

# Missed in Plain Sight

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Over  
there



School of Education  
*Stanford University*



# A past conversation

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- **Carl:** When you have students learn new ideas, why don't you tell them first, before they start practicing?
- **Dan:** *<Mumble, stall, mumble, mutter>*
- **Dan:** How about this? Students pay attention to what you tell them, and this stops them from seeing what you're talking about.
- **Carl:** That would be a good thing to show.

# So... 2 years later...

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- That's what I'll be demonstrating.
    - A form of “verbal overshadowing”.
  
  - All 3 studies take a similar form.
    - In the **tell-and-practice** condition, students are told the big idea and how to use it. They then practice on some cases. They miss the scientific structure of the cases.
    - In the **invent** condition, students work on the exact same cases, but they are not told the big idea until after. They do find the structure.
  
  - Just switching the order makes a big difference.
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# Outline

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- □ Learning to Perceive
- Missed in Plain Sight
- Why it Matters
- Why Inventing Instruction Works
- Summary

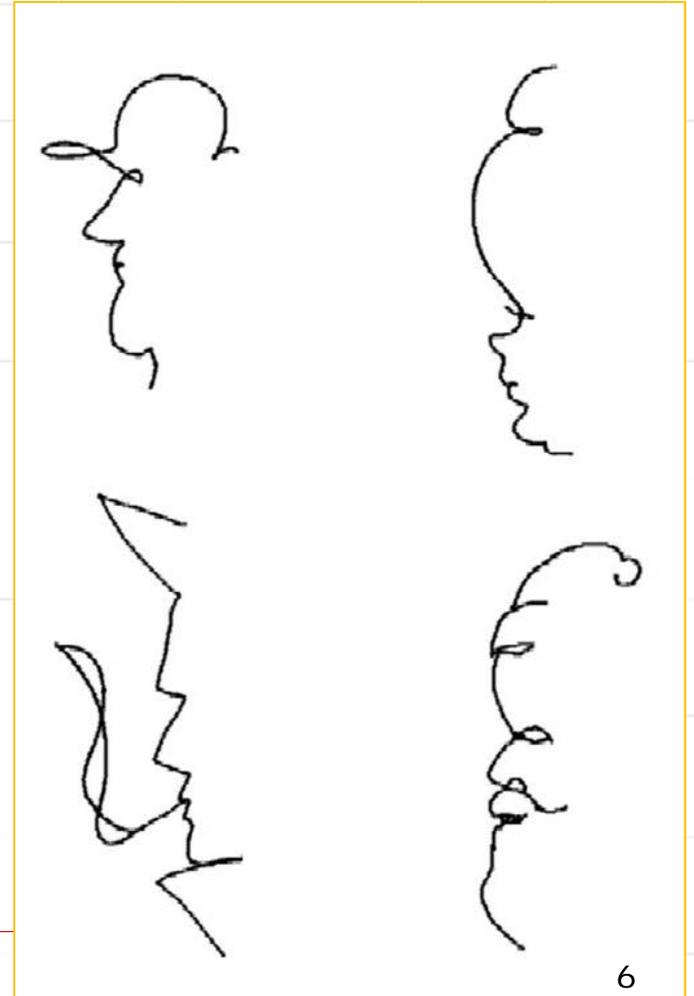
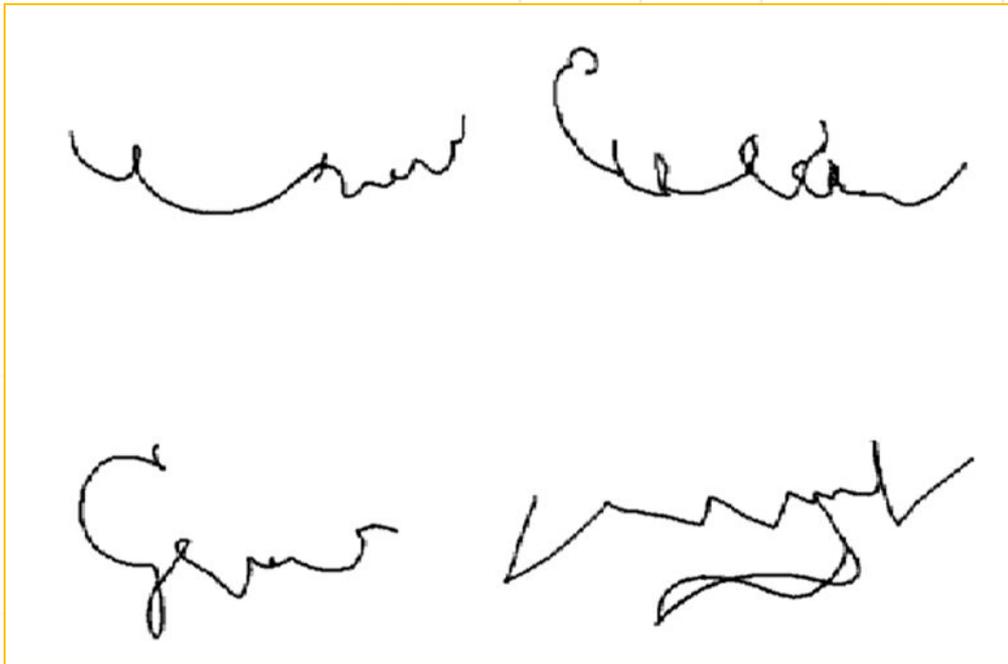
# Learning to perceive

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- A good deal of cognitive theory examines what students do with the information in their heads.
  - Problem solving, working memory, retention...
  
- A different issue is how to get students to see the information in the first place.
  - Theories of perception are most relevant here.

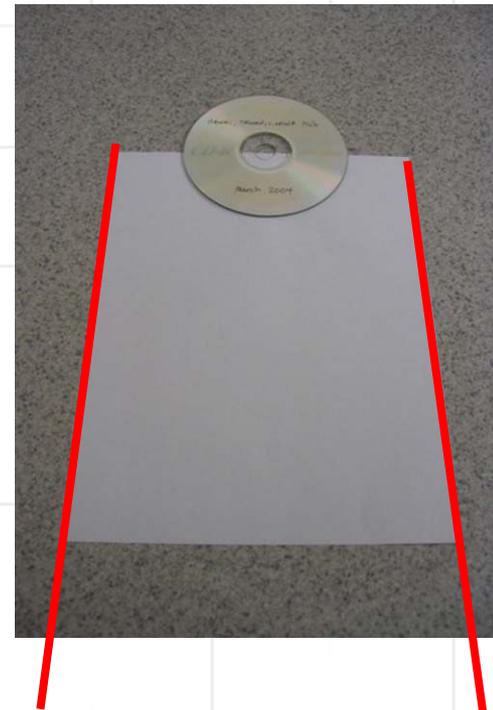
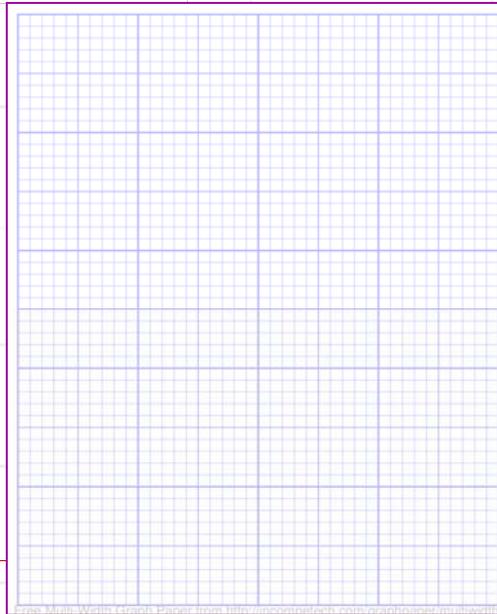
# Sensation v. Perception

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# Perception: Extracting the invariant structure amidst surface variation.

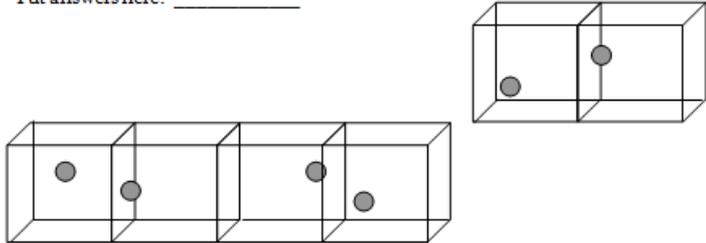
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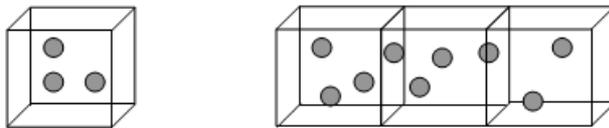
# Finding the invariant or “deep” structure is also relevant to things less obviously perceptual.

Both Have  
1:1 Ratio

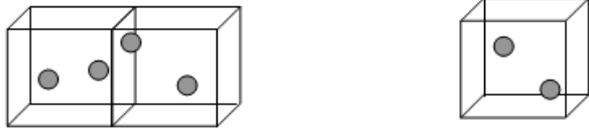
Put answers here: \_\_\_\_\_



Put answers here: \_\_\_\_\_



Put answers here: \_\_\_\_\_

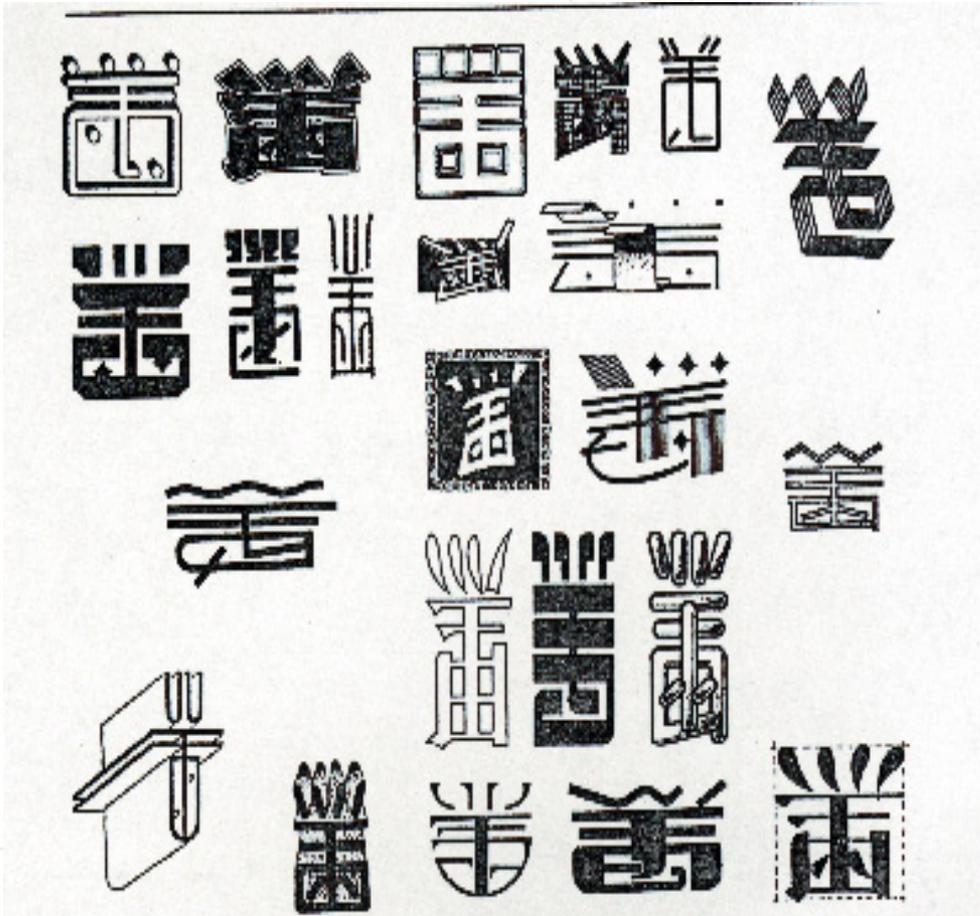


All are  
Proportions

$d=m/V$  describes  
the invariant.

# People Learn to Perceive Structure

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# How can we help students learn to perceive structure?

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- Studies have found that students do better if they receive abstract cases. The claim is as follows:
  - Novices naturally pay attention to surface features.
    - Springs and inclined planes.
  - Surface features obscure invariant (deep) structure
    - Therefore, teach more abstractly with less contextualization.
  
- Current studies challenge this story:
  - They show that instruction is the major source of variance, not the concreteness or abstractness of the materials.



