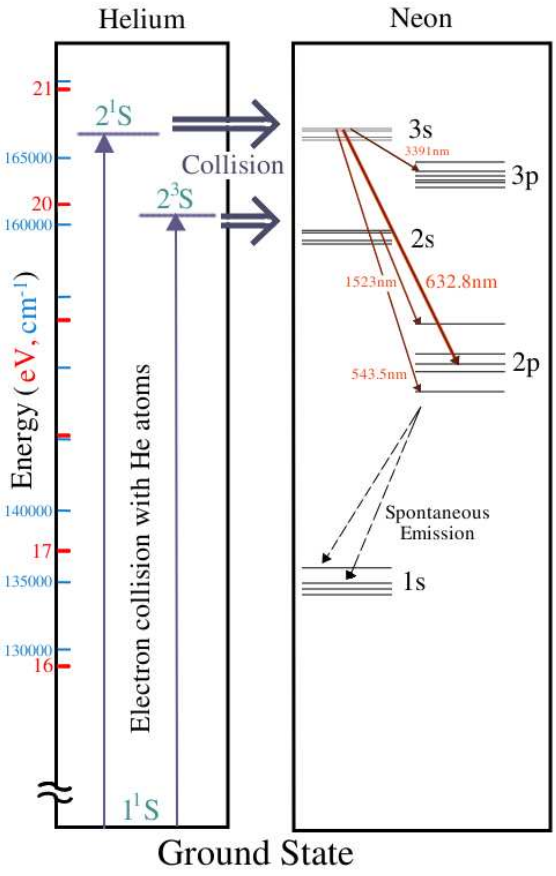


Figure 5.2: Simplified HeNe energy levels. This figure shows the energies level involved to produce population inversion for the HeNe laser.

(b) The excited $He^*(2^1S_0)$ atom collides with an unexcited Ne atom and the atoms exchange internal energy, thereby producing an unexcited He atom and excited Ne atom, written $Ne^*(3s_2)$. This energy exchange process occurs with high probability only because of the coincidental (and advantageous) near equality of the two excitation energies of the two levels in these atoms.

(c) The $3s_2$ level of Ne is an example of a metastable atomic state, meaning that it is only after a relatively long period of time - on atomic time scales - that the $Ne^*(3s_2)$ atom de-excites to the $2p_4$ level by emitting a photon of wavelength 632.8 nm. The wavelength of the emitted photon is determined by the energy difference between the two states (ΔE). Specifically, $\lambda = hc/\Delta E$ (where h is Planck's constant, c is the velocity of light). It is this emission of 632.8 nm light by Ne atoms that, in the presence of a suitable optical configuration (discussed below), leads to lasing action.

(d) The $Ne^*(2p_4)$ atom rapidly de-excites to its ground state by emitting additional photons or by collisions with the plasma tube walls. Because of

the extreme quickness of the deexcitation process, at any moment in the HeNe plasma, there are more Ne atoms in the $3s_2$ state than there are in the $2p_4$ state, and a population inversion is said to be established between these two levels. A web animation showing this entire 4 step process can found on the course web page.

When a population inversion is established between the $3s_2$ and $2p_4$ levels of the Ne atoms in the discharge, the discharge can act as an optical gain or amplification medium for light of wavelength 632.8 nm. This is because a photon incident on the gas discharge will have a greater probability of being joined by another photon in a $3s_2 \Rightarrow 2p_4$ stimulated emission process (discussed below) than of being absorbed in the complementary $2p_4 \Rightarrow 3s_2$ absorption process. This complementary process is suppressed by the rapid depletion (emptying) of the $2p_4$ state.

5.3.3 Optical Resonator

As mentioned in 5.3.2 above, Ne atoms in the $3s_2$ metastable state decay spontaneously to the $2p_4$ level after a relatively long period of time under normal circumstances; however, a novel circumstance arises if, as shown in Fig. 5.1, a HeNe gas discharge tube is placed between two highly reflecting mirrors that form an optical cavity or resonator along the axis of the discharge. When such a resonator structure is in place, photons from the $Ne^*3s_2 \Rightarrow 2p_4$ transition that are emitted along the axis of the cavity can be reflected hundreds of times between the two highly reflecting end mirrors of the cavity. These reflecting photons can interact with other excited $Ne^*(3s_2)$ atoms and cause them to emit 632.8 nm light in a process known as stimulated emission. The new photon produced in stimulated emission has the same wavelength, polarization, and phase. And the photon is emitted in the same direction, as the stimulating photon. It is sometimes useful for purposes of analogy to think of the stimulated emission process as a "cloning" process for photons, as depicted in 5.3. The stimulated emission process should be contrasted with spontaneous emission processes that, because they are not caused by any preceding event, produce photons that are emitted isotropically, with random polarization, and over a broader range of wavelengths.

