Assessing the Cognitive Levels of Math Exam Problems

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Introduction:

The taxonomy proposed by Bloom et al. (1956) comprises six levels of mastery, and is the most well-known method for categorizing the cognitive processes required to complete a task. We employ a revised version of Bloom's taxonomy (Anderson et al., 2001), which comprises the six levels listed below:

1. **Remembering**: Retrieving, recognizing, and recalling relevant knowledge from long-term memory; find out, learn terms, facts, methods, procedures, concepts.
2. **Understanding**: Constructing meaning from oral, written and graphic messages through interpreting, exemplifying, classifying, summarizing, inferring, comparing, and explaining. Understand uses and implications of terms, facts, methods, procedures, concepts.
3. **Applying**: Carrying out or using a procedure through executing, or implementing; make use of, apply practice theory, solve problems, use information in new situations.
4. **Analyzing**: Breaking material into constituent parts, determining how the parts relate to one another and to an overall structure or purpose through differentiating, organizing, and attributing; take concepts apart, break them down, analyze structure, recognize assumptions and poor logic, evaluate relevancy.
5. **Evaluating**: Making judgments based on criteria and standards through checking and critiquing; set standards, judge using standards, evidence, rubrics, accept or reject on basis of criteria.
6. **Creating**: Putting elements together to form a coherent or functional whole; reorganizing elements into a new pattern or structure through generating, planning, or producing; put things together; bring together various parts; write theme, present speech, plan an experiment, put information together in a new and creative way.

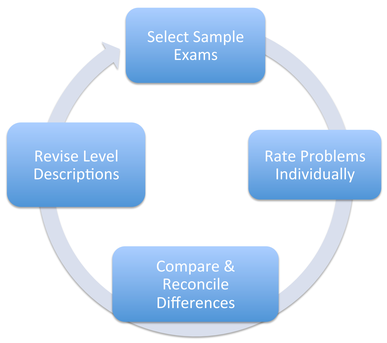
Research Question: What cognitive levels do our final exams assess, and how does the distribution of of cognitive levels vary with the year-level of the course?

Hypothesis: Upper-level courses have a higher proportion of exam problems that require cognitive levels of Analyze or higher (level 4, 5 or 6).

Methods:

First Stage: Operationalizing the cognitive levels in terms of math problems, and training of raters.

- A total of 8 past final exams at different levels were chosen at random from the Math Dept. website.
- Three cycles of problem rating and level definition revision were completed



Operationalized Level Definitions:

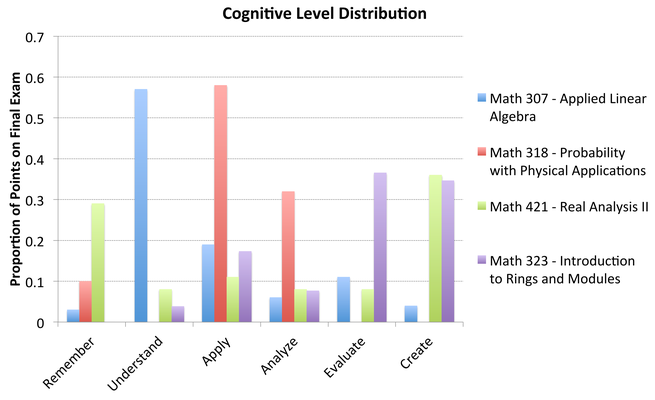
1. **Remembering:** The expected student response to a question at the remembering level is a statement of what was recalled without further interpretation or application.
2. **Understanding:** A response to an understanding question would be an interpretation or reformulation of information given in the question without integration with other knowledge.
3. **Applying:** An applying question is solved by using a known procedure, perhaps altered to be tailored to the given situation, but not altered to the point of drawing on other knowledge. Further, we include in this category tasks involving checking conditions in a definition or theorem.
4. **Analyzing:** A question at the analyze level is solved by interpreting the question in terms of knowledge gained in the course and adapting the relevant knowledge to the given question. Analyze questions are often novel questions, or are known situations phrased in novel ways.
5. **Evaluating**: An evaluating solution requires students to judge the veracity of a statement by drawing on their prior knowledge and to support this judgment with justification (e.g. stating relevant results or providing a counterexample).
6. **Creating:** Solutions to questions at the creating level involve synthesizing extant or newly-formed results into a new, coherent product.

Second Stage: Final Exam Rating

- Based on the cognitive level descriptions and operationalizations, two raters (the authors) categorized the problems on 15 final exams from the April 2014 session in UBC Math. The set was comprised of four 100-level exams, five 200-level exams, and six 300/400-level exams
- Inter-rater reliability is 74% (based on proportion of exam points that received identical classifications)
- Problems that were not categorized identically were resolved by discussion and agreement between the raters

Results:

Result 1: Within a course-level, there was a surprising degree of variability in the distribution of cognitive levels.

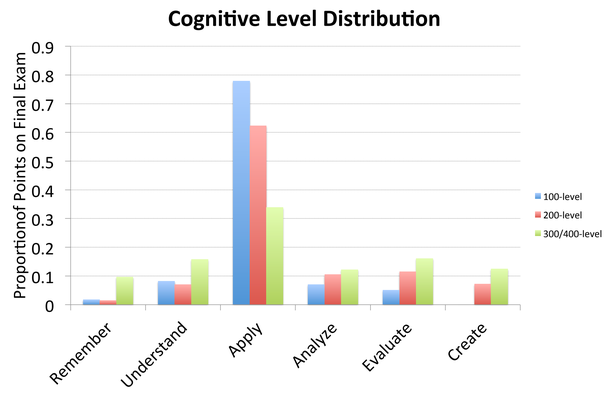


Result 2: We did not find a statistically significant difference between groups (100, 200, and 300/400-level exams) in the mean proportion of exam points categorized as higher-level (analyze, evaluate or create), as determined by one-way ANOVA ($F(2,12) = 2.350$, $p=0.138$)).

Qualitatively, there does appear to be an increasing trend in proportion of higher-level problems with course-level, so possibly a difference exists but we had insufficient statistical power.

Course Level (N)	Mean % Higher-Level Problems	Standard Deviation
100-level (4)	12.2%	6.7%
200-level (5)	29.2%	20.3%
300/400-level (6)	40.7%	25.3%

Result 3: There appears to be an overall "spreading" of the cognitive level distribution as the course level is increased



Future Work:

- Develop a tool similar to Casagrand and Semsar's "Bloom's Dichotomous Key," for quickly and consistently categorizing mathematics problems
- Categorize the problems on a larger, more representative set of final exams (possibly ALL exams from the 2013-14 academic year).
- Examine the constraints and decision-making process by which instructors choose their final exam problems. If there is a larger proportion of higher-level problems on 300/400-level exams, why?

References:

Anderson, L.W. (Ed.), Krathwohl, D.R. (Ed.), Airasian, P.W., Cruikshank, K.A., Mayer, R.E., Pintrich, P.R., Raths, J., and Wittrock, M.C. (2001) *A Taxonomy for Learning, Teaching, and Assessing: A Revision of Bloom's Taxonomy of Educational Objectives*. New York: Longman.

Bloom B.S., Engelhart, M.D., Furst, E.J., Hill, W.H. and Krathwohl, D.R. (1956). *Taxonomy of Educational Objectives: The Classification of Educational Goals, Handbook I: Cognitive Domain*. New York: David McKay Co Inc.