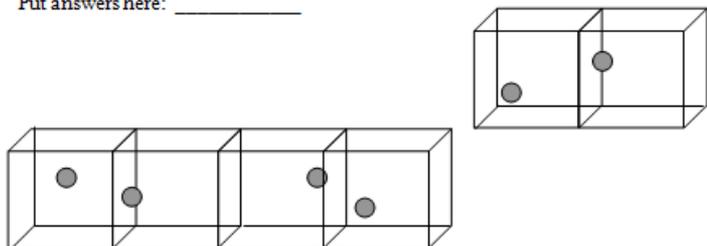
# Outline

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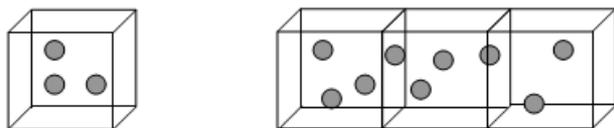
- Learning to Perceive
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# Will these examples differ in how well students learn to “see” density?

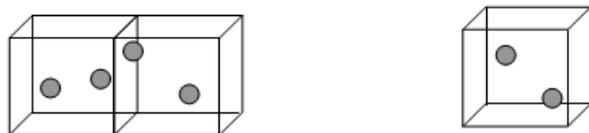
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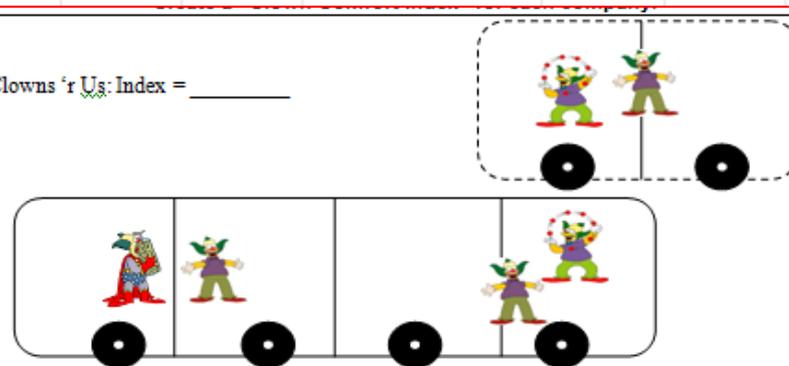
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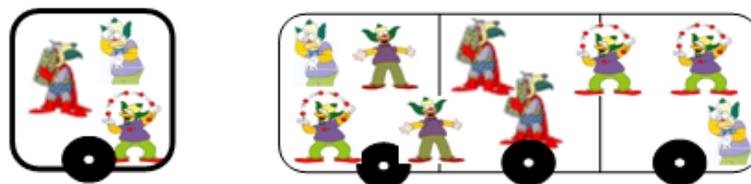
Put answers here: \_\_\_\_\_



Clowns  $r$  Us: Index = \_\_\_\_\_



Bargain Basement Clowns: Index = \_\_\_\_\_

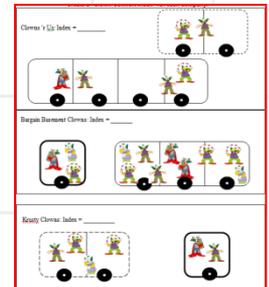
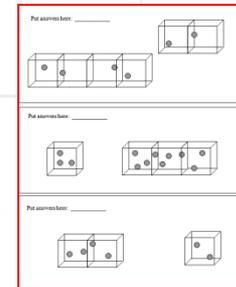


Krusty Clowns: Index = \_\_\_\_\_



# How does instruction affect learning to perceive?

- ~100 8<sup>th</sup>-graders.
  
- 2x2 Design
  - Cases: **Abstract v. Concrete**
  - Instruction: **Tell v. Invent**



## FINDING DENSITY

**Density** is how much stuff is packed into a space. Density can be the number of people in a room, the density of feathers in a pillow, and many other things.

Density is very important in chemistry. Density is a property of **matter**. Gold is denser than carbon, because more matter is packed into each atom of gold compared to each atom of carbon.

When working with density, the trick is to use the simple equation:

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} \quad \text{or} \quad D = \frac{M}{V}$$

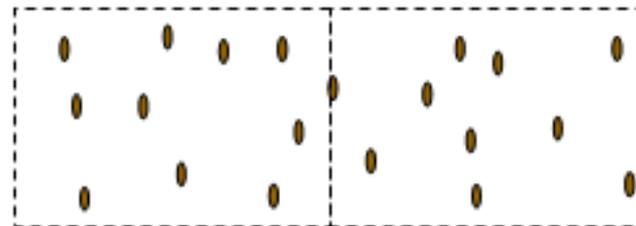
*Density is a measure of the mass of a substance per unit of volume.*

Sometimes **Mass** can be found by counting.

**Volume** is the amount of space. Volume is harder to find, because a volume can take many shapes – a sphere, a balloon, a bottle.

To make it easier, we will tell you the volume. We will measure it in fields.

In the example below, there are two fields. There are flower seeds scattered across the fields. The density is the average number of flower seeds per unit of volume.



$$\begin{aligned} \text{Density} &= \text{mass} / \text{volume} \\ &= 20 \text{ seeds} / 2 \text{ fields} \\ &= 10 \text{ seeds} / \text{field} \\ &= 10 \text{ seeds per field} \end{aligned}$$

Tell

Invent

### Inventing an Index

An **index** is a number that helps people compare things.

*Miles per gallon* is an index of how well a car uses gas.

*Batting average* is an index of how well a baseball player hits.

*Grades* are an index of how well you are doing in school.

*Star ratings* on iTunes will show how much people liked a song or album.

We want you to invent a procedure for computing one kind of index.

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### THE CROWDED MOLECULES INDEX

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Companies send molecules to factories, universities, research organizations, and so on.

To get the molecules there, each company packs the molecules into a container.

Some companies make the molecules more crowded than other companies.

People who order molecules want to know a company's "Crowded Molecule Index."

Invent a procedure for computing a Crowded Molecule Index for each company.

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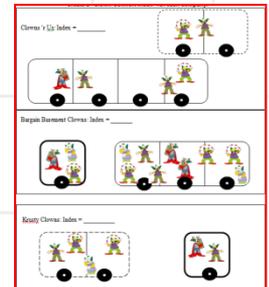
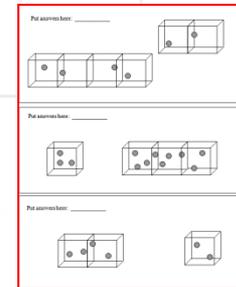
### RULES FOR THE INDEX

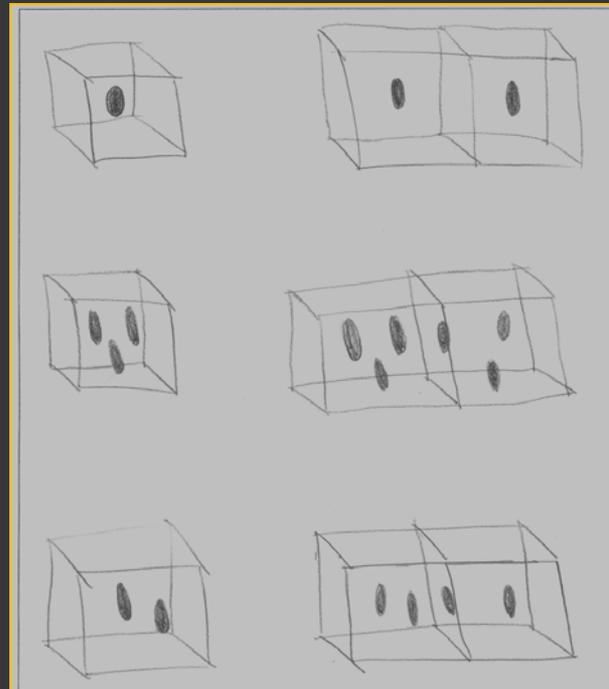
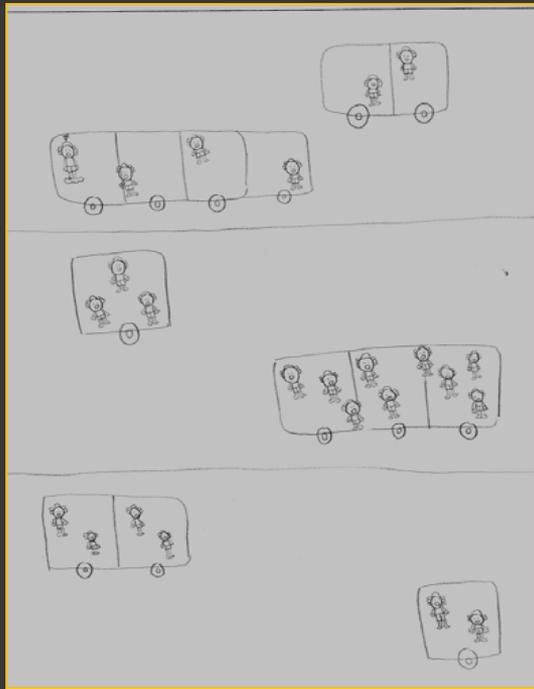
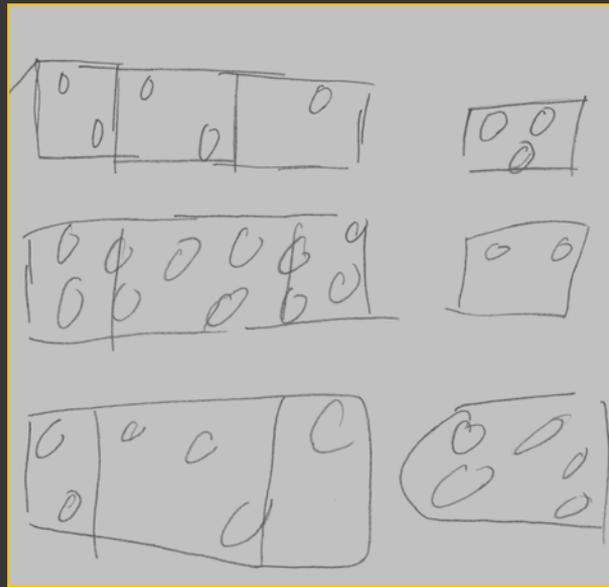
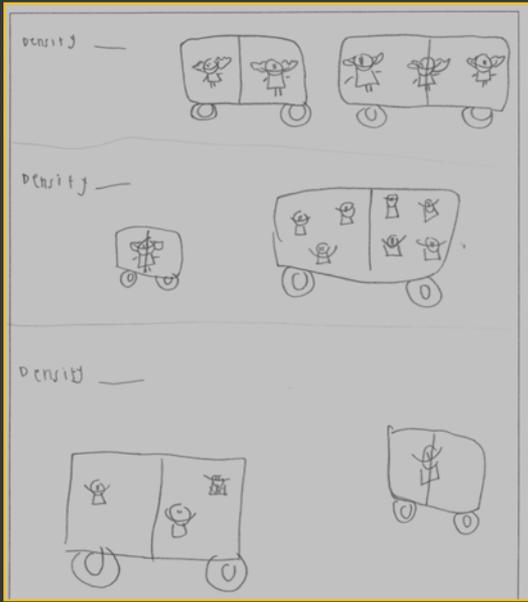
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1. **The same company always crowds the molecules the same amount, no matter how many molecules get ordered. So a company only gets a single Crowded Molecule Index.**
  2. **You have to use the exact same procedure for each company to find its index.**
  3. **A big index value should mean that the molecules are more crowded. A small index number should mean that the molecules are less crowded.**
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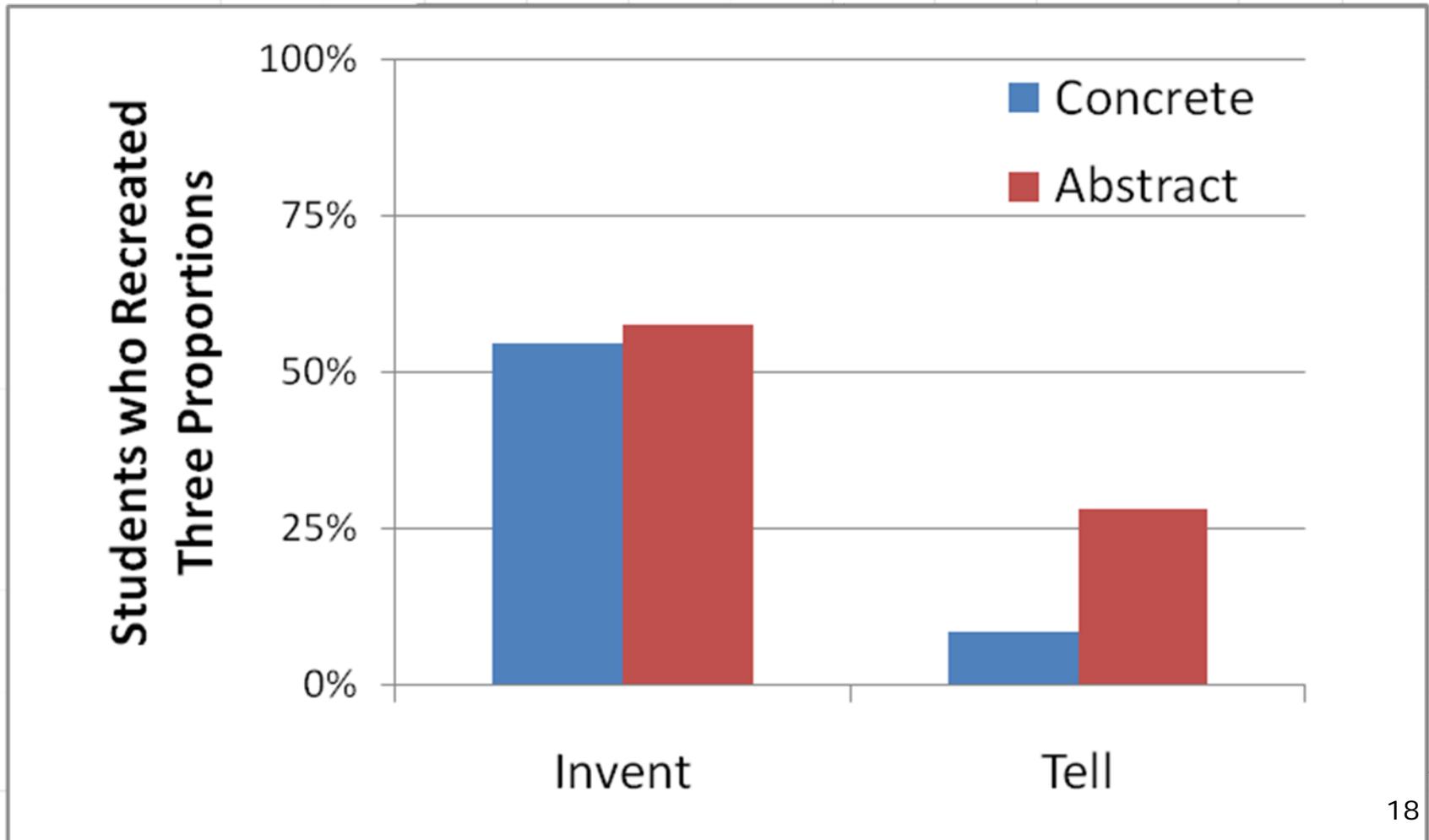
# How does instruction affect learning to perceive?

