Roles of assessment in learning in statistics and mathematics

Helen MacGillivray
School of Mathematical Sciences, QUT
Director, QUT Maths Access Centre
Australian Carrick Senior Fellow, 2007
President-elect, IASE (International Association for Statistics Education)
Visiting Fellow, UK CETL in university-wide maths & stats support
Juggling hats?

Assessment also requires balancing

“Students learn only for assessment!”

Naturally

Students think what is assessed must be what is of value

If we value learning, assessing must be for learning
Engagement of students?
This presentation

Like mathematical proofs, end product doesn’t reflect evolution to it

- Overall comments on assessment
  - General HE & stats educ lit
  - Criteria, standards, objectives

- 4 “stories” illustrating aspects of assessment in different contexts
  - Intro data analysis: service & core, teaching data investigation
    - Components of assessment balanced over different objectives
  - Intro prob & dist’nal modelling: core & semi-service; unpack, analyse, extend, link with data
    - Assessment for learning problem-solving
  - 2nd year linear algebra: core & semi-service; looks towards industry problems, applied research, computational maths
    - Balance of continuous & tests, theory, applications & computing
  - 2nd year engineering unit: 1/3 data analysis; 1/3 dist’ns; 1/3 comp maths
    - Balancing components, workload, objectives

- Comments throughout on: alignment with objectives, balance, integrated assessment & learning packages, group work, collaborative work, plagiarism
From general HE literature …..

- *If learning really matters most, then our assessment practices should help students develop .. skills, dispositions, and knowledge…..*


- **Students study more effectively when they know what they are working towards….. Students value assessment tasks they perceive to be ‘real’**


*Objectives of learning & assessment must be clear*
…..reflected in statistics education literature

- In a survey of US statistics educators, of all areas of statistics education, assessment practices have undergone the least reform Garfield et al (2002)
- Calls for statistics educators to assess what they value (Chance, 2002)

- Explicit aligning of assessment with objectives features in both the general higher education (James et al, 2002) and statistics education literature (Gal and Garfield, 1998).
Aligning of assessment with objectives

Like mathematical proofs (& this presentation!), an iterative process

Components of assessment objectives to produce an assessment, teaching & learning package that is integrated, balanced, developmental, purposeful, with structured facilitation of student learning across the student diversity

- This needs identification of
  - Purpose of the learning
  - What the cohort are bringing to their learning
  - How the students manage their learning
  - The students’ perception of its roles for them

For wide range of backgrounds, programs, motivations, study skills……
Recent pressures for staff in tertiary assessment

- Seeking balances & paths amongst:
  - Formative, summative, flexible, continuous, rich, authentic
  - Generic graduate capabilities
  - Work-integrated learning
  - Criteria & standards referenced assessment
  - HE fads, generalisations & arbitrary rules

- Plus challenges of:
  - Avoiding over-assessment
  - Politics of pass rates & attrition & standards
  - Increasing diversity of student cohorts
  - Instant gratification generation
  - Workloads – students & staff
Criteria & standards referenced assessment

The term ‘criteria-referenced assessment (CRA)’ is often interpreted as meaning verbal descriptors of standards

Not so

- in criteria & standards-referenced assessment it is the configuration (Kaplan, 1964) or pattern of performance’ Sadler (1987) which is used for ranking or reporting a level of achievement ....

- Good packages have inbuilt configuration or pattern of performance

- Configuration comes from
  - construct of formative & summative assessment aligned with objectives & learning across cohort
  - construct of timing, types & weights of tasks

- Exemplars help to identify characteristics of each component of assessment, with verbal descriptors for salient criteria
Statistical Data Analysis 1

science, maths, surveying, educ…..: approx 500 pa

- Theme is basic statistical data concepts and tools & using them in real data investigations.
  - Separate phases – tools & building blocks of procedures, concepts and procedural skills
  - Synthesis – choosing, using, interpreting, combining in whole data investigations

- Structure, examples & learning experience built around real data investigations from first ideas through to report
  - Planning, collecting, handling, graphing, summarising, commenting on …. data
  - Categorical data – chisq tests; principles of testing hypotheses; p-values
  - Revision of normal; standard errors; confidence intervals and tests for 1 & 2 means, proportions, variances. Tolerance intervals
  - ANOVA & exp’tal design (via software): interaction (2-way), multiple comparisons, checking assumptions. Unbalanced data
  - Multiple & polynomial regression (via software): interpretation, diagnostics, re-fitting
Learning & assessment package

- Computer-based practicals on datasets from past student projects
- Worksheets with full solutions
- Fortnightly quizzes of fill-in-gaps & short response type; out Sunday, in by Friday: best 5 out of 6 contribute 10%
- Workfolder containing their ongoing work on the worksheets and their marked (collected) quizzes: 3%
- Whole semester group project in planning, collecting, analysing & reporting data investigation in context of group choice: 20%
- In-semester test (similar to quizzes 1-4): 10%
- End of semester exam (similar to quizzes 1-6, more on 5, 6): 57% *

