The Basic Skills Test in Mathematics

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Abstract

The data from past Calculus exams at UBC show that 75% of students' mistakes are related to basic high-school material. It appears that students often understand the concepts from Calculus, but their basic algebraic tools fail them when they are solving problems. As a result of this observation, in 2007 the Department of Mathematics introduced a Mathematics Basic Skills Test as part of the course requirements in two of the Calculus courses designed for students with no prior knowledge of Calculus.

The goal of the test has been to show students where their basic weaknesses are, with the hope that this would lead to an early remedial. More recently, however, the department has developed the need to have an effective tool that can be used to place students in the appropriate mathematics class. In order to decide if the current test provides useful predictive information, a study has been performed in which data from the 2007 and 2008 Basic Skills Tests have been collected and analysed to see if the test is a good predictor of students' performance in first-semester Calculus. The study shows that the Basic Skills Test has a better correlation to course grades than high-school grades and grades in the BC Provincial Exam, and potentially a higher capacity to predict students at risk of failing.

What is the UBC Mathematics Basic Skills Test?

The Basic Skills Test (BST) is a test that covers some high-school mathematics considered essential for passing a university-level Calculus course.

The test comprises 30 questions, the majority of which are multiple-choice questions, but it also includes some short-answer problems.

The topics covered in the test are Algebra (about 50% of the questions), Analytic Geometry, and Functions (each about 25% of the questions).

The setting

Courses involved:

- MATH 180 – Differential Calculus
  (with Applications to Physical Sciences and Engineering)
- MATH 184 – Differential Calculus
  (with Applications to Commerce and Social Sciences)

MATH 180 and MATH 184 are 4-credit Calculus courses intended for students with no prior knowledge of Calculus. Prerequisites are a score of 67% or higher in one of MATH 002 and Principle of Mathematics 12.

The Test – Some Statistics

In 2007 and 2008 the BST was offered twice in the first month of the term. Students who scored 19 or less out of 30 on the first test were required to write the second test. The best score out of two would account for 5% of their final grade in the course. In 2008 students who scored less than 20 out of 30 in both tests were advised to enrol in MATH 110, a new 6-credit Calculus course, however enrolment in MATH 110 was limited to 80 students.

In 2008, in MATH 180 32% of the students had to write the second test, 57% of them improved their scores on the second test. In MATH 184, 33% of the students had to write the second test, 61% of them improved their scores on the second test.

2008 BST (best of 2) Descriptive Statistics		
Course	**MATH 180**	**MATH 184**
Mean	22.31	21.46
Standard Error	0.19	0.19
Median	23	22
Mode	22	22
Standard Deviation	4.00	4.25
Sample Variance	15.97	18.05
Kurtosis	0.25	0.85
Skewness	-0.52	-0.59
Minimum	6	4
Maximum	30	30
Count	454	512

Correlation with Course Grades

The correlation between BST scores and course grades are estimated using the Pearson correlation coefficient (*r*). There seems to be no significant difference between the degree of correlation between course grades and the scores on the first test and course grades and the best score out of two tests. Unless otherwise states, here we present correlation analysis based on the BST best of two scores (BST).

Below are the correlation coefficients measuring the correlation between course final grades (CA grades) and

- BST (BST and CA grades)
- High-school grades (MATH 12 and CA grades)
- BC Provincial exam grades (PM 12 and CA grades)

The Basic Skills Test appears to be as good as or a better predictor of success in first-semester Calculus than high-school grades and grades in the BC Provincial Exam.

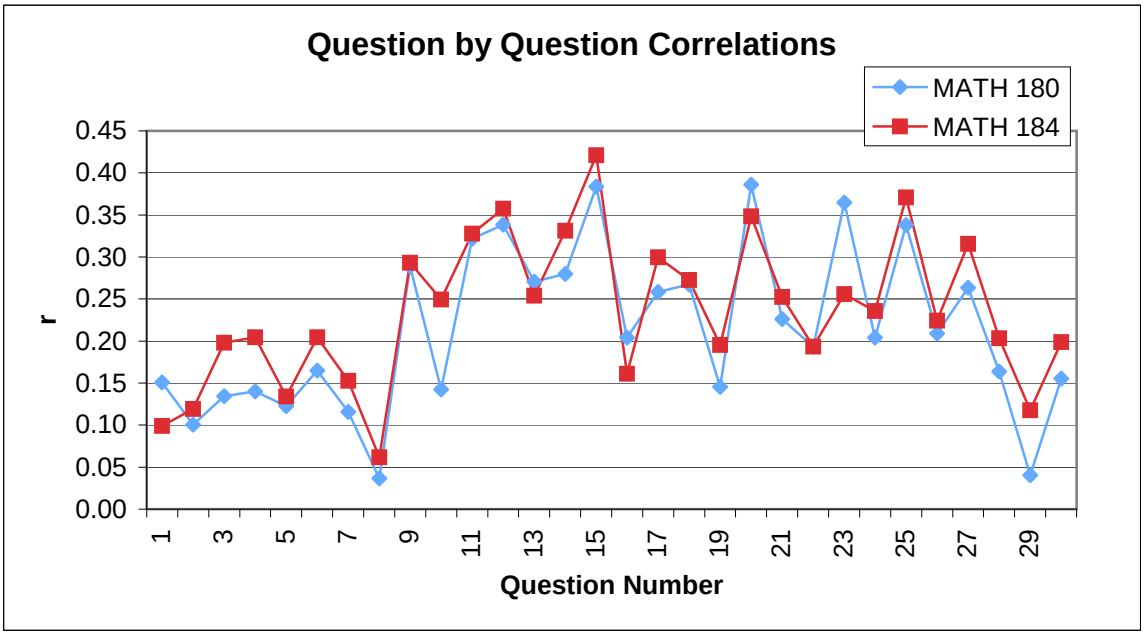
		N	***r***	***r²***
MATH 180	**2007**			
	BST vs CA grades	424	0.55	0.30
	MATH 12 vs CA grades	417	0.44	0.19
	PM 12 vs CA grades	293	0.42	0.18
	2008			
	BST vs CA grades	446	0.60	0.36
	MATH 12 vs CA grades	417	0.54	0.29
	PM 12 vs CA grades	339	0.59	0.35
MATH 184	**2007**			
	BST vs CA grades	500	0.60	0.36
	MATH 12 vs CA grades	485	0.43	0.18
	PM 12 vs CA grades	329	0.52	0.27
	2008			
	BST vs CA grades	513	0.64	0.41
	MATH 12 vs CA grades	451	0.50	0.25
	PM 12 vs CA grades	344	0.53	0.29