As stimulated emission processes occur along the axis of the resonator a situation develops in which essentially all $3s_2 \Rightarrow 2p_4$ Ne* decays contribute deexcitation photons to the photon stream reflecting between the two mirrors. This photon multiplication (light amplification) process produces a very large number of photons of the same wavelength and polarization that travel back and forth between the two cavity mirrors. To extract a light beam from the resonator, it is only necessary to have one of the two resonator mirrors, usually called the output coupler, have a reflectivity of only 99% so that 1% of the photons incident on it travel out of the resonator to produce an external laser beam. The other mirror, called the high reflector, should be as reflective as possible. The small diameter, narrow bandwidth, and strong polarization of the HeNe laser beam are determined by the properties of the resonator mirrors and other optical components that lie along the axis of the optical resonator

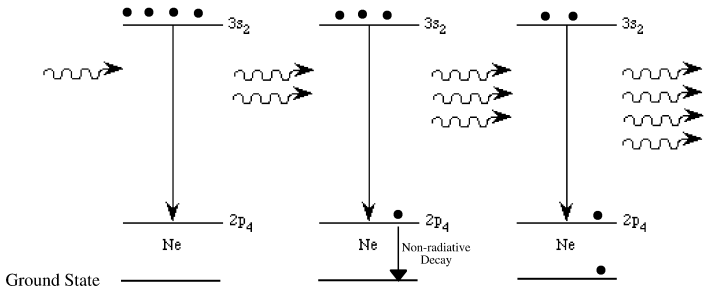


Figure 5.3: Stimulated emission. When a population inversion between the $3s_2$ and $2p_4$ is present, an incident photon will stimulate an electron (circles) to fall from the higher to the lower energy level and realize a "cloned" photon. In this diagram one can also note the importance of having a long-lived upper state ($3s_2$) and a short-lived lower state ($2p_4$); otherwise it is difficult to maintain a population inversion

5.3.4 Resonator Theory

Spherical-Mirror Resonators

An optical resonator composed of two planar mirrors ($R_1 = R_2 = \infty$) is stable for any mirror separation so long as they have been perfectly aligned. The difficulty with this arrangement is that in practice planar mirrors are extremely sensitive to misalignment; they must be perfectly parallel to each other and perfectly normal to the incident light rays. This sensitivity can be reduced by replacing the planar mirrors with spherical ones. The trade off, however, is that spherical-mirror resonators are only stable for specific geometric configurations. These mirrors can be either concave ($R < 0$) or convex ($R > 0$).

Limiting yourself to ray optics, and specifically to the methods of paraxial matrix-optics, it is possible to determine that the region of stability for any spherical-mirror resonator is given by;

$$0 \leq \big(1 + \frac{d}{R_1}\big)\big(1 + \frac{d}{R_2}\big) \leq 1 \tag{5.2}$$

where d is the optical cavity length, and R_1 and R_2 are the radii of curvature for the two mirrors. Typically, the two middle terms are written in terms of the ***g* parameters**

$$g_1 = 1 + \tfrac{d}{R_1} \quad \text{and} \quad g_2 = 1 + \tfrac{d}{R_2}$$

It is left as an exercise to demonstrate that this result is valid. You should record this derivation in your lab book. A good starting point for this analysis in located in your text book (Saleh and Teich, *Fundamentals of Photonics*).