Quizzes, test, exam: exemplars + exemplar processes
Quizzes & test formative & summative; exam summative
Assistance given for quizzes - most important aspect is DOING them

*For a few years also an optional essay on how statistics revolutionised science in the 20th century: 10% if improved overall result. Dropped because (i) almost never improved result (ii) attracted students who could least afford the time. Objective not worth student & staff effort
Research on numeracy/maths & statistical reasoning of cohort


- Numeracy & level of maths stood out as most important predictors of general statistical reasoning
- Fish question greatest discriminator between core & advanced maths preparation

A farmer wants to know how many fish are in his dam. He took out 200 fish and tagged each of them. He put the tagged fish back in the dam and let them get mixed with the others. On the second day, he took out 250 fish in a random manner, and found that 25 of them were tagged. Estimate how many fish are in the dam.
Own choice group project

- Teaches & assesses data investigation & synthesis of procedure choice & interpretation
  - Other assessment can focus on operational knowledge & skills - tools & building blocks of procedures, concepts and procedural skills
- Group because task needs a group
- Feedback on proposal + ongoing help; they propose – we advise
- Use of past datasets in class demonstrations and practicals
- Access to past projects, including assessments, and model reports
- Each group receives a written assessment report with comments & marks for the 3 criteria
  - Criteria, standards & exemplars. Formative & summative
Own choice group project

- **Criteria**
  - (i) Identifying context and issues; planning and collecting of data; quality of data and discussion of context/problems
  - (ii) handling, processing, preparing & understanding data & issues; exploring and commenting on features of the data
  - (iii) using statistical tools for statistical analysis and interpretation of the data in the context/issues

- **Group problems?**
  - They form groups, we help as necessary using pracs
  - Dropouts after week 8 can cause some problems but solvable

- **Plagiarism?**
  - Projects retained & designated “published”. To copy = 0

- **Contributions balance? Not a problem with right emphasis on project as learning experience**
  - Almost never in (i)
  - Seldom in (ii); allocation of tasks helps in (ii) & (iii);
  - Leaders in (iii) tend to learn more, need less revision for exam & do better…..and learn by helping others
Just a few recent titles

- Still time for play?
- How long can you suck?
- Talking your ear off
- Gym junkies
- Gifted hands
- Ah McCain you’ve done it again
- An analysis of alcohol induced loquaciousness
- Investigation into student internet usage
- Maritime museum usage
- pH of river
- Optical illusions

- Voluntary student unionism: to join or not to join
- We love muffins
- Human curiosity
- Holding breath
- Usage of the 15 min workstations in the GP library
- Strength of our athletes
- Where are all the single people?
- Seed germination
- The big news about breakfast
- Music and the people
Graphs 2006 sem 1

Scatterplot of total exam vs project_20

Scatterplot of total exam vs quiz mark_10

Scatterplot of total exam vs week10_1_06

Low +ve relationship

Moderate +ve relationship

High +ve relationship
Statistical modelling 1
All maths programs, maths electives, maths educ: approx 120

- **Builds** skills and foundations in concepts & thinking in
  - Intro probability, conditional arguments, distributional and stochastic modelling for
  - applications in a wide range of areas, from communication systems and networks to traffic to law to biology to financial analysis

- **Analyzes** prior understanding/
  misunderstandings

- **Links with data**, observation and simulation

- **Links with and consolidates 1st yr calculus & algebra skills**

  *Whole approach is problem-solving & modelling*


  It’s time to apply this in teaching probability & distributions
Formative components of assessment

- Initial general probability reasoning questionnaire (PRQ) to seed thought & discussion (introduced 2004)
- Class activities, simulations, selected computer modules, worksheets with unlimited help
- Each topic has preliminary experiences or exercises or discussion points (development completed 2005)
  - prior knowledge, foundations & seeds
  - perceive, unpack, analyse, extend

“Using what we already knew to learn other stuff was really good and helped us learn other stuff”

_A student definition of constructivism perhaps?_
Formative/summative & summative components: all oriented to problem-solving

- Four assignments based on class activities, examples and worksheets, with problems in authentic contexts **20% before 2006; 16% in 2006** (Assistance available. Collaboration – yes; straight copying rare)

- Group project. 2 everyday processes that could be Poisson (free choice); data collected; Poisson-ness investigated by combination of tests and graphs **10%**.