Analysis of Correlations to Course Grades for Individual BST Questions

Is there a subset of questions in the BST that contribute more significantly in predicting success in first-semester Calculus?

An analysis of the correlations between scores on individual questions and course grades indicates that some questions in the 2008 BST 1 are more strongly correlated with course grades than others.

Poorly correlated	Average *r*	More strongly correlated	Average *R*
Question 1	0.12	Question 12	0.35
Question 2	0.11	Question 15	0.4
Question 5	0.13	Question 20	0.37
Question 7	0.13	Question 25	0.35
Question 8	0.05		
Question 29	0.08		



Test Diagnostics for Individual BST Questions

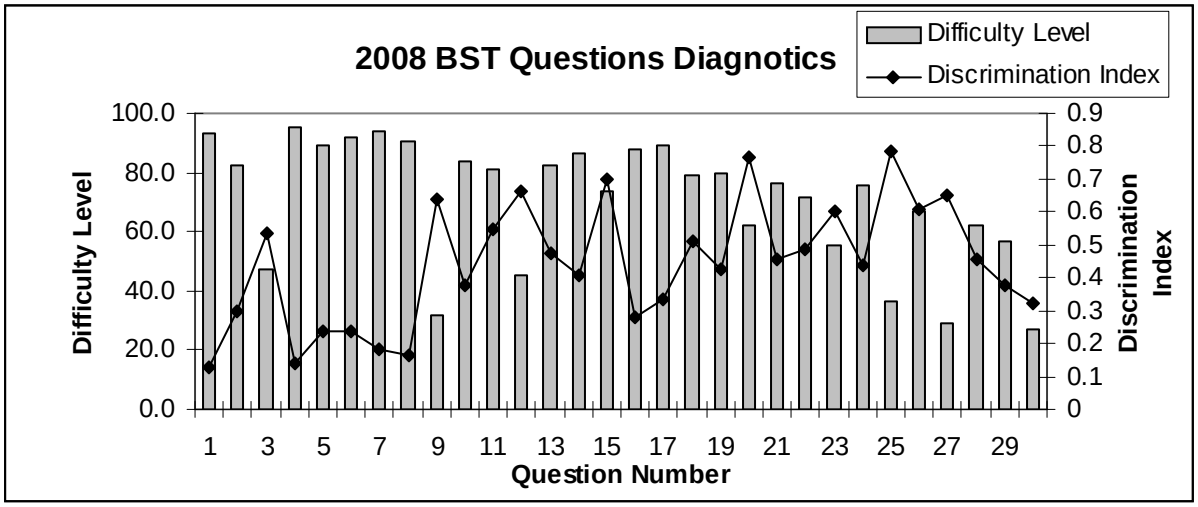
- Do the questions on the BST have the appropriate difficulty level?
- Can the BST be used to discriminate between students who master basic skills in Mathematics and those who don't?

Typical test diagnostics on individual test items include a measure of the level of difficulty of each test item, and an estimate of the ability of each test item to discriminate between students who do well on the test overall and those who don't. The following parameters have been calculated for each question in the 2008 BST 1:

- *Difficulty level (p)*: Percentage of students who answered the question correctly
- *Discrimination Index (DI)*: A measure of the performance on a question for students who did well on the test overall relative to those who did poorly. DI values range
  - < 0.40 indicate excellent discrimination (very good item)
  - 0.30-0.39 indicate good discrimination (reasonably good item)
  - 0.10- 0.29 suggest fair discrimination (item needs to be improved)
  - > 0.10 indicate poor discrimination (item should be revised or not used again)

Results:

Based on average *p* and DI values, we found that Questions 1, 4–8, and 16 all have a very high *p* value (or low level of difficulty, ~ 90%), and a low discrimination index (DI range 0.13–0.28). This suggests that these test questions are likely to provide little information on which students know the content and which who do not, decreasing the reliability of the test score. In addition, note that Questions 1, 5, 7, 8 have also low correlation coefficients, suggesting that little predictive information is associated with these questions.