- ~100 8<sup>th</sup>-graders.
  
- 2x2 Design
  - Cases: **Abstract v. Concrete**
  - Instruction: **Tell v. Invent**
  
- Procedure
  - Students did cases on Day 1
  - Students redrew the cases on Day 2





# Students received credit if they included three unique proportions.



# The Blind leading the Blind.

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- Experts often have a blind spot.
    - We forget that students do not see the same structure we do.
    - The students use  $d=m/V$  to solve the problems, so they must see the structure, right?
  
  - Students also have a blind spot.
    - People don't know they are missing what they cannot see.
    - They applied  $d=m/V$  fine, so they never realized there was more to be seen across the cases.
  
  - The Invent students had to notice structure to get going.
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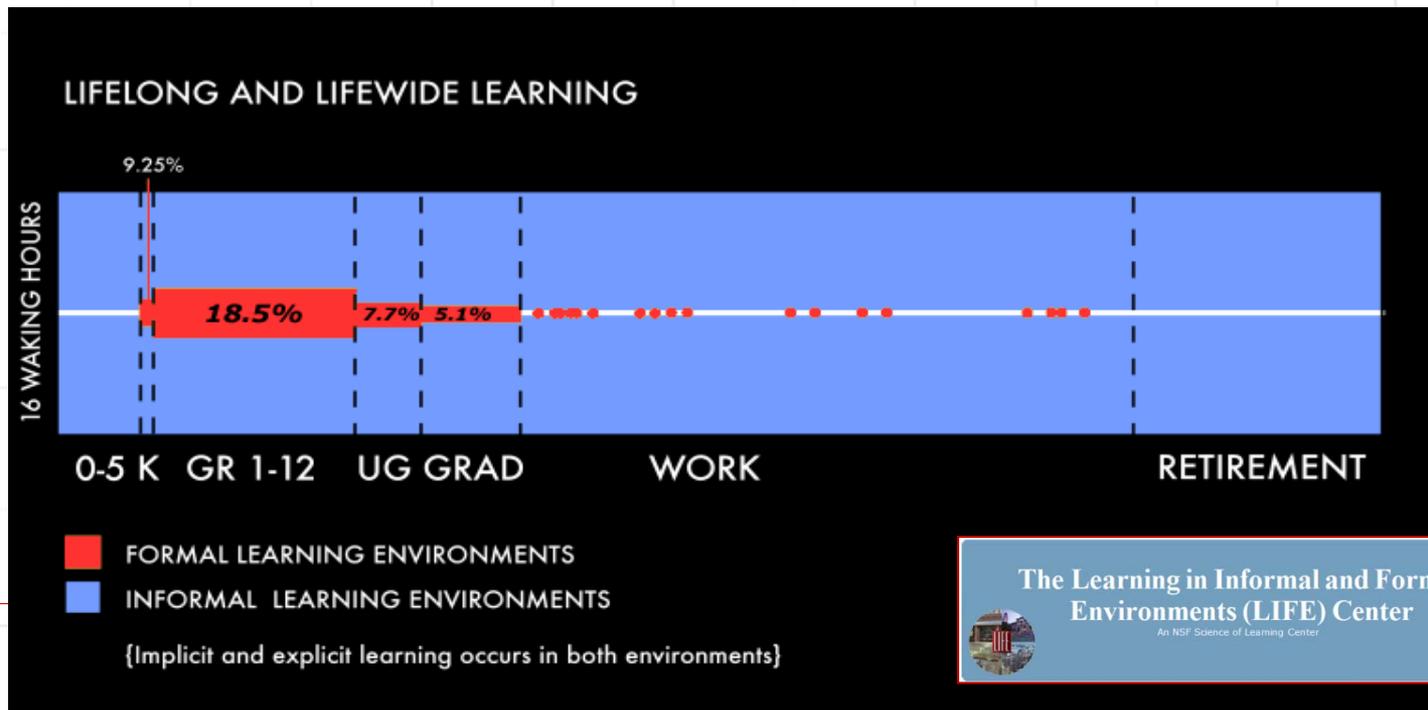
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# Why It Matters

- These “perceptual” issues play out in transfer.
- Transfer involves using learning in a new context.



# Example of (negative) Transfer

(courtesy of Brian Ross)

- Students learned:
  - Combinations using cars as example.
  - Permutations using marbles as example.

- Post-test:

		<u>Correct Solution</u>	
		Combinations	Permutations
	Marbles	<b>P</b>	<b>P</b>
<u>Problem Cover Story</u>		<b>C</b>	<b>C</b>
	Cars		

# Transfer is important.

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- School presupposes that students transfer from class to class, year to year, school to home, home to school.
- Whitehead called lack of transfer, *Inert Knowledge*.
  - People know “it,” but they do not use it when they should.
- Successful transfer depends on two things:
  - (1) Knowledge of general principles, skills, strategies, etc.
  - (2) Knowledge of contexts to which they can apply.
  - Students often learn (1) but not (2). Example of clinical psychologists.
- For marbles and cars, students never saw the structure of permutations and combinations.
  - Students saw math problems about marbles and cars.
  - They transferred based on what they perceived.

# Why People Fail to Transfer

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- Many claim that transfer is rare.
  - Our proposal: The way we teach has a lot to do with it.
- Typical Instruction model: Tell-and-Practice.
  - 1. Students are told what to do or think.
    - Lecture; worked example; written instructions; etc.
  - 2. Students practice on a set of cases.
    - Word problems; visual problems; readings; etc.
- Students focus on the procedure or idea they are told.
  - They encode procedure and the obvious surface features of cases.
  - Never notice the deep structures that generalize across cases.
  - No deep structures, no transfer.

# Developmental Context of the Research

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- Let's see how this plays out in a fuller instructional context.
    - Eight-graders learning density and speed.
  - What do density and speed have in common?
    - Deep structure of ratio:  $D=m/V$ ,  $S=d/t$
  - Ratio and Proportional Reasoning
    - By about 7<sup>th</sup>-grade, most can reason about proportional problems.
      - Which bag is a better bet to get a black marble?
        - Bag 1: 2 blacks and 6 whites.
        - Bag 2: 3 blacks and 10 whites.
  - Doesn't mean they use ratio to understand science.
    - Still a tenuous concept, but a big idea for science at this age.
-

# Effects of Tell-&-Practice on Transfer

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- Topic: Speed and Density – deep ratio structure.
- Four science classes totaling 120 high-diversity 8<sup>th</sup>-graders.
  - Each class split (stratified random assignment) into two rooms.
    - Half received Tell-and-Practice treatment.
    - Half received “Control” treatment (Invent).
- Study spanned 28 days (but only 4 days of instruction).
  - Class level explanations.
  - Small group seat work.
  - Assessments done in “test” mode.

# Tell & Practice

# Inventing

Monday

Density Worked Example  
Cases

Inventing Introduction  
Cases



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When working with density, the trick is to use the simple equation:

$$D = \frac{M}{V} \quad \text{or} \quad \text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

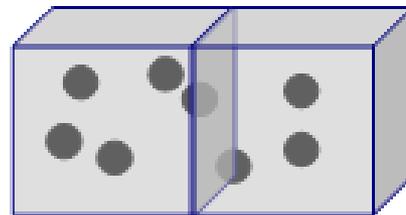
*Density is a measure of the mass of a substance per unit of volume.*

Sometimes **Mass** can be found by counting.

**Volume** is the amount of space. Volume is harder to find, because a volume can take many shapes – a sphere, a balloon, a bottle.

To make it easier, we will tell you the volume. We will measure it in cubes.

In the example below, there are two cubes. There are 8 objects spread across the cubes. Density is the average number of objects per unit of volume.



$$\begin{aligned} \text{Density} &= \text{mass} / \text{volume} \\ &= 8 \text{ objects} / 2 \text{ cubes} \\ &= 4 \text{ objects} / \text{cube} \\ &= 4 \text{ objects per cube} \end{aligned}$$

## Tell & Practice

An **index** is a number that helps people compare things.

*Miles per gallon* is an index of how well a car uses gas.

*Batting average* is an index of how well a baseball player hits.

*Grades* are an index of how well you are doing in school.

*Star rating* is an index of how efficient an electrical appliance is.

We want you to invent a procedure for computing one kind of index.

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### THE CROWDED CLOWNS INDEX

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Companies send clowns to parties, circuses, amusement parks, sporting events, and so on.

To get the clowns to the event, each company packs the clowns into a bus.

Some companies make the clowns more crowded than other companies.

The more crowded the clowns are, the grumpier they will be.

People who order clowns want to know a company's crowded clown index.

Invent a procedure for computing a crowded clown index for each company



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### RULES FOR THE INDEX

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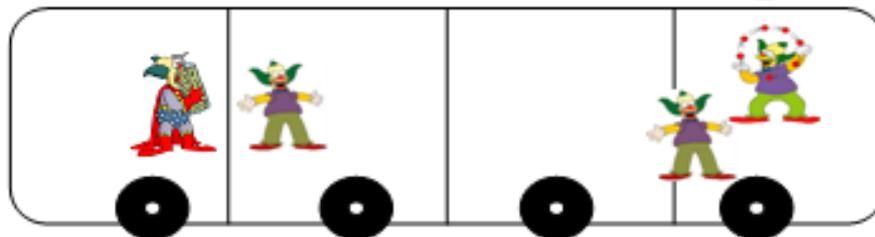
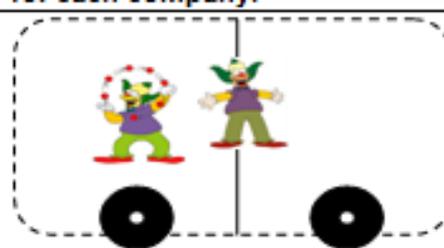
**1. The same company always crowds the clowns the same amount, no matter how many clowns get ordered. So a company only gets a single crowded clown index.**

**2. You have to use the exact same procedure for each company to find its index.**

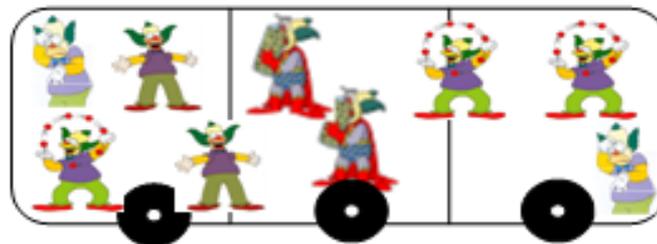
**3. A big index value should mean that the clowns are more crowded. A small index number should mean that the clowns are less crowded.**

# Inventing an Index

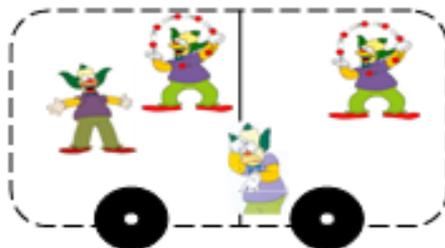
Clowns 'r Us = \_\_\_\_\_



Bargain Basement Clowns: = \_\_\_\_\_



Krusty Clowns = \_\_\_\_\_



- The Cases:
- a) Practice Density.
  - b) Invent an Index.