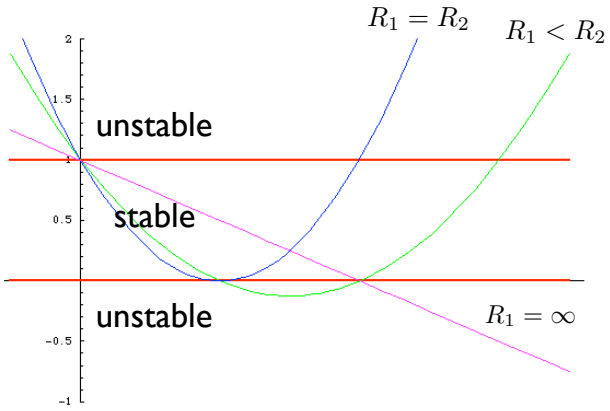


Figure 5.4: Plot of the middle term in expression (5.2) as a function of the mirror separation, d, for various mirror combiations (i.e. values of R_1 and R_2). The cavity is stable at all locations, d, for which the value of the term is between 0 and 1 (denoted by red lines).

Frequency Dependence of the Cavity Length

An important, but yet unmentioned, consequence of photon amplification is that in order for it to happen the photons must always be in phase (**why?**). This means that the light wave found in the laser cavity must be a standing wave, *i.e.* there can only exist a half integral number of wavelengths between the mirrors. Recalling that the wavelength λ is related to the frequency f by the relation $f = c/\lambda$ prove that the allowable laser frequencies are given by;

$$f_m = m\frac{c}{2L} \tag{5.3}$$

where L is the length of the laser cavity, and m is the number of half-wavelengths that fit between the mirrors.

From this you would expect lasing to occur only for very specific mirror distances. While in theory this is true, in practice lasing action can be obtained more or less continuously throughout the regions of stability (mirror separation) for a particular set of mirrors. **Explain why this is the case.**

5.4 Important Safety Rules

Do not be apprehensive of this lab. If you are careful the danger involved in this lab is extremely minimal. However, if you fail to heed the following warnings bad things will happen.

5.4.1 Do Not Touch the High Voltage Electrodes

There is a plastic casing surrounding the laser tube, there is no reason for you to remove this casing or to place your hands within the confines of it. People have died by mishandling laser power supplies. You won't, but there is no reason to test this theory.

5.4.2 Do Not Stare Into The Beam

This is not a high power laser beam; if for some reason the beam does impact near the area of your eyes your 'blink reflex' should be enough to protect you. This does not mean you should place your head/eye in the path of the beam to see where it is going. A small index card (provided) is a much better means of observing the path of the beam. It is also important to watch for stray reflections off of mirrors or other reflective surfaces. This means that any **watches, rings, or bracelets should be removed** before beginning this experiment. Laser goggles are provided should you feel uncomfortable.

5.4.3 Do Not Touch Any Optical Surface

It does not take many impurities on the optical surfaces to prevent lasing action from occurring. Scratches, fingerprints, and even dust on the cavity elements can prevent a laser from working. If you suspect that lasing is not occurring due to one of these causes, do not attempt to clean the optics yourself. Ask a TA and they will do it for you.

5.4.4 Do Not Remove the External Mirror Mount from the carriage

The height of the external mirror mount has been precisely adjusted so that the HeNe laser can lase with only the alignment of the two adjustment screws. If this height changes, the laser will not work. If you need to remove this external mirror mount, simply loosen the carriage and remove the entire assembly. In order to replace the mirror, carefully loosen the set screw and remove the 1" disc to which the mirror is glued. Carefully place the disk into the appropriately labeled box for storage. If you do remove the external mirror mount or inadvertently change its height, let a TA know that it needs to be replaced and adjusted.

5.4.5 Do not spend more than 15 minutes trying and failing to get the HeNe laser to lase

If you spend more than 15 or 20 minutes trying to get the HeNe to lase, there may be something more fundamentally wrong with the laser than alignment - like an optic is dirty or the external mirror mount height has been changed. You should not continue in vain but rather find a TA to help you diagnose and fix the problem.

5.5 Measuring the Wavelength

Using the supplied mirror, direct the (pink) fluorescence from the Brewster window into the monochromator (see the appendix for instructions on usage). Slowly tune the monochromator from 580 nm to 640 nm, noting the location and amplitude of the spectral lines. **How does this result compare to what you expect? What do you expect? Once you have successfully aligned the laser, as explained in the proceeding sections, repeat the**

monochromator measurement with the output from the laser. How (and why) is this different from the measurement of the fluorescence?