A Grade Predictor Model

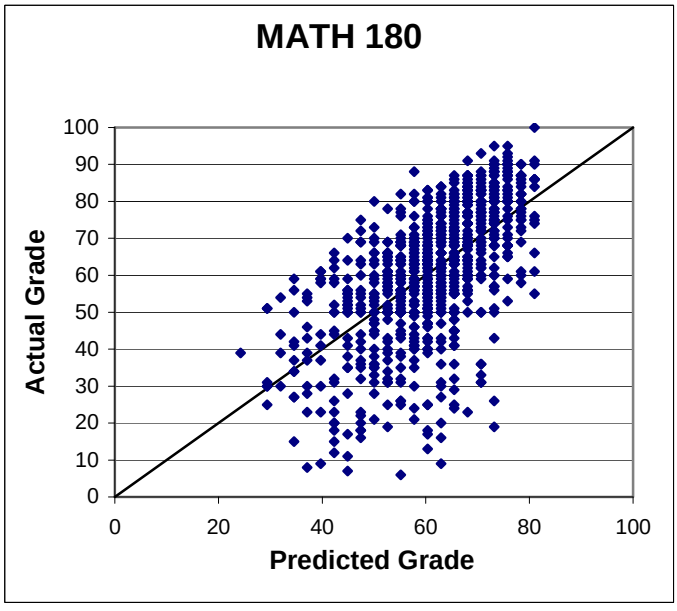
Based on BST data, we developed a simple linear regression model to predict the grade that incoming students were likely to receive in their first semester of Calculus. The model has the form

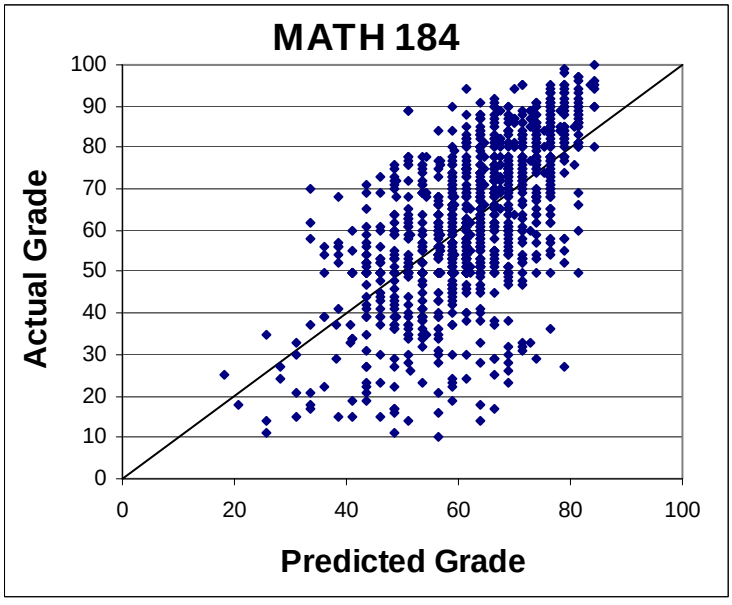
$$Y = a * BST + b$$

where *BST* is the BST best of two score and *Y* is the predicted course grade. A statistical analysis (*t*-test) indicated that grades for MATH 180 and MATH 184 students in 2007 and 2008 do not differ significantly, thus both data sets were collapsed into one and used to construct the model:

Course	***A***	***b***	r^2
MATH 180 (N = 870)	2.5767	3.6492	0.37
MATH 184 (N = 1013)	2.5335	8.1159	0.38

The standard deviation, a measure of the deviation of the actual grade from the predicted one, is σ = 16.7 for MATH 180, and σ = 14.9 for MATH 180.





A Predictor Instrument in a Multi-section Course: Are Instructor Effects Significant?

UBC first-year Calculus courses are large multi-section courses taught by a variety of instructors. They differ in teaching experience, academic background and interests.

Based on the BST predicting model, students' performance in each section of MATH 180 and MATH 184 was analysed and significant differences were identified. After interviewing the instructors on their experience teaching the course, and a sample of students randomly selected from their sections, we were able to link some elements of their teaching practices to higher/lower than expected student performances.

Sections that on average performed better than expected were characterized by

- higher levels of interaction between students and instructor
- material and assessment tools (homework, midterm exams, in-class examples) that were well matched with the level of expectations of the final exam
- higher degree of integration between lecture material and outside classroom activities (problem-solving workshops)

It was noticeable how in the sections that performed worse than expected students did not show appropriate expectations of their learning: when faced with harder than expected tests they tended to, at least in part, blame the instructor, without taking fully responsibility for their failure.