# Tell & Practice

# Inventing

Monday

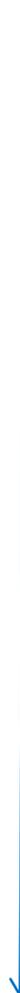
Density Worked Example  
Cases

Inventing Introduction  
Cases

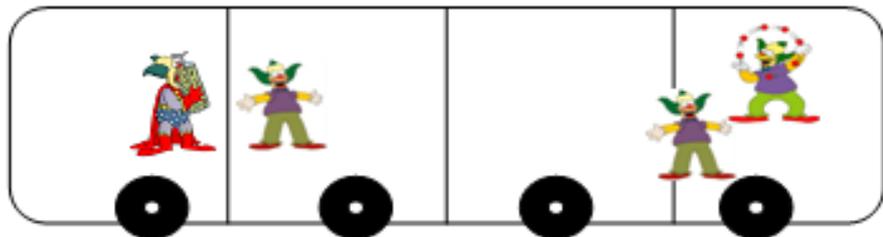
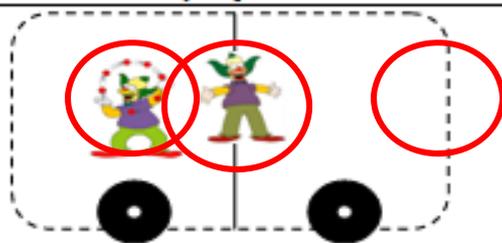
Memory Test

Memory Test

Tuesday



Clowns 'r Us = \_\_\_\_\_



# Coded Drawings for:

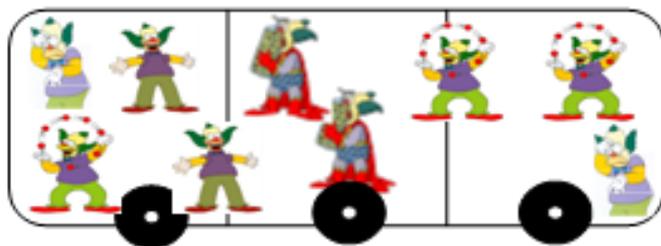
Deep Structure  
Proportionate Ratios

(3 pts possible)

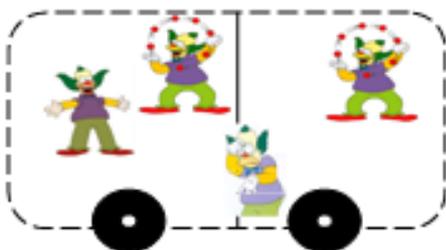
## Surface Features

- Clown elaboration
- Clowns on lines
- Bus outline
- Wheels (not on count)
- Excessive text recall
- “Krusty clowns...”

Bargain Basement Clowns: = \_\_\_\_\_



Krusty Clowns = \_\_\_\_\_

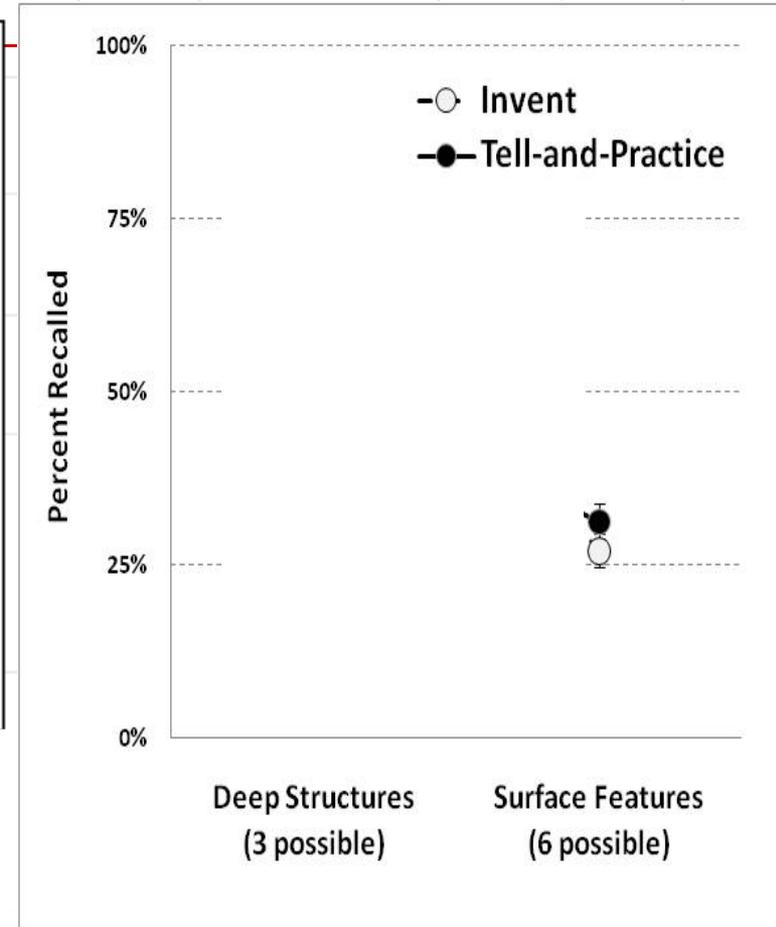


(6 pts possible) 32

# Encoding Results

(b) Samples of recall 24 hrs later.

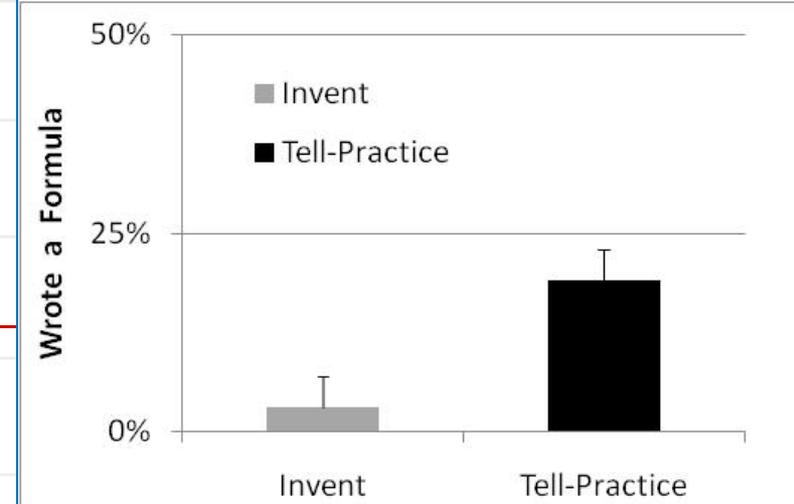
	High Structure	Low Structure
High Surface		
Low Surface		



T&P paid attention to what they were told.  
 They solved the crowded clowns fine.  
 They just did not see the structure.

# Review of First Results

- Much like earlier study.
- Tell-&-Practice missed the problem structure.
  - Paid attention to what they were told.
  - They learned the structure of the formula but not its reference.
- Invent condition encoded deep and surface features.
  - Surface and deep were not trading off. (Surface not blocking deep.)
    - ( $r = -.08$ )
- Instruction had consequences for transfer.



## Tell & Practice

## Inventing

Monday

Density Worked Example  
Cases

Inventing Introduction  
Cases

Memory Test

Memory Test

Tuesday

Lecture on Ratio in Physics  
Speed Example + Cases

Invent Intro + Cases



- Popcorn poppers
  - Speed

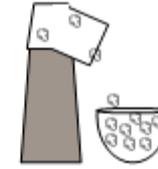
Poppomatic Popcorn = \_\_\_\_\_



12 seconds



18 seconds



24 seconds



Done

HotPops = \_\_\_\_\_



20 seconds



30 seconds

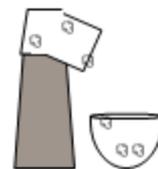


50 seconds



Done

HipHop Popping Corn = \_\_\_\_\_



6 seconds



12 seconds



20 seconds



Done

## Tell & Practice

## Inventing

Monday

Density Worked Example  
Cases

Inventing Introduction  
Cases

Memory Test

Memory Test

Tuesday

Lecture on Ratio in Physics  
Speed Example + Cases

Invent Intro + Cases

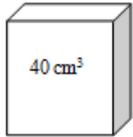
Friday

Density Example + Cases  
Speed Example + Cases

Invent Intro + Cases  
Invent Intro + Cases

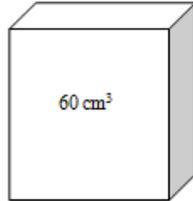


Diamond Jim's -----



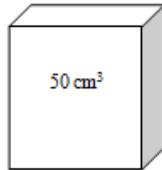
40 cm<sup>3</sup>

160 grams



60 cm<sup>3</sup>

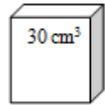
240 grams



50 cm<sup>3</sup>

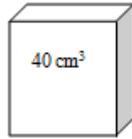
200 grams

Gold miner gems -----



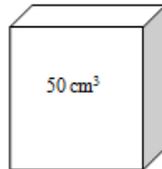
30 cm<sup>3</sup>

150 grams



40 cm<sup>3</sup>

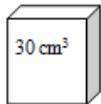
200 grams



50 cm<sup>3</sup>

250 grams

Jewelry Jane -----



30 cm<sup>3</sup>

30 grams



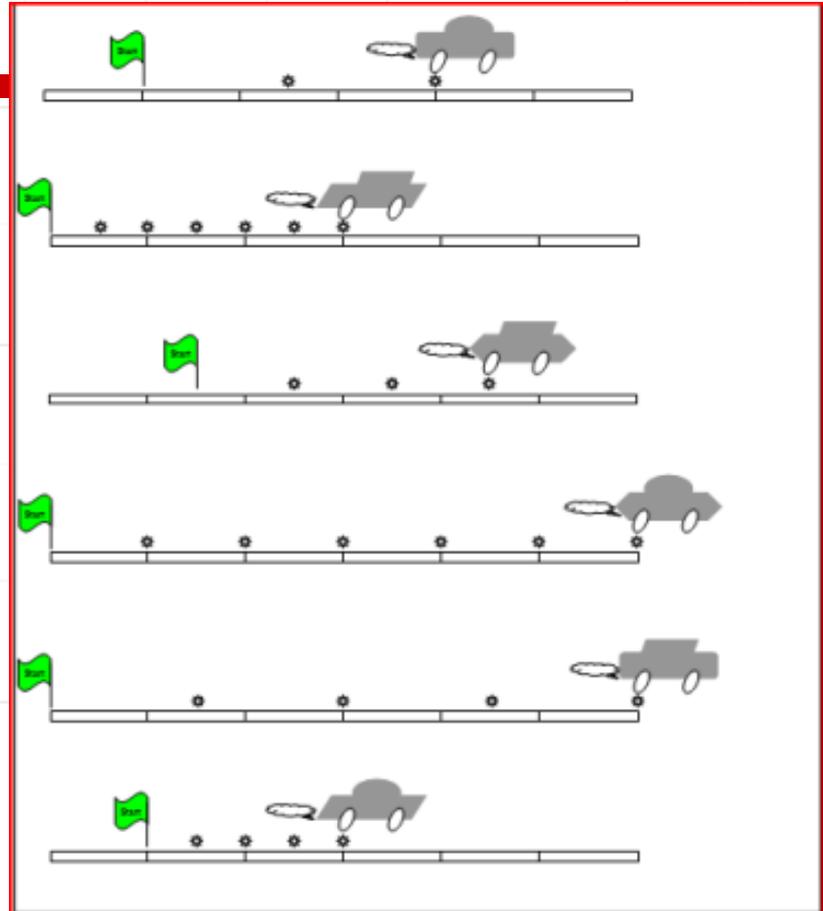
10 cm<sup>3</sup>

10 grams



20 cm<sup>3</sup>

20 grams



# By end of first week...

---

- Tell & Practice had received:
  - 4 analogs with instruction and worked examples.
    - Density and Speed
    - Continuous and Discrete
  - Explicit instruction that ratio is an important, common structure in physics.