5.6 Aligning the Beam

The components of the lab have been left in place such that the alignment of the beam should not be overly tedious. The most important element is to remember not to adjust the height of the mirror or remove the mirror post from the post holder for any reason. Any changing of the mirrors should be done with the mirror mount left in place, this makes changing the mirrors a more delicate task but is far outweighed by the subsequent difficulties one would have in aligning the mirror if it were done otherwise.

There is a small screw in the mirror mount used to hold the mirror in place. In order to remove a mirror, loosen this screw while making sure to keep the mirror in place with your fingers. Do not touch the mirror surface, only the black disk that the mirror is attached to. Gently remove the mirror and place it in the mirror holder. To put a mirror in, just do the opposite as described above. Initial alignment should be done with a mirror with a radius of curvature in the range $40 < R < 65$ cm.

Referring to Fig. 5.1, now that the mirror is in place, translate the rail carriage so that the mirror is about 10 cm from the Brewster window. Make sure that the carriage is clamped tightly to the rail, as any stray motion may cause difficulties in the alignment process. There is a fixed aperture immediately in front of the Brewster window, you will use this as a guide to ensure that the incident light rays to the mirror are being reflected back along the same path. When the aperture is fully closed you should see the reflected light somewhere on the aperture surface. If it is not there check with an index card to see where the reflection is going. Using the x and y mirror mount controls only, slowly make adjustments such that the reflection returns back through the opening of the aperture. Lasing may not occur immediately, if this is the case slowly fine tune the position of the reflected beam in the opening. It will be obvious to you when lasing is taking place as there will be a bright red glow from the external mirror as well as an output beam from the OC mirror. Once you have the laser working, adjust the mirror until you have a beam of maximum intensity (brightest beam).

If you are having difficulty getting the mirror aligned, it may be necessary to place a second aperture close to the external mirror. If the post holder containing the second aperture is not already attached to a rail carriage, do so. Place the carriage within the laser cavity, just in front of the external mirror (see figure 5.1). Close the first aperture and make sure that the bright spot is located at the center of the second aperture. If this is not the case, you may need to adjust the height of the second aperture. Try to be as exact as possible, if it is not aligned properly then it will prevent you from obtaining lasing rather than helping you. Once it is aligned, proceed in the same manner as above, except that now you are attempting to make the reflection pass through the openings of both apertures. When starting it might be useful to only have one aperture closed at a time, as this will make the reflection more pronounced and easier to align.

5.7 Spatial modes of a spherical mirror Laser

A laser resonator is capable of supporting more than one spatial mode. Each of these modes has associated with it a specific optical frequency, and are therefore only stable for certain configurations of mirror curvatures and separation.

It is important to note that simply because a mode is capable of lasing at a given mirror configuration does not necessarily mean that it will. Modes undergo a gain competition within the gain medium whereby the one with the highest gain wins out and holds the others below threshold. In an ideal laser this results in a single dominant mode, however in ordinary lasers several modes may lase at the same time. In the case of a gas laser, the atoms in the tube are moving at different velocities along the cavity axis. This velocity distribution results in a similar distribution in the resonant frequencies of the atoms. If modes are produced with frequencies far enough apart one mode may obtain gain independent of the other, since these modes aren't competing with each other they are capable of coexisting.

It is typical in gas lasers such as the HeNe for the gain to be concentrated near the axis of the tube. As a consequence any modes that have a maxima on this axis tend to dominate. However, if a small object (such as a thin wire) is placed such that it impedes the gain at the axis center, then only those modes with a node (zero) occurring at the location of the object will lase. Thus by translating the object laterally across the profile of the beam it is possible to obtain spatial mode control.

5.7.1 Mode Control

Note: the mirror marked "HR" seems to work best for this "mode control" experiment involving the wire.

If you have not already done so, place the polarizer in the output beam line so that you can control the intensity of the beam hitting the camera. The image that you see on the camera will probably not resemble a perfect gaussian TEM_{00} mode. As you adjust the x and y components of the external mirror you should notice that the pattern on the camera (or as seen on the far wall changes). **Using the CCD camera or your artistic abilities, capture some of these patterns and print them out for your lab book and explain their occurrence.**

Re-adjust the mirror to the position of maximum intensity. Slowly open and close one of the apertures (making sure that the other is fully open), again making note of the appearance of the beam. **What happens and why?**

As stated above, mode control may be obtained by placing a thin wire in the open air section cavity of the laser (see Fig. 5.1). Since the mode control is easier in a cavity deep in the regime of stability, make sure that the external mirror is about 20 cm (or less) from the Brewster window. Place a rail carriage containing the shortest post holder on the rail roughly midway between the first aperture and the mirror. Insert the translation stage with the 25μm wire in the post holder and adjust it such that the laser beam is impeded by the wire.