- End of semester exam. Problem-solving based on activities, worksheets, assignments; ranging from simple to slightly complex in life-related authentic contexts. Students design & bring in own summaries (4 A4 pages) **70% before 2006; 66% in 2006**
Some examples from group projects

- Australian Rules (football) grand final
- Time spent on phone
- Pedestrian traffic in mall
- Time to be served icecream
- Occurrences of “Harry” per page in a Harry Potter book
- Traffic on a pedestrian bridge
- Distribution of leaves on tiles
- Behaviour of ants
- Arrivals & service at library
- Distances between coffee shops
- Service in “fast” supermarket checkout
- Time between customers wearing high heels.
- Time between changes of a baby’s nappy
New assessment component in a problem-solving environment


“an emotionally and cognitively supportive atmosphere where students feel safe to explore, comfortable with temporary confusion, belief in their ability and motivation to navigate stages.”

- Formative assessment & assignments designed for managed optimal learning but students needed greater persuasion to learn through trying (’ave a go ....)

- Some topics identified as most needful of immediate involvement of students in active problem-tackling in an environment that maximises engagement & learning
Tutorial group exercises, 2006

- 4 practicals structured for immediate “hands-on” learning.
- Groups allocated; different groups for each practical.
- No compulsion to complete exercise; credit for participation.
- Assistance available as required.
- Full collaborative work required, with groups ensuring that explanations were shared within the group.
- Participation in each of these four special tutorials contributed 2% to the overall assessment.
Evaluation of new component

- Qualitative
  - Tutors and students voted experiment success.
  - The tutorials were buzzing, and early departures were practically non-existent.
  - Student opinion was that four was the ideal number.
  - Other tutorials benefited significantly.

- Quantitative
  - Assignments provide exemplars for exams
  - Data support that assignments most important in predicting exam (as desired!)
  - In 2005, assignments score depended on group project & PRQ score
  - In 2006, assignments score depended only on tut group exercises score for participation – strategy worked!
2nd year linear algebra unit
maths+others e.g. maths educ, physics, eng – approx 80-90

- Mixed student cohorts with often bimodal results
  - Balance of theory and practice?
- Some changes in continuous assessment – did they help or impede student learning?
  - Challenge of student engagement
- Interface of first and second level courses
  - first level courses respond to school/tertiary interface
  - first year units – which are best predictors?

- The examples and learning experiences in unit are motivated by higher level needs in mathematics generally & particularly computational mathematics, & by applications based on experience with industry problems.
Assessment package, 2003 & 2005

- **2003**
  - 21% continuous assessment
    - 3 Maple group assignments totalling 21%
    - mid-semester exam 15%
    - final examination 64%.

- Lecturer’s observations plus feedback:
  - Maple group assignments too heavy for 7%
  - Students needed more structured help with their learning

- **2005**
  - 40% continuous assessment
    - 2 Maple group assignments totalling 24%
    - 3 “homework” quizzes totalling 16%
    - final examination 60%. Similar in style, format and level to 2003
Analysis of data: assessment components

- For both continuous assessment programs, a test-type component and a Maple group assignment component combined as best predictors of exam

- Exam has applications but no actual Maple use, providing support of the claims in the literature, that both theory and practice contribute to overall learning and understanding in linear algebra

- Reassurance that the change in the continuous assessment program is not detrimental to performance, and appears to assist in learning

- Lecturer’s concerns about high marks in the 2005 continuous assessment program are reflected by only 25% of the variation in exam marks being explained

- **but the challenge of how to grade the continuous assessment can be tackled with confidence in the program’s facilitation of student learning across the theory and practice components of the unit**
Analysis of data: $1^{\text{st}}$ year predictors

- **Formal prerequisites**
  - $1^{\text{st}}$ level calculus unit and
  - $1^{\text{st}}$ level introductory linear systems and analysis unit, with the brief synopsis
    - *linear systems and matrices; vector algebra; coordinate systems; introduction to abstract algebraic systems; complex numbers; first and second order differential equations.*

- Entry to $1^{\text{st}}$ yr units via advanced mathematics in senior school or equivalent $1^{\text{st}}$ yr unit.
- Alternative prerequisites $1^{\text{st}}$ yr engineering maths
- Other compulsory $1^{\text{st}}$ year units for maths degree are an introductory unit in computational mathematics, Statistical Data Analysis 1 & Statistical Modelling 1.
Analysis of data: 1st year predictors

Data are complex because of different pathways. But best single predictor amongst 1st yr units, of performance in 2nd yr linear algebra in 2003 & 2005* was Statistical Modelling 1.

Synthesis of techniques and problem-tackling with new contexts, theory and applications appears to be the common thread linking these unlikely partners.