## Tell & Practice

## Inventing

Monday

Density Worked Example  
Cases

Inventing Introduction  
Cases

Memory Test

Memory Test

Tuesday

Lecture on Ratio in Physics  
Speed Example + Cases

Invent Intro + Cases

Friday

Density Example + Cases  
Speed Example + Cases

Invent Intro + Cases  
Invent Intro + Cases

Transfer Test #1

Transfer Test #1

Monday

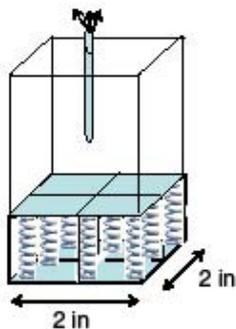
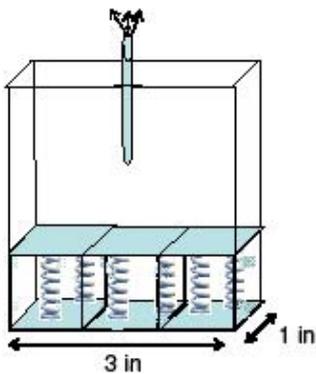
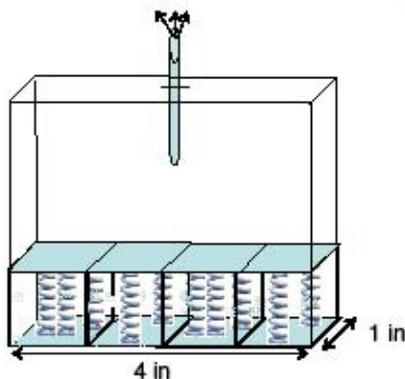
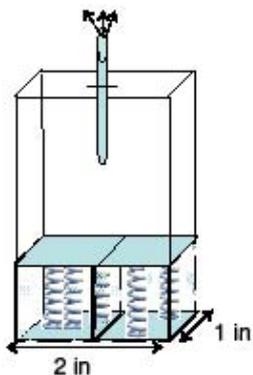


Name \_\_\_\_\_

Period \_\_\_\_\_

Two companies make aerosol containers for people who produce paint, bug spray, air freshener, and other products. Aerosol cans have pressure that pushes the spray out through the nozzle. Describe the *pressure* that each company uses in their aerosol cans.

Company #1

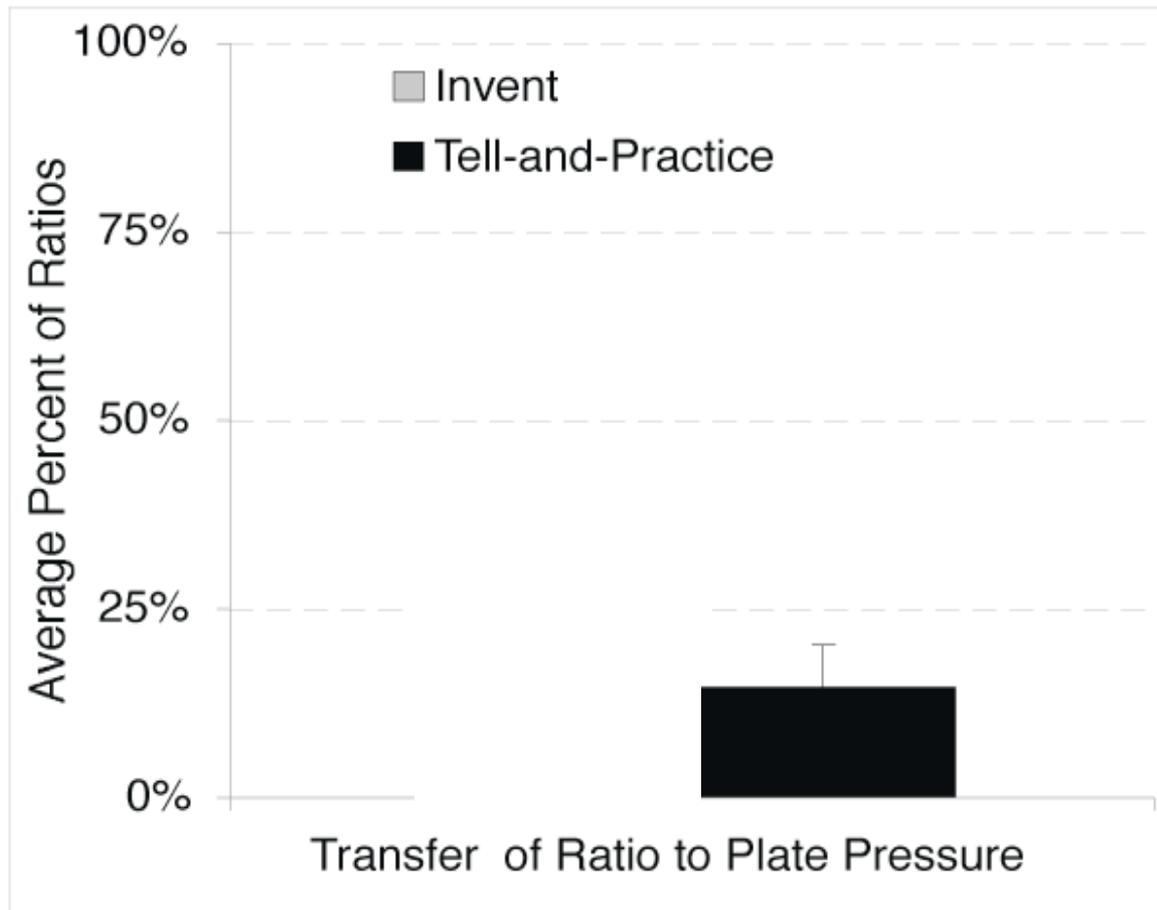


“Describe the pressure that each company uses in their aerosol cans.”

Surface pressure on plates is ratio of Springs (force)/Area.

Coded for use of ratio in description.  
(4 pts = 100%)

# Transfer Results



# Refining the point...

---

- The lack of transfer was not due to telling *per se*.
  - Telling people things is important!
  
- Transfer failed in this case, because direct instruction shortcut the inductive process of finding structure of cases.
  
- Telling is OK if it happens after students have a chance to engage structure.

## Tell & Practice

## Inventing

Monday

Density Worked Example  
Cases

Inventing Introduction  
Cases

Memory Test

Memory Test

Tuesday

Lecture on Ratio in Physics  
Speed Example + Cases

Invent Intro + Cases

Friday

Density Example + Cases  
Speed Example + Cases

Invent Intro + Cases  
Invent Intro + Cases

Transfer Test #1

Transfer Test #1

Monday

Word Problem Practice

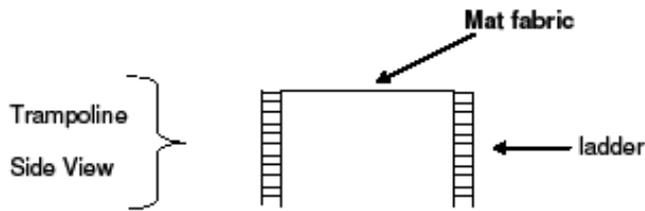
Lecture on Density, Speed,  
& Ratio in Physics

Word Problem Practice

3 Weeks Later  
(Monday)

Transfer Test #2

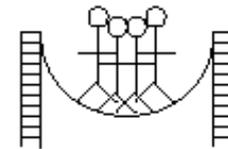
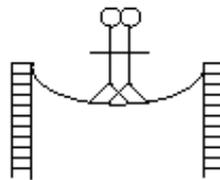
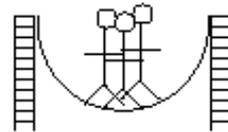
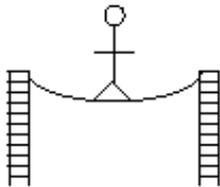
Transfer Test #2



Trampolines are made with mats using different fabrics.

Stiffer mats make the trampoline bouncier.

Determine the stiffness of the mat fabric for each trampoline.

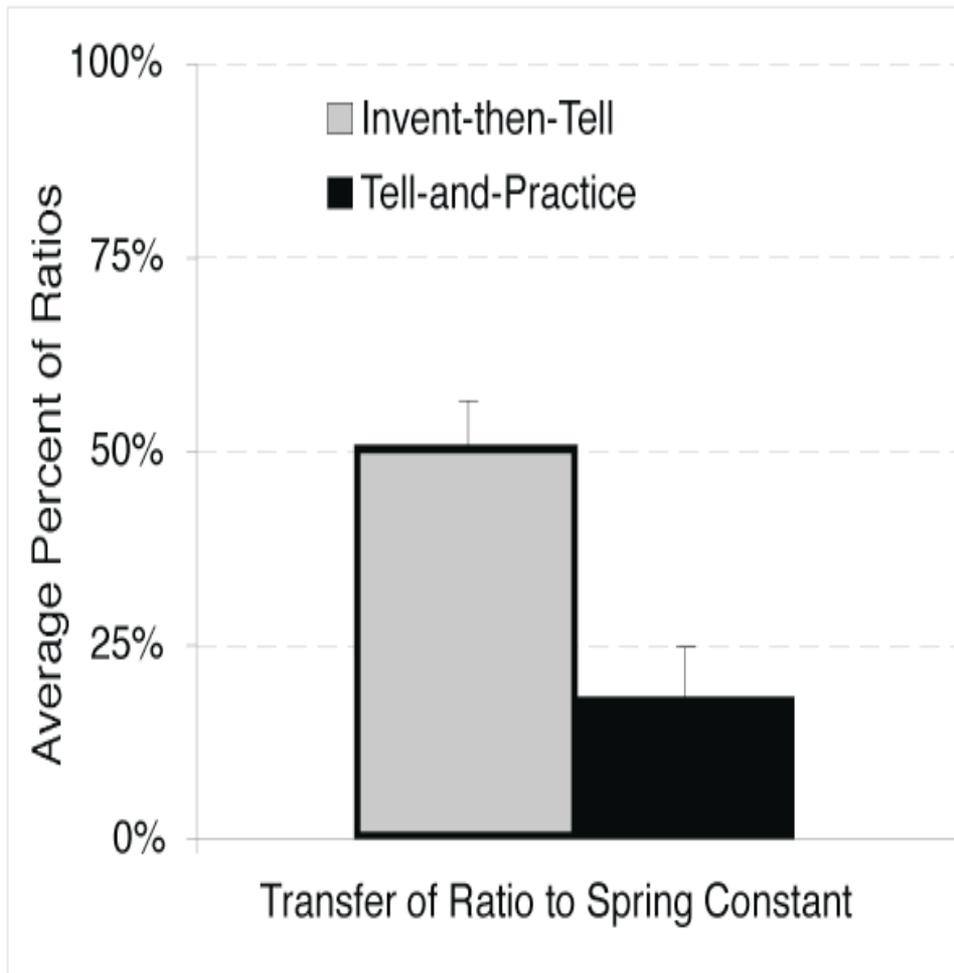


“Describe the stiffness of the mat fabric for each trampoline.”

Application of spring constant ratio of stretch by people (weight)

No mention of “companies” or paired cases.

Coded for use of ratio in description.  
(4 pts = 100%)



Delayed Transfer Rates

## Tell & Practice

## Inventing

Monday

Density Worked Example  
Cases

Inventing Introduction  
Cases

Memory Test

Memory Test

Tuesday

Lecture on Ratio in Physics  
Speed Example + Cases

Invent Intro + Cases

Friday

Density Example + Cases  
Speed Example + Cases

Invent Intro + Cases  
Invent Intro + Cases

Monday

Transfer Test #1  
Word Problem Practice

Transfer Test #1  
Lecture on Density, Speed,  
& Ratio in Physics

Word Problem Practice

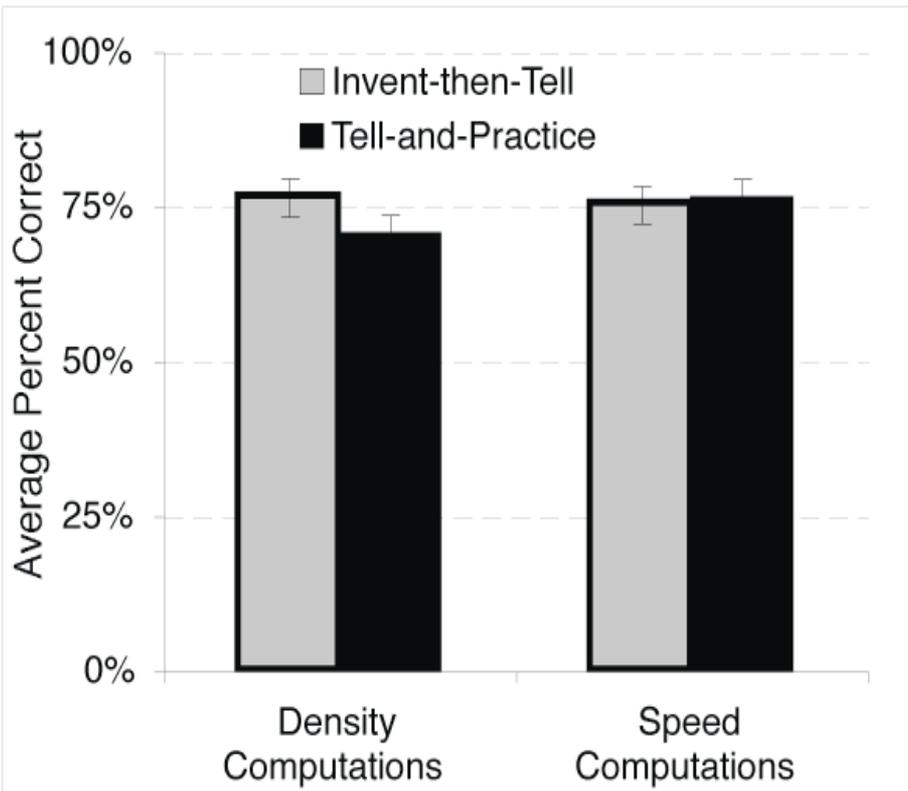
3 Weeks Later  
(Monday)

Transfer Test #2  
Word Problem Test

Transfer Test #2  
Word Problem Test



1. Time = 3 hours. If speed = 36 mph, what is the distance traveled?
2. Brenda packs 120 marshmallows into 4 soda cans. Sandra packs 300 marshmallows into 11 soda cans. Whose soda cans are more densely packed?
- 3...