By translating the wire across the profile of the beam you should be able to isolate individual spatial modes at different positions of the wire. Capture some of these modes with the CCD camera, print them and put them in your lab book. The theoretically expected spatial modes are families of Hermite-Gaussians (see

Saleh and Teich,*Fundamentals of Photonics* for the function form). Plot your observed modes with the theoretical expected spatial mode functions.

The wire is mounted on a translation stage with 20μm tics on the micrometer knob, this will allow you to quantitatively prove that each mode is maximized when the wire is at one of its nodes. To do this you need to measure the precise distances (with the micrometer) between isolated distinct modes, the length of the cavity, as well as the distance between the wire and the external mirror. These numbers plugged into the standard expression for the Hermite-Gaussian modes should produce the required result [1]. This is a lengthy analysis that is best done outside of the lab.

5.8 Determining Mirror Curvatures

By adjusting the length of the optical cavity, and carefully observing where lasing occurs, you should be able to determine the curvature of the internal mirror. Do this using a concave curved mirror (something in the range $40 < (-R) < 60$ cm) *a*nd the planar mirror. Note, you should start with the planar mirror since in this case there is only one unknown and you will have a linear term (as a function of d) in the stability inequality, expression (5.2). Using the value obtained for the radius of curvature of the internal mirror, you should theoretically predict the regions of stability for all the available mirrors in the lab. The available mirrors are as follows;

- Planar Mirror
- -1000 mm to -2000 mm Concave Mirror
- -250 mm to -300 mm Concave Mirror
- -40 mm to -45 mm Concave Mirror
- $+1000$ mm to $+2000$ mm Convex Mirror

Warning: most of the mirrors available in this lab to you are actually concave ($R < 0$), but be aware that their radius of curvature may be mis-labeled as a positive number. In fact, the " $+1000$ mm to $+2000$ mm" mirror is the only mirror with a positive curvature ($R > 0$ - i.e. a convex mirror) available in this lab.

Which of them could be used to produce lasing action? Is there one mirror that cannot be used to realize lasing?

5.9 Power Distribution

For this measurement, you will use the power meter. The detector head has a 25 μm pinhole mounted over the face and you will need to position it outside of the cavity so that the beam hits the pinhole. Start by removing the wire holder from the post holder and use the post holder to mount the power meter immediately to the right of the output coupler (OC) mirror. Make sure that the

[1] You will find it useful to look at the sections on Hermite-Gaussian beams and optical resonators in Saleh and Teich's *Fundamentals of Photonics*

output beam is aligned with the pinhole and that you have a measurable signal on the meter. Insure that the output beam is a TEM_{00} mode (this may require adjustment of the aperture in front of the Brewster window and/or small adjustments of the OC alignment). Translate the pinhole across the beam (beginning at the outer edge of the beam) taking distance and power measurements at precise intervals. Plot a graph of power vs. distance. **What kind of function is produced?**

5.10 Polarization

Place the Melles-Griot polarizer in the exit beam. Adjust the polarization angle and note the effect on the beam. **Why does this happen? Does this result make sense with respect to the Brewster window orientation?**

5.11 Wavelength measurement of the Laser Output

This is to remind you to repeat the monochromator measurement with the output from the laser. **How (and why) is this different from the measurement of the fluorescence?**

Appendix A

CCD Camera Operation

A.1 The HeNe and Fourier Lab CCD Camera

For several experiments in this course you will use the CCD cameras to capture images or movies. Make sure to copy all files to your own directories or transfer them to a usb stick as the files will be periodically deleted.

A.1.1 Capturing Image and Video Files

On the computer, select 'Run Command' from the start menu, type **xawtv** and press 'Enter' (alternatively select *Applications* → *RunCommand* and type 'xawtv').