* Note: changes in the 1st year units since then have probably changed this
2nd year engineering maths unit
all engineering programs - approx 450-520

- Unit “new” in 2007 but composed of sections common across previous engineering units

Content in 2007:
- Statistical data investigations & analysis (1/2 unit)
- As in Statistical data analysis 1; as given in all eng programs since 1994
- Introductory numerical analysis (1/4 unit)
- Introduction to random variables & distributional modelling, including linear combinations of normals, goodness-of-fit & introduction to reliability (1/4 unit)
Level of unit

First year work in Science and Maths
- Statistical data analysis 1
- Numerical component extract from 1st yr unit
- Intro rv’s & distributions extract from Statistical Modelling 1

But
- It’s different
- It’s not straight calculus/algebra & any of these that are needed must be at fingertips in new contexts because of amount of material
- The statistics (both parts) full of new concepts & new ways of thinking
‘tis always thus in Australian eng courses

Because
■ of the philosophy of Australian eng courses (whether new, old or middling)
■ engineering needs the most technical maths faster than any other discipline

AND
■ engineering needs the most maths generic skills faster than any other discipline

Advantages of stats being in 2nd year eng are….
(i) they’re 2nd years in some ways & they have better maths thinking than most other disciplines
(ii) they start reflecting on their studies (I’ve been listening to & observing 2nd (or 3rd) year eng students for over 30 years)

Disadvantages of stats being 2nd or 3rd year eng are….
(i) they think they’re 2nd years in every way
(ii) it’s stats & they’re eng students
(iii) many tend to think it’s less important than other units
Learning & assessment package
focus is on learning by doing

- Computer-based practicals on datasets from past student projects
  - Weeks 1-6 on statistical data analysis
- Worksheets with full solutions
  - For all sections: 15 worksheets in total

- Stats (¼): five quizzes of fill-in-gaps & short response type 14%
- Whole semester group project in planning, collecting, analysing & reporting data investigation in context of group choice 20%
  - As for all eng since 1995 & as in Stat data Analysis 1
- Numerical analysis (¼): assignment 6%
- End of semester exam (based on quizzes & w’sheets): 60%
  - Ensures overall coverage correctly proportioned

Quizzes, assignment, exam: exemplars + exemplar processes
Project: criteria, standards & exemplars

Quizzes, project & assignment formative & summative
Assistance given for quizzes – most important aspect is DOING them
Exam summative
Assessment data: stats quizzes

Stats quizzes designed for efficient & effective learning.

Evidence of value over years & units.

Plot good – why?

Strategy introduced late ’90’s in an MBA ½ unit with highly diverse cohort with FT jobs. Then developed further in eng unit when data analysis became a ½ –unit module; strategy used to decrease time demands so as to keep the full project.

Unexpected & amazing side effect in eng unit was drastic reduction in copying. Students still worked together but argued/explained instead of copying. Similar effects observed in Statistical Data Analysis 1.
Assessment data: stats project

Project teaches & assesses synthesis of planning, thinking, understanding, choice of procedures and interpreting output.

Practicals designed to provide learning for project as well as for unit content.

Engineering projects about same standard over past decade.

Areas of choices 2007:
Most popular was transport! 21% on some type or aspect of transport. 17% observational (usually on people); 16% experimental; 12% food/drink;
8% work or study related; 5% each on computing, media, sport, surveys; 3% each on house prices/rentals & on other prices/retail
Assessment data: numerical assignment, overall

Plot indicates problems – why?

Dotplot of numerical

Dotplot of final
Assessment data: data quests on exam vs data quizzes, project

2007: Relationship good – a bit too much variation

2002, elect engs: excellent relationship. Less variation, partly because half size of 2007 class

Consistent over years & units; relationship & variation as it should be. Some relationship but project assesses different objectives
Assessment data: num. quests exam vs num. assign; dist’n quests exam vs dist’n quiz

These two need consideration – why?

Too much collaboration: why?
Change in engineering course
inequitable backgrounds
Assignment not difficult but long & detailed
Too much other assessment because of new eng faculty rule

The students seemed to be drowning in assessment in other units weeks 8-11. In weeks 10-12 they tried, with many valiantly doing last quiz. Many were glad to be able to do project weeks 12, 13, but had difficulty engaging with new work.
Conclusions: Assessment for learning

- Each item/task/component has role in integrated, balanced, developmental, purposeful learning package

- Learning objectives ↔ assessment
  - What is of value in this item/task/component?
  - How do we learn & assess this objective?

- Structured for facilitation & management of student learning across the cohort diversity
  - What balance of formative/summative does this task have?
  - Is this task manageable & correctly weighted for purpose?
  - Are the purpose & criteria of task clear within package?
  - Do we know enough of students’ pasts, presents & futures?
  - Have we clearly communicated on collaborative & individual work?

- Above assist in preventing plagiarism

And explore, analyse, interrogate & interpret DATA
Thank you for your attention

Questions, comments, debate, …..?