Correct Answers for  
for Simple Word Problems

## Tell & Practice

## Inventing

Monday

Density Worked Example  
Cases

Inventing Introduction  
Cases

Memory Test

Memory Test

Tuesday

Lecture on Ratio in Physics  
Speed Example + Cases

Invent Intro + Cases

Friday

Density Example + Cases  
Speed Example + Cases

Invent Intro + Cases  
Invent Intro + Cases

Monday

Transfer Test #1  
Word Problem Practice

Transfer Test #1  
Lecture on Density, Speed,  
& Ratio in Physics

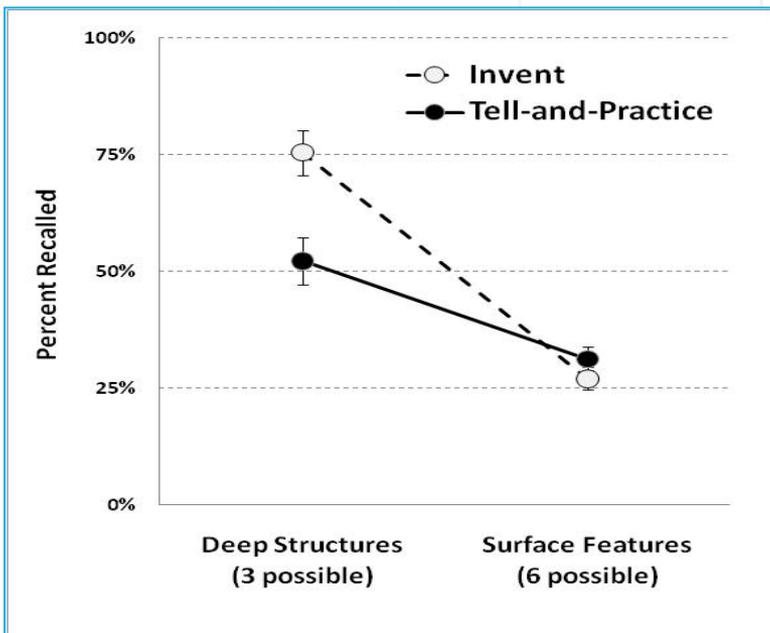
Word Problem Practice

3 Weeks Later  
(Monday)

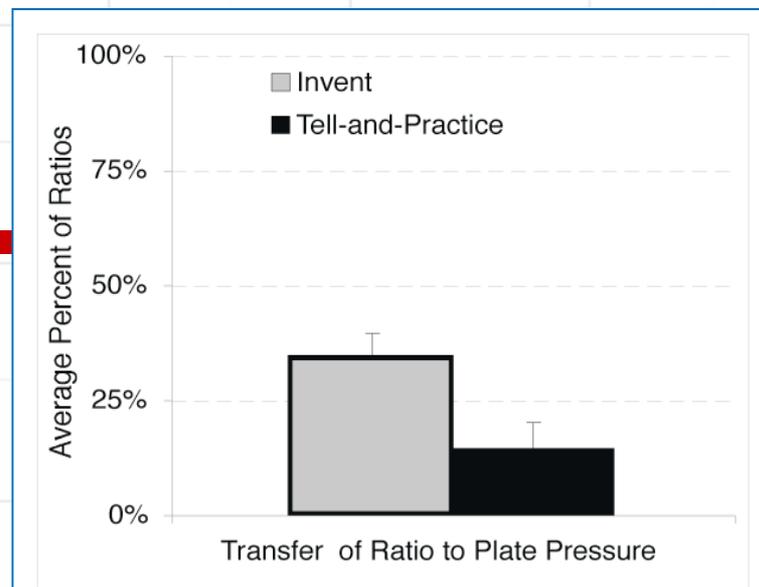
Transfer Test #2  
Word Problem Test

Transfer Test #2  
Word Problem Test

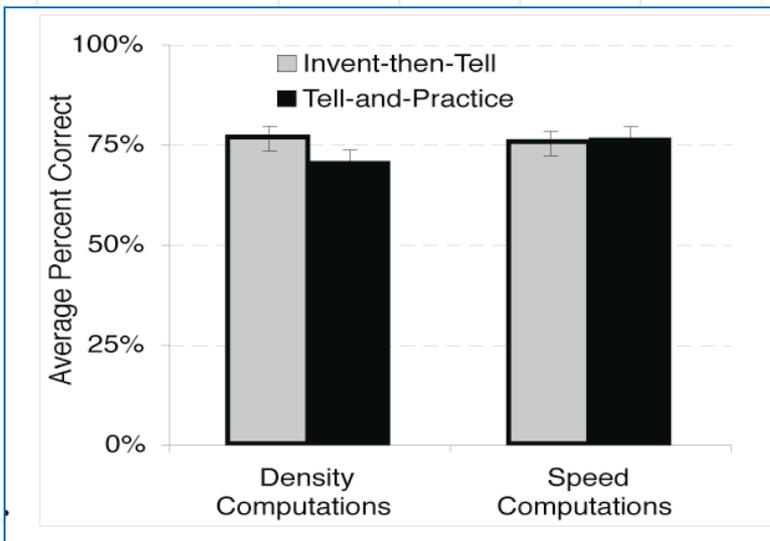




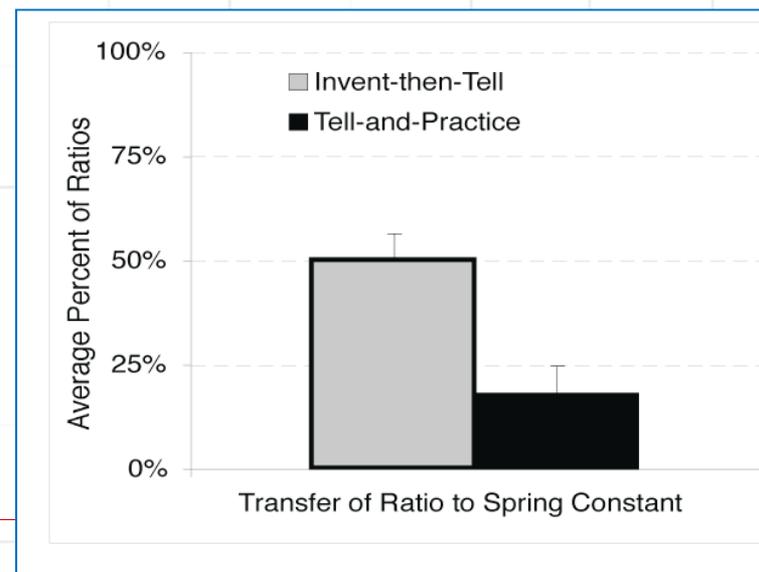
Memory 24 hours later



Transfer before Invent is "told"



Delayed word problem test



Delayed transfer after Invent is "told"

# Typical Instruction Earns a C- for Transfer

---

- Articles in Web of Science using transfer over last 5 years.
- 75% used Tell-and-Practice for treatment and control.
  - Of those that used only Tell-and-Practice, 40% did not bother to mention the method of instruction in the abstract.
- Impressive numbers indicate entrenchment.
  - Tell-and-Practice instruction is the average.
  - It makes sense that surface features appear to be a problem.
- But average transfer research still earns below a “C”
  - A bad way to find the psychological invariants of learning is to examine one context – tell and practice.

# Outline

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- Learning to Perceive
- Missed in Plain Sight
- Why it Matters
- □ Why Inventing Instruction Works
- Summary

# What makes Inventing effective?

---

- Many alternatives to tell-and-practice.
  - Inquiry-, project-, problem-based.
  
- Inventing differs.
  - Does not replace direct instruction.
  - Complements it.
  
- Elements of good invention activity...

# Contrasting cases.

- Highly structured cases make tasks brief.
  - They can help students discern differences, and find deep structures amidst them.

Clowns 'r U;: Index = \_\_\_\_

Bargain Basement Clowns: Index = \_\_\_\_

Krusty Clowns: Index = \_\_\_\_

# Inventing compact representations

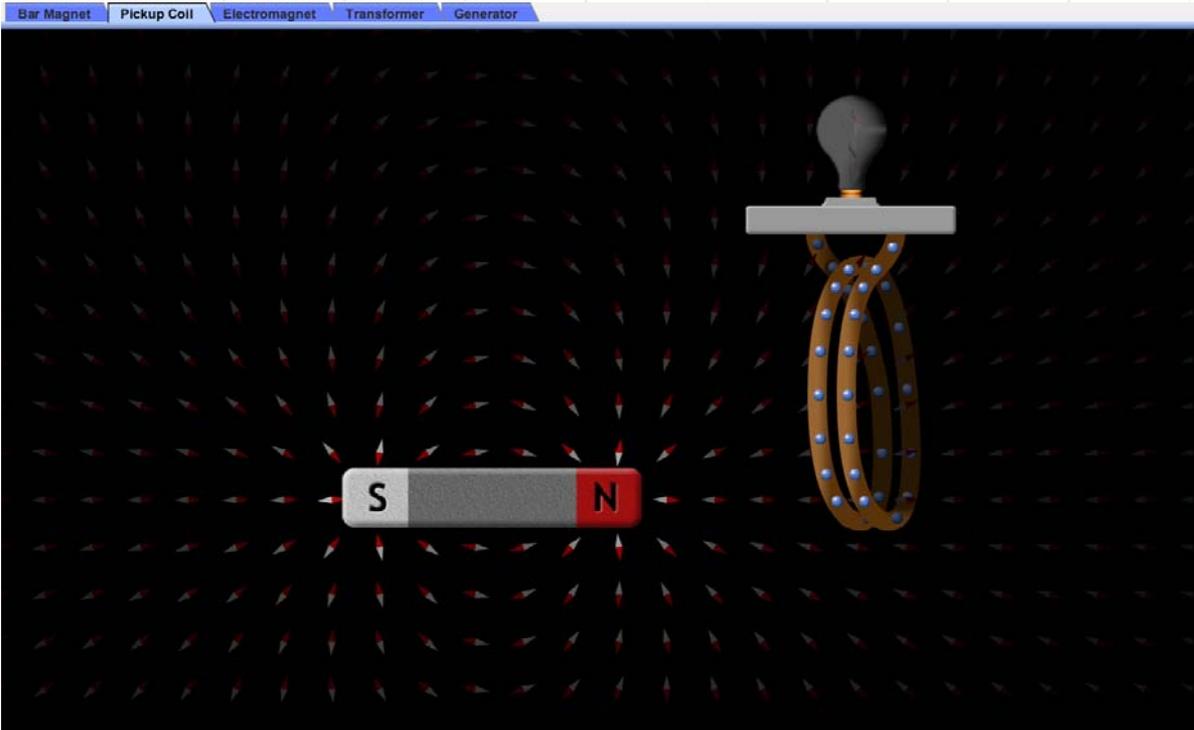
---

- Contrasting cases important but not sufficient.
  - Tell-and-Practice received same contrasting cases.
  - Inventing yielded 3 to 4 times the level of transfer.
  
- Inventing activity
  - Creating one representation for all the cases orients students to deep structure that generalizes across cases.
  - Students do not naturally look for the generalization.
    - A brief interlude...