Right click on the image window to open the options menu

Ensure the "TV norm" is set to "NTSC"

Adjust the brightness and contrast by clicking in the respective scroll bars

- Left clicking will increase the brightness/contrast
- Right clicking will decrease the brightness/contrast

Select "Grab Image (JPEG)" to save an image. The image will be named *snap_unknown_timestamp_nr*.jpg To change the base name (from *snap* to something else), initialize the program by typing **xawtv -o** followed by the new base.

For example, initializing with the string **xawtv -o HeNeModes** will cause images to be saved as $HeNeModes_timestamp_nr$.jpg

Recording a movie requires that "Capture" is set to "grabdisplay"

- Select "Record Movie (avi)"
- Set the movie driver to "Microsoft AVI (RIFF) format"
- Set the "movie/images filename" to *base*.avi
- Set the audio format to "No Sound"
- Set the sample rate to "44100"

- Set the video format to “MJPEG (AVI)”
- Set the frames per second to “12.0 fps”

Make sure to copy all files to your own directories or transfer them to a USB stick as the files will be deleted periodically from the machines.

Appendix B

Extended Fourier Analysis

Use the following example of Fourier spectra to compare the observed Fourier spectrum of the mesh to theory.

B.1 Plane Wave

A plane wave that is propagating along the $z-$axis has the following Fourier transform:

$$E(x,y) = A_0 e^{ik_z z - wt} \equiv E_0[f(z,t)] \tag{B.1}$$

Now we will drop f(z,t)

$$F[E(x,y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E_0 e^{-2\pi i(k_x x + k_y y)}\mathrm{d}x\mathrm{d}y = E_0\delta(k_x)\delta(k_y) \tag{B.2}$$

You can verify that the Fourier transform of a constant is a delta function by taking the inverse Fourier transform of a delta function:

$$F^{-1}[\delta(k_x)] = \int_{-\infty}^{\infty}\delta(k_x)e^{2\pi i k_x x}\mathrm{d}x \tag{B.3}$$

and since the delta function has the fundamental property that

$$\int_{-\infty}^{\infty} f(x)\delta(x-a)\mathrm{d}x = f(a) \tag{B.4}$$

it follows that

$$F^{-1}[\delta(k_x)] = e^{2\pi i(0)x} = 1 \tag{B.5}$$

therefore

$$F[1] = F\big[F^{-1}[\delta(k_x)]\big] = \delta(k_x) \tag{B.6}$$

Thus the transverse Fourier tansform of a plane wave is a point on the focal plane at $(k_x, k_y) = (0,0)$.

B.2 slit

A plane wave passing through a slit in one dimension of width w can be described by a rectangular function:

$$E(x,y) = E_0 \text{rect}(\frac{x}{w}) = \begin{cases} E_0 & |x| < w/2 \\ 0 & \text{else} \end{cases} \tag{B.7}$$

Thus in the Fourier transform plane the spectrum should be:

$$F[E(x,y)] = E_0 \int_{-\infty}^{\infty} \int_{-w/2}^{w/2} e^{-2\pi i(k_x x - k_y y)} \mathrm{d}x\mathrm{d}y \tag{B.8}$$

$$= E_0 \delta(k_y) \int_{-w/2}^{w/2} e^{-2\pi i k_x x} \mathrm{d}x \tag{B.9}$$

$$= E_0 \delta(k_y) \frac{1}{-2\pi i k_x} e^{-2\pi i k_x x} \Big|_{-w/2}^{w/s} \tag{B.10}$$

After some manipulation, this can be shown to be

$$= E_0 \delta(k_y) \frac{\sin(\pi k_x w)}{\pi k_x} \tag{B.11}$$

Using the definition $\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$, the above equation further reduces to

$$F[E(x,y)] = E_0 w \delta(k_y) \text{sinc}(w k_x) \tag{B.12}$$

B.3 Grid

A grid is a periodic sequence of N slit functions with width w and separation s in the $x-$direction. Thus;

$$E(x,y) = E_0 \sum_{n=1}^{N} \text{rect}(\frac{x}{w} - \frac{n \cdot s}{w}) \tag{B.13}$$