John Shemwell



Cathy Chase

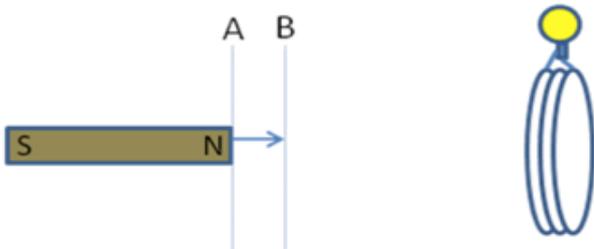


100 Stanford Undergraduates  
in Lab section.

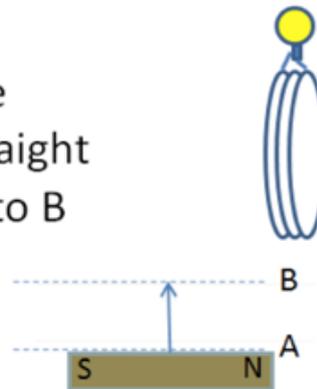
They had already received a  
lecture on Faraday's Law

**Predict → Observe → Explain**  
**Vs.**  
**General Explanation**

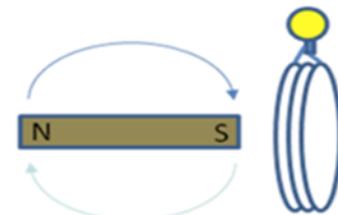
Moving the magnet slightly  
closer from far away



Moving the  
magnet straight  
up from A to B



Flipping the polarity  
of the magnet



# Results

□ Predict → Observe → Explain

- Stayed at surface of each instance, one at a time.
- “Magnet moved closer and changed the field.”

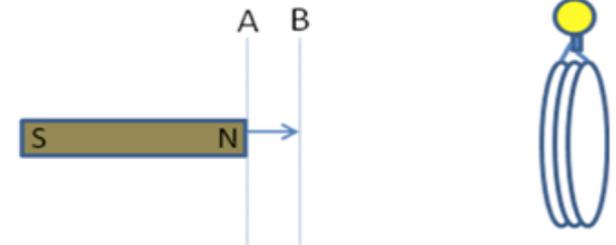
□ General Explanation

- Had to find deep structure amidst the cases.
- “A change in x-vector of field causes...”

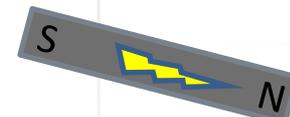
□ Differences at posttest.

- POE: 12%
- GE: 45%

Moving the magnet slightly closer from far away

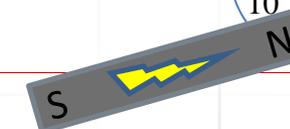


A: Starts out on



10°

10°



B: Starts out off



# Replication with small variations.

---

- Topic: Speed and Density – deep ratio structure.
- Four science classes totaling 140 high-diversity 8<sup>th</sup>-graders.
  - Lower achieving kids
    - One class 25% special needs.
    - One class tracked “low”
- Other differences
  - No class level discussions (except lectures).
  - Cut instructional time by 25%.
  - Removed Transfer Test #1 (for time reasons).
  - Videotaped 12 pairs in each condition.
  - Added a factor using the transfer problem (discuss later)

## Tell & Practice

## Inventing

Monday

Density Worked Example

Cases

Inventing Introduction

Cases

Tuesday

Memory Test

Lecture on Ratio in Physics

Speed Example + Cases

Density Example + Cases

Memory Test

Invent Intro + Cases

Invent Intro + Cases

Wednesday

Speed Example + Cases

Word Problem Practice

Invent Intro + Cases  
Lecture on Density, Speed,  
& Ratio in Physics

Friday

Word Problem Practice

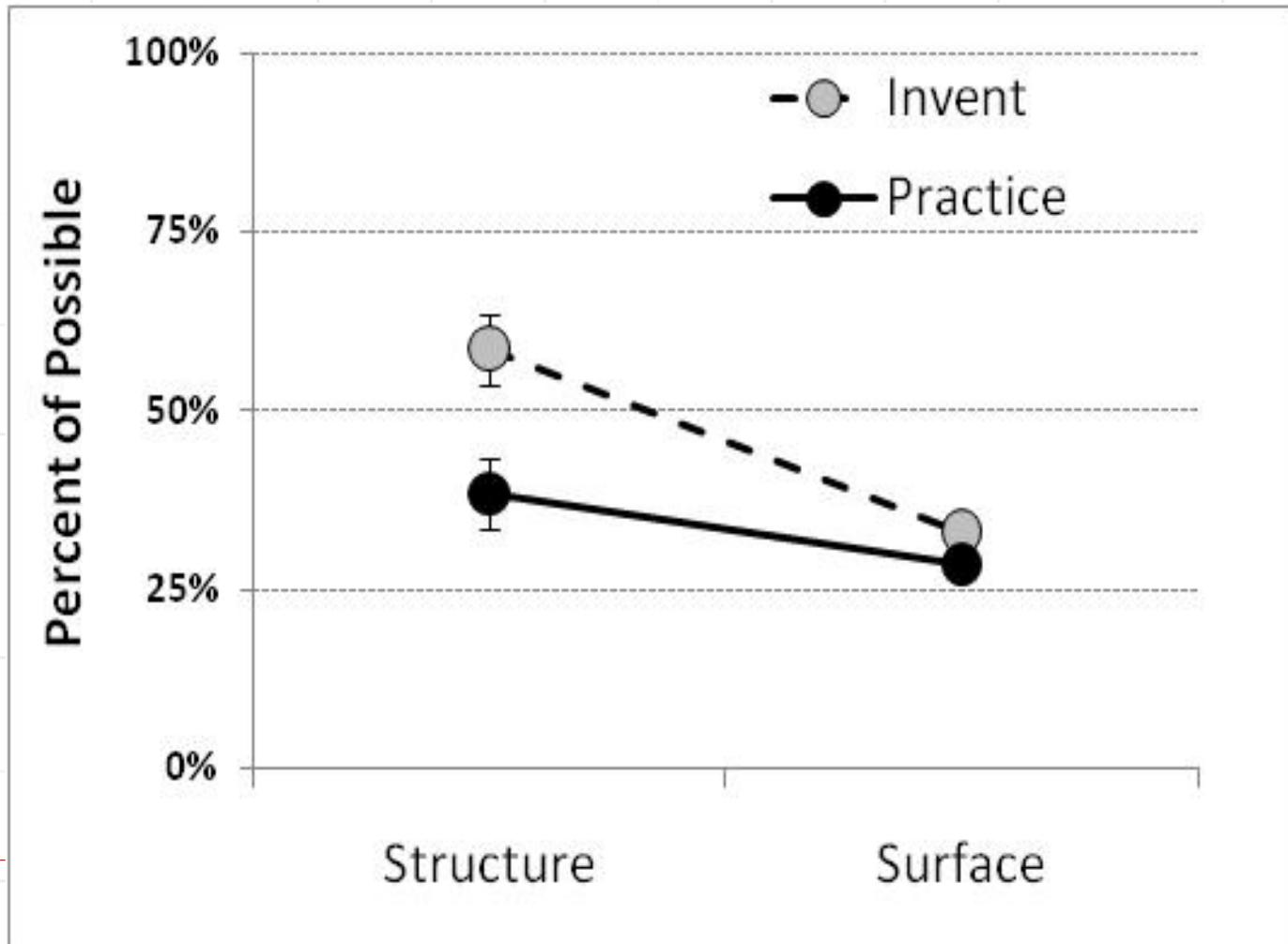
Word Problem Practice

1 Week Later  
(Friday)

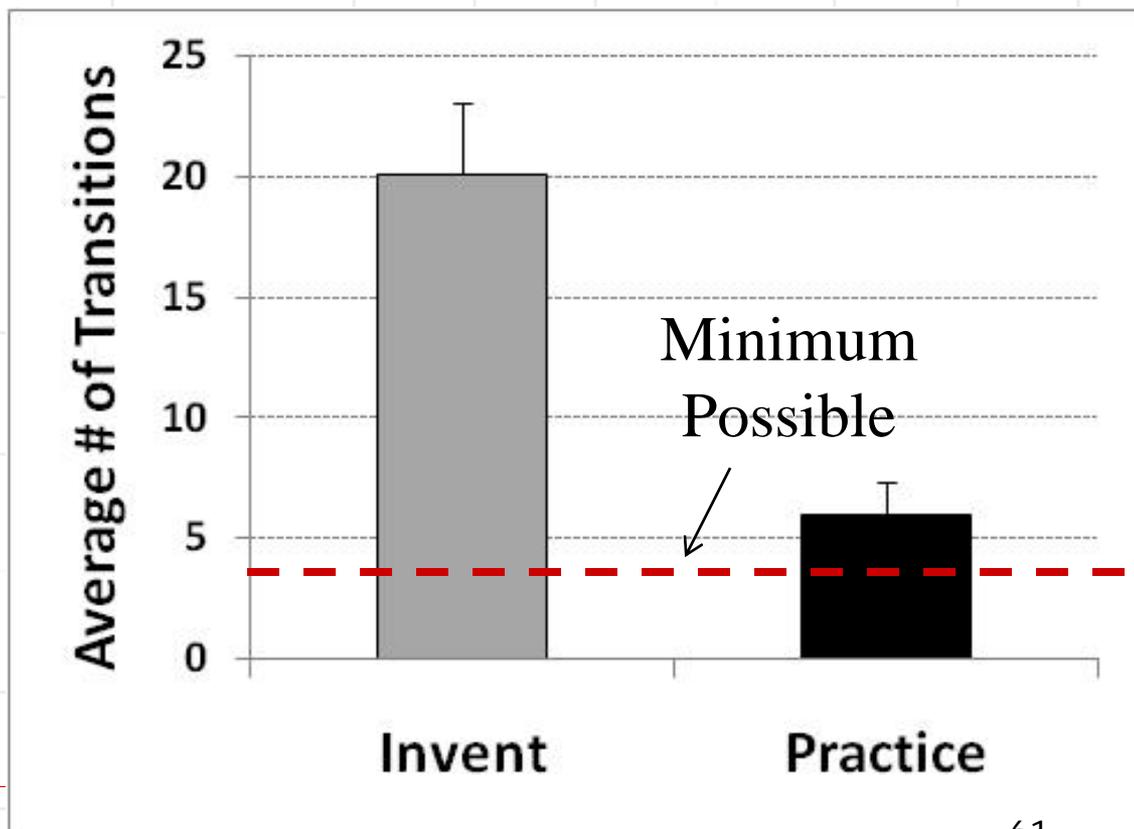
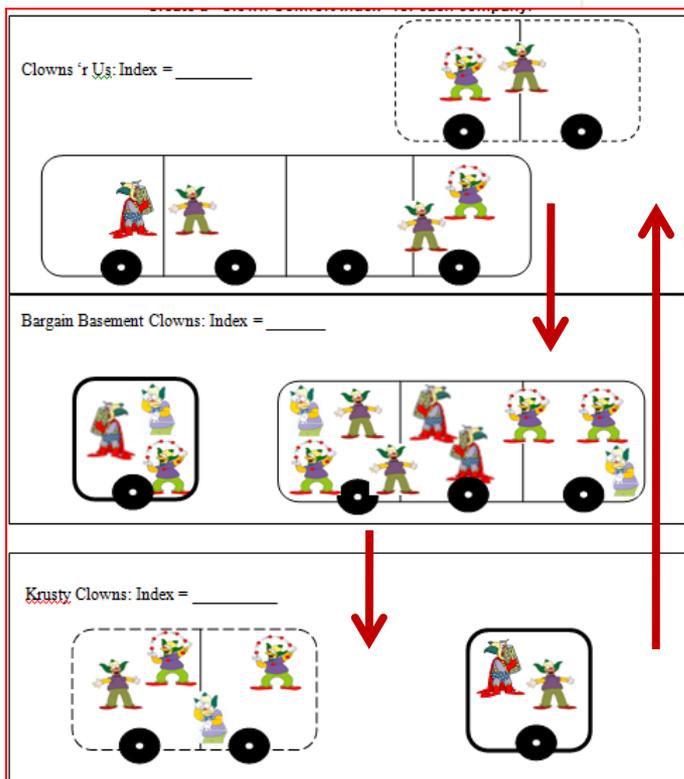
Transfer Test  
(single v. quad)  
Word Problem Test

Transfer Test  
(single v. quad)  
Word Problem Test

# Memory for Crowded Clowns



# Invent students compared across cases, Tell-and-Practice students did not.



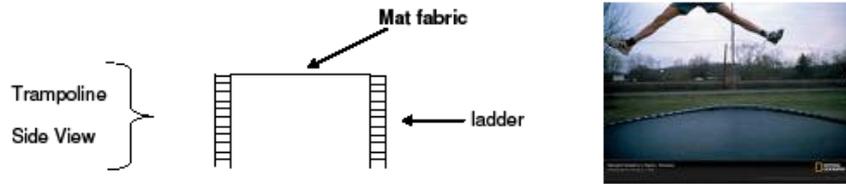
# What transferred for the Invent students?

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- (a) Strategy to look for structure across cases?
  - Students learned to analyze across cases, and transfer problem had multiple cases.
  
- (b) Concept of ratio in physics problems?
  - Students understood value of ratio and looked for its application.
  
- Used the transfer task to find out.