This can be expressed as a convolution of a series of delta functions with the rectangular function:

$$E(x,y) = E_0 \Big[\sum_{n=1}^{N} \delta(x - n \cdot s) \Big] * \text{rect} \frac{x}{w} \tag{B.14}$$

where the $*$ represents a convolution. The Fourier transform of a convolution product [1] is

$$F\big[(E_1 * E_2)\big] = F[E_1] \cdot F[E_2] \tag{B.15}$$

Therefore in the Fourier transform plane the spectrum should be

[1] See the Appendix in Fundamentals of Physics

$$F[E(x,y)] = F\Big[\ \sum_{n=1}^{N} \delta(x - n \cdot s)\ \Big] \cdot F\Big[E_0 rect(\frac{x}{w})\Big] \tag{B.16}$$

The second Fourier transform is the same as the slit from the last example. For the first term, the summation can be moved outside of the Fourier transform:

$$= \sum_{n=1}^{N} F[\delta(x - n \cdot s)] \cdot \big(E_0 w \delta(k_y)\mathrm{sinc}(wk_x)\big) \tag{B.17}$$

Using equation B.4

$$= \sum_{n=1}^{N} e^{-2\pi i n s k_x} E_0 w \delta(k_y)\mathrm{sinc}(wk_x) \tag{B.18}$$

The infinite series of exponentials can be rewritten:

$$\sum_{n=1}^{N} e^{-2\pi i n s k_x} = e^{-\pi i s k_x (N+1)} \frac{\sin(\pi N s k_x)}{\sin(\pi s k_x)} \tag{B.19}$$

so;

$$F[E(x,y)] = E_0 \delta(k_y) w \mathrm{sinc}(wk_x) \cdot e^{-\pi i s k_x (N+1)} \frac{\sin(\pi N s k_x)}{\sin(\pi s k_x)} \tag{B.20}$$

Thus the intensity is

$$I(k_x,k_y) = |E_0^2| \delta(k_y) w^2 \mathrm{sinc}^2(wk_x) \frac{\sin^2(\pi N s k_x)}{\sin^2(\pi s k_x)} \tag{B.21}$$

Appendix C

The Monochromator and Photomultiplier: description and usage

C.1 Introduction

A monochromator is a device used to isolate a small wavelength interval of a spectrum. It consists of an entrance slit, a dispersive element (*e.g.* a grating), and an exit slit which allows only a selected portion of the spectrum to pass to a detector. The photon detector in this lab is a photomultiplier tube. The instrument can be used to study the wavelengths and intensities of spectral lines emitted by a source.

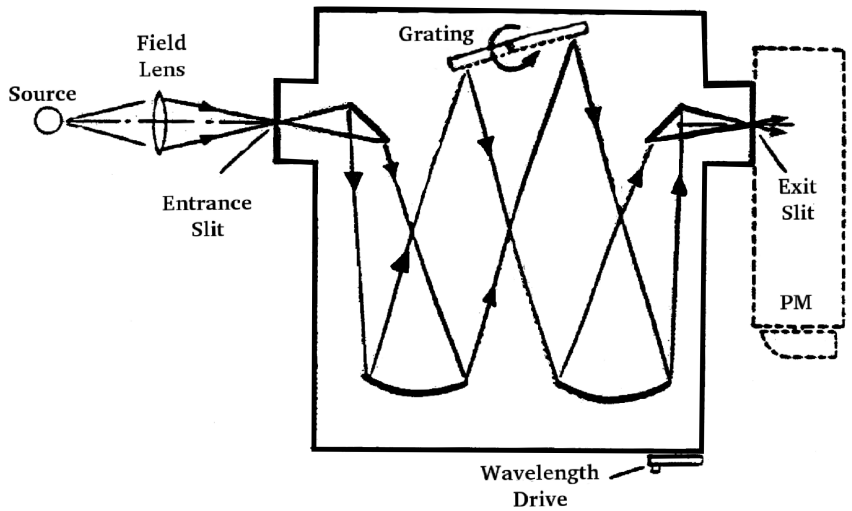


Figure C.1: The Czerny-Turner Monochromator (the field lens is optional).

C.2 The Czerny-Turner Monochromator

Although the particular model of monochromator used here is small, the design and operation is identical to that of larger models. This particular arrangement of mirrors, slits, and grating is referred to as the Czerny-Turner arrangement. The dispersive element is a diffraction grating.

The light path through the instrument is shown in Fig. C.1. The light from the source falls on the entrance slit. If necessary the light intensity can be increased with a field lens. A spherical mirror then collimates the beam of light and illuminates the grating. The grating disperses different wavelengths at different angles and one of these wavelengths is focussed onto the exit slit by the other spherical mirror. Since the slits are very narrow (typically 0.1mm or less), only a small range of wavelengths emerges from the exit slit. The instrument width of this monochromator is approximately 1nm, but for more expensive instruments a width of 10^{-2}nm is easily achievable.

C.3 The RCA 931A Photomultiplier Tube

The photomultiplier tube is a vacuum tube device used in industry to measure light signals, especially those from transient light sources. (A transient light source is one which only emits for a short period of time, such as a spark). The active element of a PM is the photo-cathode, which liberates electrons when struck by light. These electrons form a current pulse, which is amplified by a series of dynodes. Because the photon energy depends on the colour of the light, the energy (and number) of liberated electrons will also. This changes the dynode avalanche amplification which results in a current pulse amplitude dependent on the colour of the light. The last dynode is called the anode and is connected by an anode resistor to ground (see Fig. C.2). A voltage is therefore developed across the anode resistor which, in the ideal case, is proportional to the intensity of the incident light. The schematic construction and operation of the photomultiplier tube is shown in Fig. C.2.

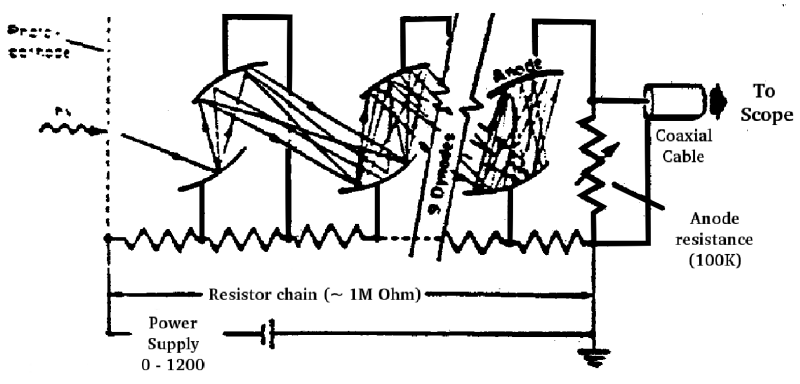


Figure C.2: Schematic of the RCA 931 Photomultiplier Tube.

The linearity of the PM response must be established before accurate in-

tensity measurements can be made. This is done by plotting the amplitude of the PM signal as a function of the intensity of the incident light. Ideally the graph should be linear, but at large signals a deviation from linearity is usually observed.

C.4 Monochromator alignment

The transmission through the monochromator is highly dependent on the angle of the light with respect to the monochromator. To line up the monochromator, first make sure the gain and the supply voltage on the PMT are set to ZERO and remove it from the monochromator. Set the monochromator to 500 nm and then take the slits out the front and back and look in through the back of the monochromator (i.e. put your eye where the PMT would normally be). Move the light source around until you can see light through the monochromator and then put in the first slit and optimize the position of the lamp with this first slit in place. Put in the second slit and repeat the above step. When you are done, put the PMT back in place and turn the supply on and adjust the gain setting until you get a measurable signal (about 1V). The supply voltage to the PMT can be increased if more gain is needed, but do not exceed 1.2 kV. While you can use the voltmeter to measure the voltage output of the PMT, in most cases where the PMT output noise is high and you want a reliable measurement, you will need to use a fan to chop the light going to the monochromator, and you will use the oscilloscope to measure the PMT response to the dark and light periods.

C.5 Spectral Response of the PM-Monochromator Combination

It was pointed out in Section C.3 that the photomultiplier does not respond equally to light of different wavelengths; neither does the monochromator. The combination of the two is very wavelength sensitive. For many experiments involving comparison of intensity measurements at various portions of the spectrum, it is necessary to know the spectral response of the entire system.

To measure this response, light from a known continuum source is passed through the system and the PMT signal, $S(\lambda)$, is measured at many wavelengths over the complete spectrum. If the intensity of the source, $I_w(\lambda, T)$ at a temperature of T, is known as a function of wavelength, λ, then the response of the system as a function of wavelength is given by,

$$R(\lambda) = \frac{S(\lambda)}{I_w(\lambda, T)} \tag{C.1}$$

Once the response function is known, the intensity of any other source at wavelength λ may be determined by measuring $S_2(\lambda)$ and dividing by $R(\lambda)$.