# Strategy?

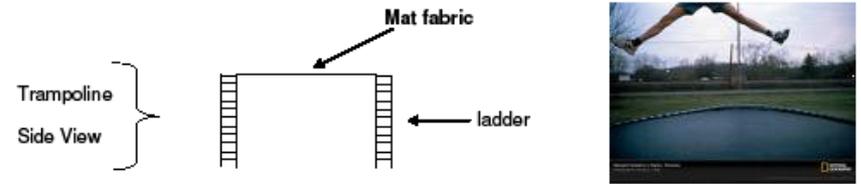
## Four Trampolines



Trampolines are made with mats using different fabrics.

Stiffer mats make the trampoline bouncier.

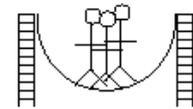
Determine the stiffness of the mat fabric for each trampoline.



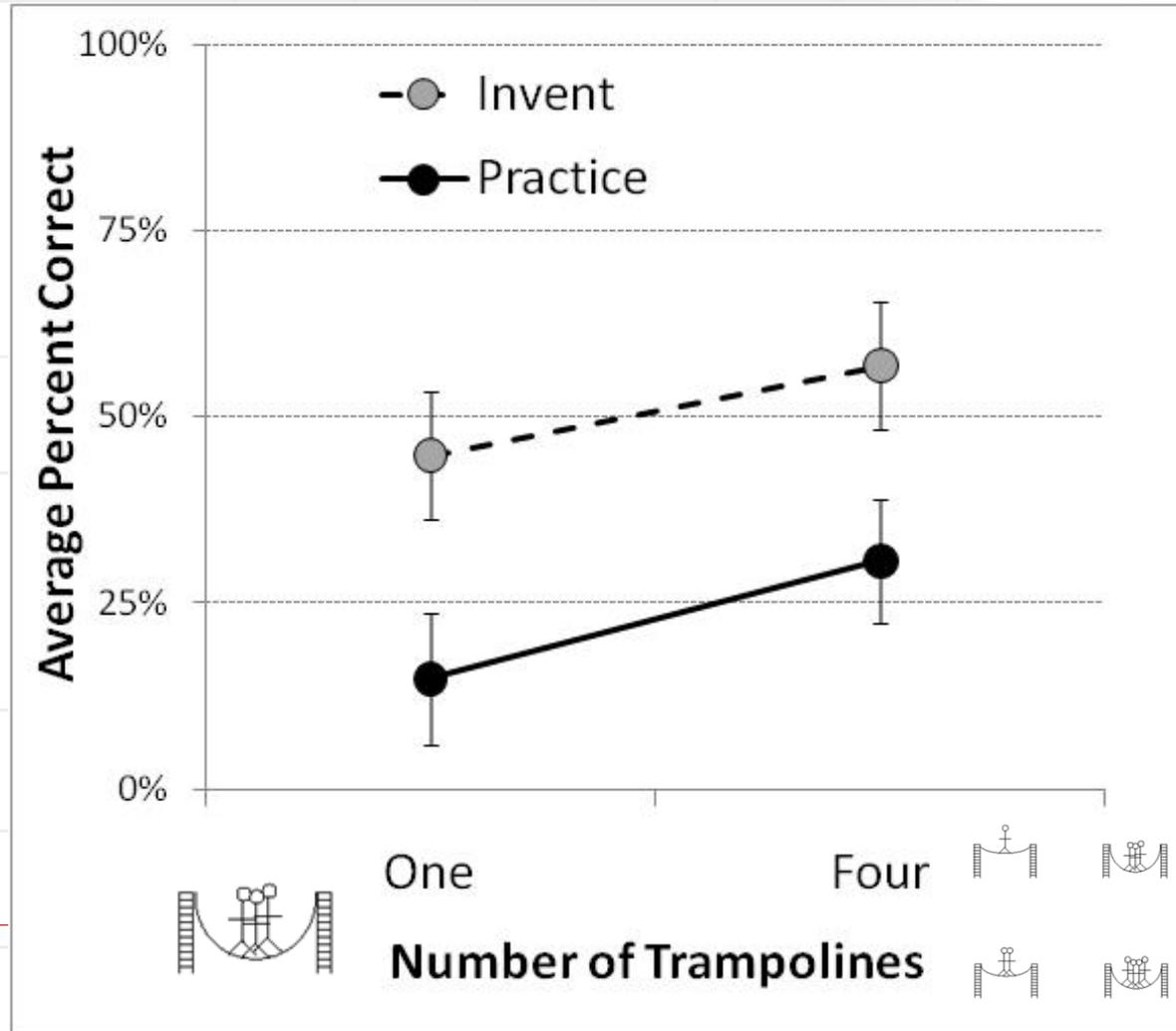
Trampolines are made with mats using different fabrics.

Stiffer mats make the trampoline bouncier.

Determine the stiffness of the mat fabric for each trampoline.



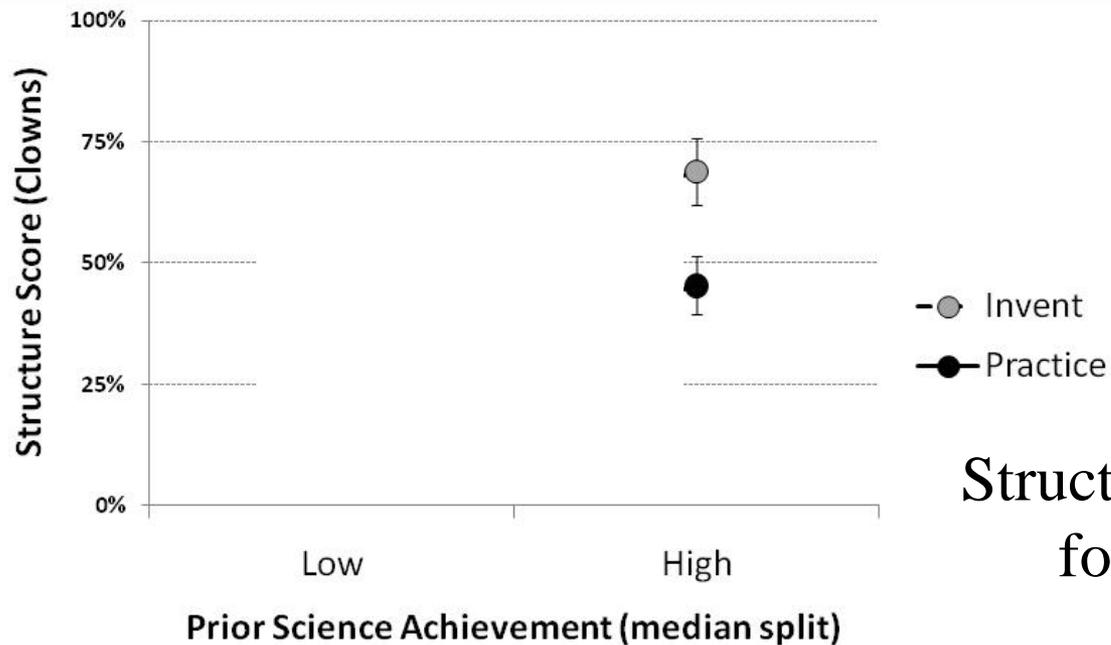
# Appears to be ratio concept.



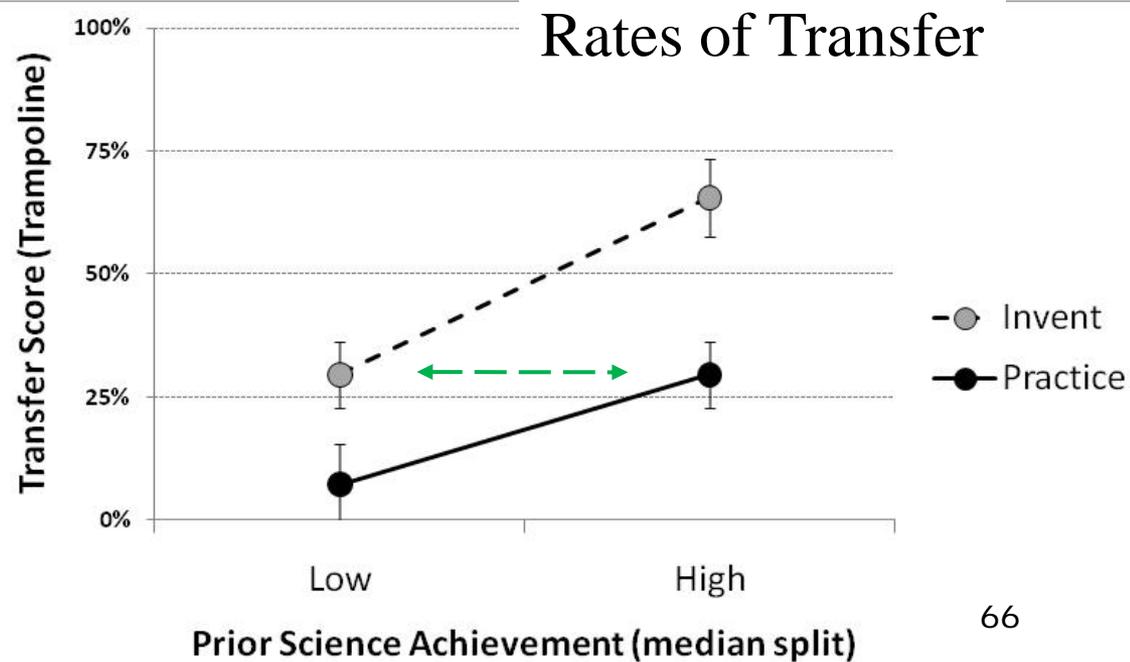
# One last mop-up concern.

---

- A common complaint:
  - Open activities are good for high achievers, but low achievers should get direct instruction.
    - Open activities are too confusing or hard
  
- Compared results for high and low achievers.



## Structure Encoding for Clowns



# Outline

---

- Learning to Perceive
- Missed in Plain Sight
- Why it Matters
- Why Inventing Instruction Works
- □ Summary

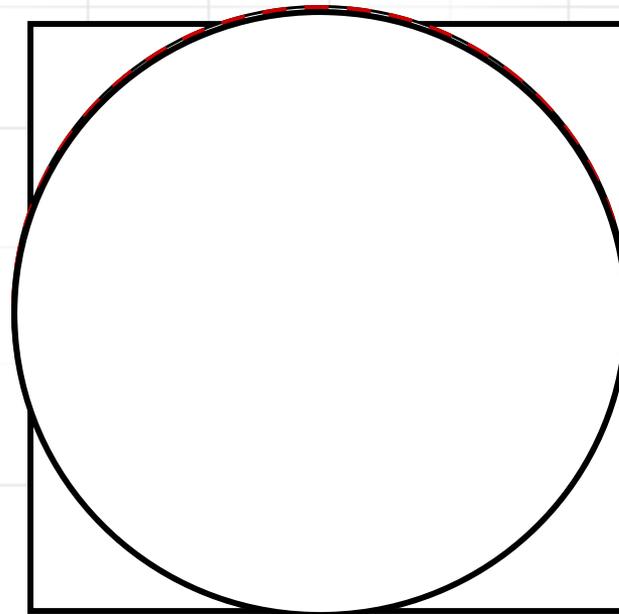
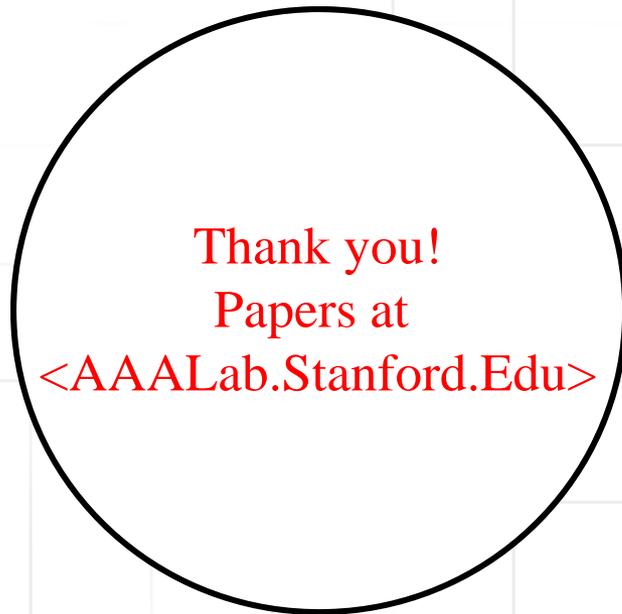
# Summary: a little bit of knowledge is a dangerous thing.

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- Telling students an answer too soon, means they learn just what you told them.
- They may not learn what you are talking about.
- It is useful to have them search for the “invariant” before you tell them.
- Remember, just because you see it, it doesn't mean you perceive what is important...

People always see something, so they often do not know there is more to be had.

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Circle    Biggish    Empty    Solid Black Line    Left Side of Screen